PhD in Design, Modeling and Simulation in Engineering
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# Reduced Order Homogenization for Multiscale Analysis of Nonlinear Composites

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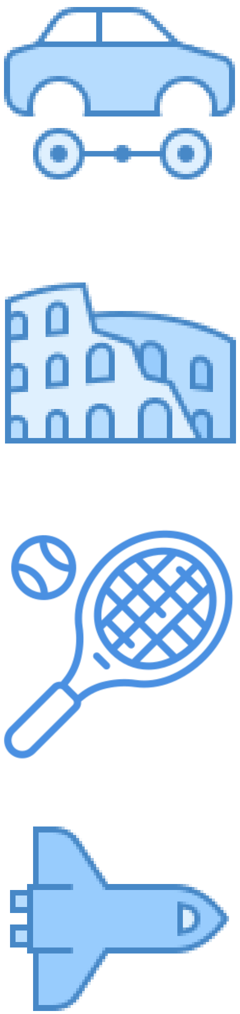
# FRAMEWORK



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## Heterogeneous materials

Usually regarded as composite materials or **Composites,** this class of materials is characterized by a **heterogeneous microstructure** in which two or more constituents are combined in order to reach improved properties.

- Natural and artificial composites
- Large amount of engineering applications
- Constituents with different shapes, dimensions, material properties and many possible different arrangements
- Complex inner geometries

# FRAMEWORK

## PROs and Cons

- **Improved mechanical properties**
- Hot topic in scientific and industrial research
- Increasing usage
- Towards **Metamaterials**

- Increasing performances and safety requirements
- Constituents characterized by **nonlinear behaviour**
- Structural response depending from the inelastic phenomena arising at the microstructure

### Numerical Analysis

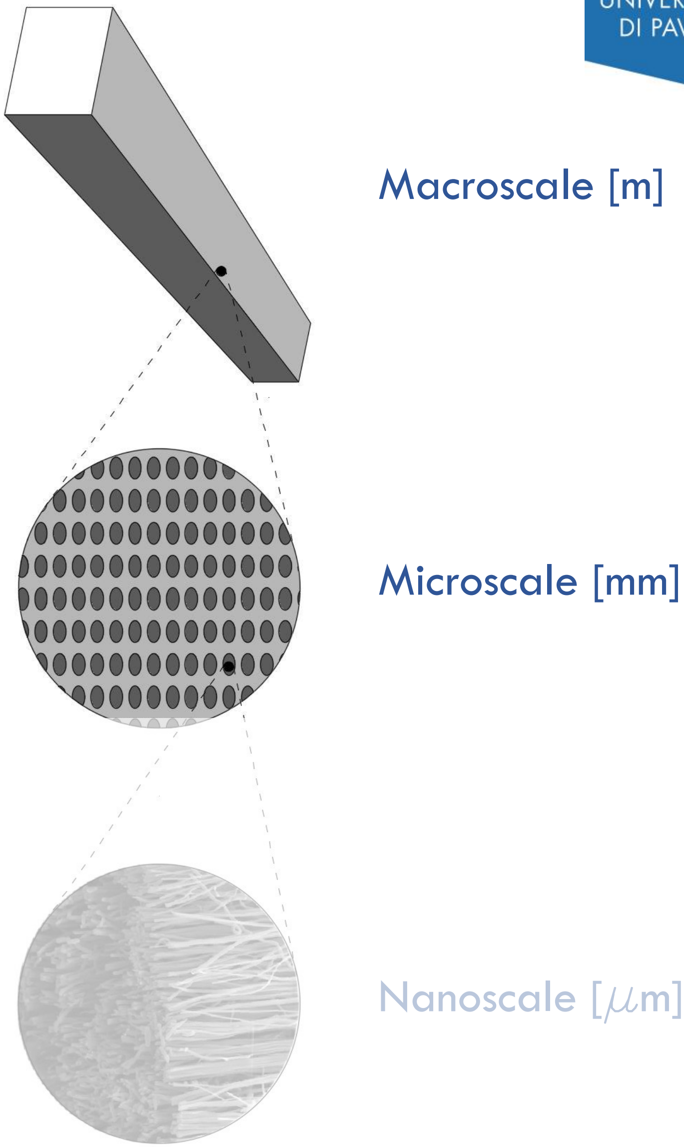
Microscopic structure have to be considered in order to understand how the nonlinearities occurring in the microstructure influence the overall behavior of the composite material.

# MOTIVATION

## Observation Scales

- Structural or Macroscopic scale having characteristic dimension L. Structural elements can be considered as a homogeneous material, mechanical properties at this scale mimic the overall properties of the composite material.

- Microscopic scale or Microscale, having characteristic size l<<<L . The micro-structural constituents and their complex arrangement can be easily identified.



# MOTIVATION

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## Modeling strategies for Nonlinear Composites

**Macroscopic modeling:** heterogeneous structure a fictitious homogeneous continuum.

- stress and strain fields are considered as average fields
- phenomenological approach

☺
- Easily implemented in the framework of FEA (coarse mesh with respect to inhomogeneites dimensions)
- Inexpensive calculations

☹
- Impossible to consider the different constituents
- Inaccurate

**Microscopic modeling:** discontinuities between the different constituents of the heterogeneous material are considered.

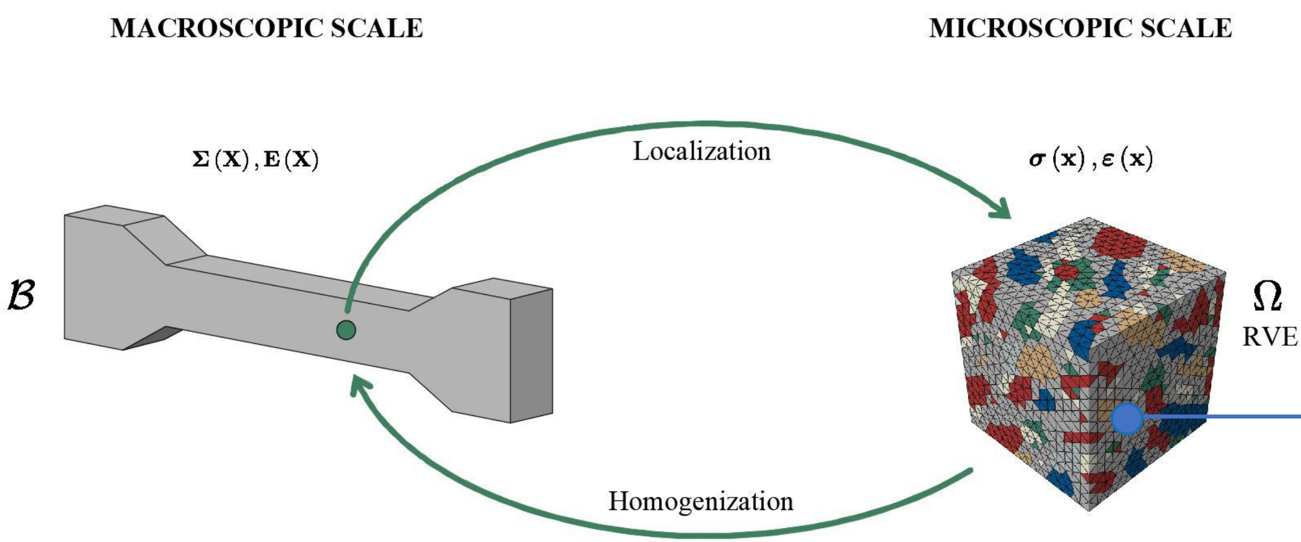
☺
- Captures the local phenomena
- High accuracy

☹
- Very high computational burden (high number of history variables)
- memory and computational time issues

# MOTIVATION

## Modeling strategies for Nonlinear Composites

**Multiscale Analysis:** a modeling approach considering both the microscale and the structural or macroscale, also known as **two-scale technique.**



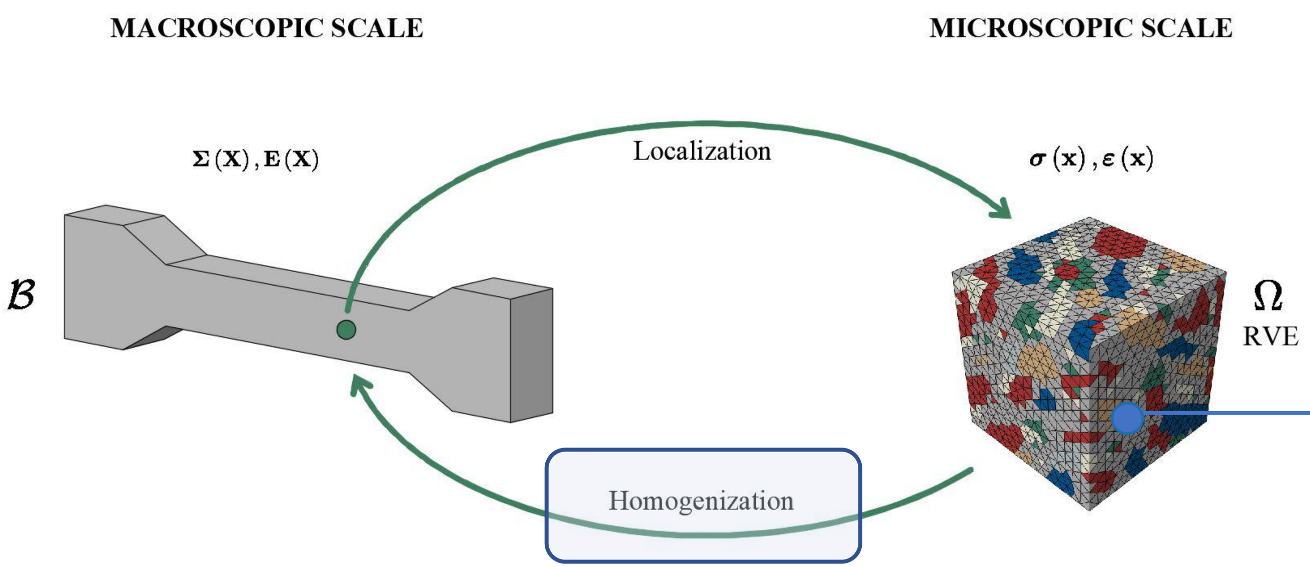

- Based on the Principle of Scales Separations

Nonlinear constituents

# MOTIVATION

## Modeling strategies for Nonlinear Composites

**Multiscale Analysis:** a modeling approach considering both the microscale and the structural or macroscale, also known as **two-scale technique.**




- Based on the Principle of Scales Separations

  Nonlinear constituents

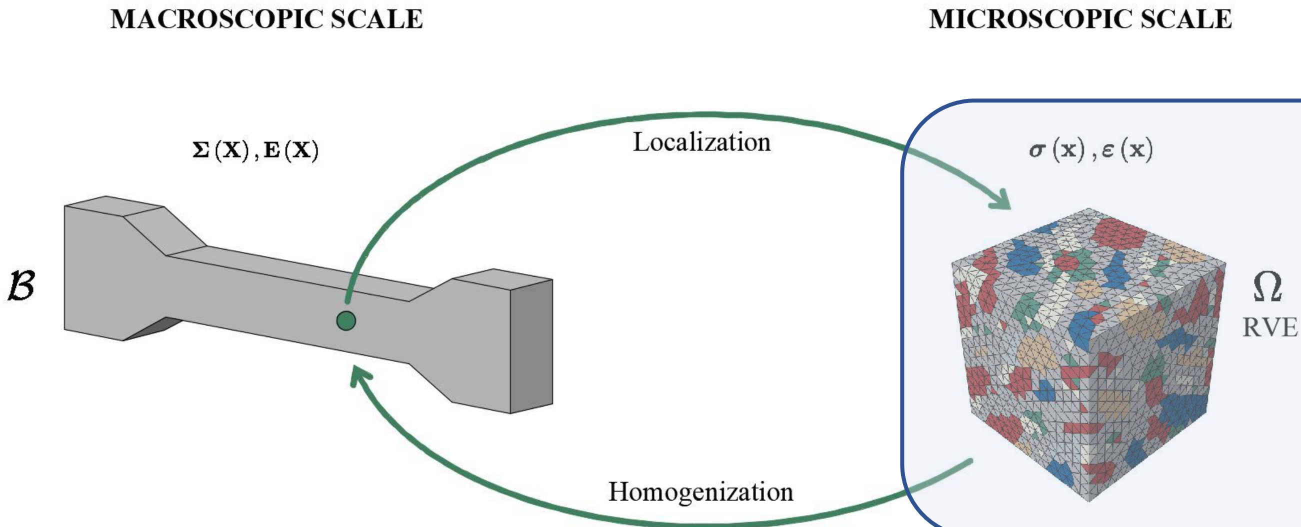
- **Analytical Homogenization Schemes** like the Hashin-Shtrikman (HS) variational principle introducing a reference material: very low number of unknowns, limited accuracy

- **Computational Homogenization Schemes** like the well known $FE^2$: high accuracy but prohibitive computational effort

# MOTIVATION

## Reduced Order Models (ROMs)

- Numerical tool for **reducing the computational burden in computational homogenization** procedures




- **NO nonlinear FEA** required to solve the microstructural BVP on the RVE

- **A well known ROM is the Transformation Field Analysis,** based on the use of eigenstrains in order to consider the inelastic deformation arising from the material nonlinearity

# OBJECTIVES

The main goals are

- The introduction of Reduced Order Models as an efficient numerical tool for lowering the computational cost in Computational Homogenization
- Developing a novel Hashin-Shtrikman based Reduced Order Homogenization Scheme (PWUHS) for studying the micromechanical response of composites having nonlinear constituents
- Implementing the proposed Homogenization Scheme in the framework of Multiscale Analysis to provide a software which is a reasonable compromise between efficiency and numerical accuracy
- The application of the proposed Reduced Order Homogenization for the Multiscale Analysis of 3D-Printed Composites

# OUTLINE

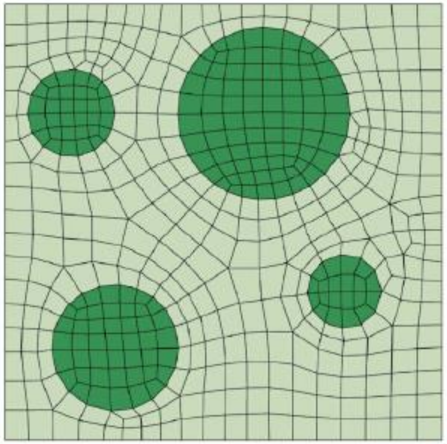
- The PWUHS Reduced Order Model
  - PWUHS Homogenization Scheme
  - Numerical Procedure
  - Numerical applications
  - Remarks
- PWUHS comparison to PWUTFA
  - Equivalence between PWUHS and PWUTFA
  - Numerical applications
  - Convergence study
  - Remarks
- Multiscale Analysis using PWUHS
  - Experimental validation
  - Auxetic composites
  - Implementing the Multiscale Procedure
  - Multiscale Analysis of Auxetic Honeycombs
- Concluding remarks

# PWUHS Homogenization Scheme

## Macroscopic Problem

$$
\begin{aligned}
&\mathbf{E} = \mathbf{B}\,\mathbf{U} && \\
&\mathbf{B}^T\,\boldsymbol{\Sigma} + \mathbf{b} = \mathbf{0} && \text{in} \quad \mathcal{B} \\
&\mathbf{N}\,\boldsymbol{\Sigma} = \mathbf{t} && \text{on} \quad \mathcal{S}_t \\
&\mathbf{U} = \mathbf{U}^* && \text{on} \quad \mathcal{S}_u
\end{aligned}
$$

- $\boldsymbol{\Sigma}$ not directly obtained from $\mathbf{E}$ ⟶ Solve Microscopic BVP

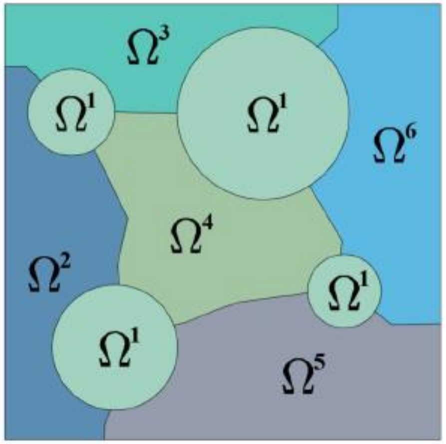


Ω

RVE

## Microscopic Problem

- Average Strain $\boxed{\mathbf{E} = \dfrac{1}{A}\displaystyle\int_\Omega \varepsilon\, dA}$ Scale transition
- Subsets division $\Omega = \displaystyle\bigcup_{j=1}^{m} \Omega^j$ ⟶
- Strain in each subset $\varepsilon^j = \mathbf{E} + \boxed{\hat{\varepsilon}^j}$ Strain fluctuation
- Stress in each subset $\boxed{\boldsymbol{\sigma}^j = \mathbf{C}^j\left(\varepsilon^j - \boldsymbol{\pi}^j\right)}$ Nonlinear material
- Average Stress $\boldsymbol{\Sigma} = \dfrac{1}{A}\displaystyle\int_\Omega \boldsymbol{\sigma}\, dA$



# PWUHS Homogenization Scheme

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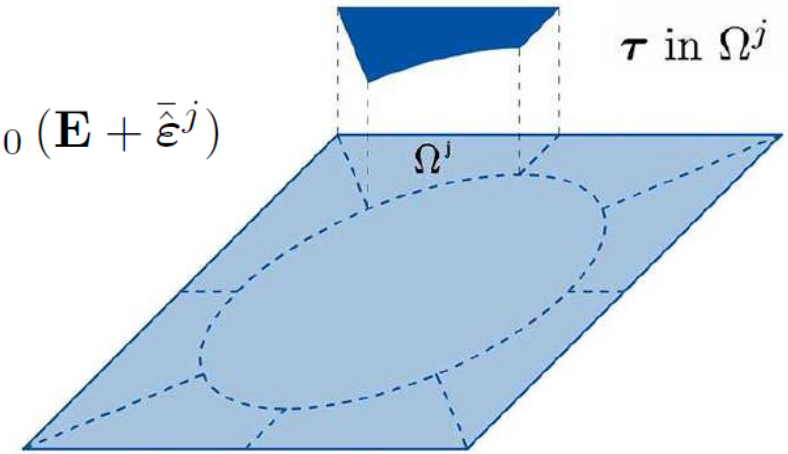
- Introduction of an elastic reference material (Hashin-Shtrikman formulation)
  - Uniform elasticity matrix $\mathbf{C}_0$ of the reference material
  - Coupling via an eigenstress, the **polarization stress** $\boldsymbol{\tau}^j(\boldsymbol{x}) = \boldsymbol{\sigma}^j(\boldsymbol{x}) - \mathbf{C}_0\,\boldsymbol{\varepsilon}^j(\boldsymbol{x})$
  - Piecewise Uniform distribution of the polarization stress (**PWUHS**)

- Constant polarization stress $\boldsymbol{\tau}^j$ in each subset $\Omega^j$ with $j = 1, .., m$.




  - Average polarization stress $\bar{\boldsymbol{\tau}}^j \;=\; \bar{\boldsymbol{\sigma}}^j - \mathbf{C}_0\left(\mathbf{E} + \bar{\hat{\varepsilon}}^j\right)$
  - Polarization stresses $\mathbf{T} \;=\; \left\{\bar{\boldsymbol{\tau}}^1, \ldots, \bar{\boldsymbol{\tau}}^m\right\}^T$
  - Periodic strain fluctuation $\hat{\varepsilon}^j(\boldsymbol{x}) = \boldsymbol{\Gamma}^j(\boldsymbol{x})\mathbf{T}$
  - Average strain fluctuation $\bar{\hat{\varepsilon}}^j = \bar{\boldsymbol{\Gamma}}^j\mathbf{T}$

# PWUHS Homogenization Scheme

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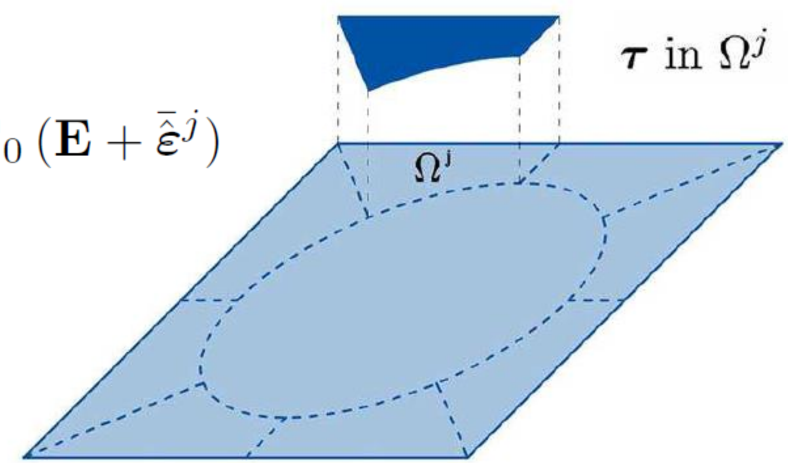
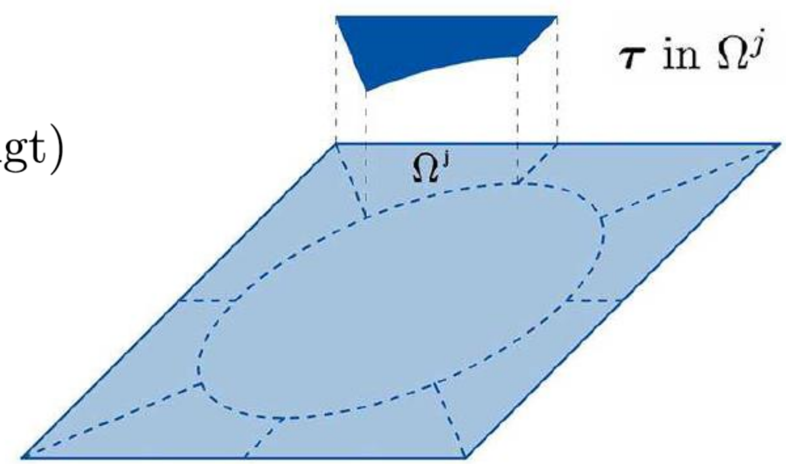



Average Localization Matrices

# PWUHS Homogenization Scheme

## Precomputations on the elastic reference material

- Construction of $m$ average Localization Matrices $\bar{\mathbf{\Gamma}}^j$
  - 3x $m$ micromechanical elastic analyses (FEAP)
  - a unit value is assigned to only one of the 3x $m$ polarization stress components in $\mathbf{T}$
- Choice of the reference elasticity matrix $\mathbf{C}_0 = \bar{\mathbf{C}}$
  - elastic matrix of the composite $\bar{\mathbf{C}}$
  - Voigt Homogenization theory $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{Voigt})$
  - FE homogenization $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{FE})$



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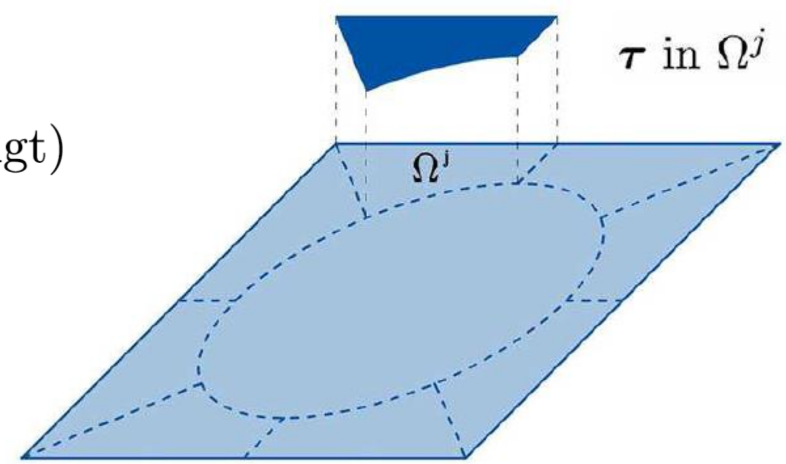
  - Voigt Homogenization theory $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{Voigt})$
  - FE homogenization $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{FE})$

3 additional precomputations assigning a unit value to $\mathbf{E}$ and averaging the stresses



# PWUHS Homogenization Scheme

## Updated secant modulus approach

- Correction of the elasticity matrix

$$\mathbf{C}_0 = \mu_0 \begin{bmatrix} \frac{1-\nu_0}{1-2\nu_0} & \frac{\nu_0}{1-2\nu_0} & 0 \\ \frac{\nu_0}{1-2\nu_0} & \frac{1-\nu_0}{1-2\nu_0} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}.$$

  - Updated secant shear modulus $\mu_0^t = \frac{1}{2}\frac{\|\boldsymbol{\Sigma}'\|}{\|\mathbf{E}'\|}.$
  - Scaling factor $f_0 = \frac{\mu_0^t}{\mu_0}.$
  - Correction of the average reference elastic matrix $\mathbf{C}_0^t = f_0 \mathbf{C}_0$
  - Correction of the average localization matrix $\bar{\boldsymbol{\Gamma}}^t = \frac{1}{f_0}\bar{\boldsymbol{\Gamma}}.$

# PWUHS numerical procedure

**Algorithm 1** PWUHS Homogenization Scheme

- **Offline stage (Precomputations):**

  Perform $3 \times m$ elastic analyses, get localization tensors $\bar{\mathbf{\Gamma}}^j$

- **Online stage at the typical time step $t$:**

1: Assign $\mathbf{E}$
2: With the history variables $\mathbf{\Pi}_n$ and $\boldsymbol{\alpha}_n$ at $t_n$, a trial state is evaluated in all the subsets (prediction)
3: **if** $f^j \leq 0$ for $j = 1, ..., m$ **then**
4: `exit` (elastic step)
5: **else**
6: Get residual $\mathbf{R}$ (correction)
7: **if** $|\mathbf{R}| > tol$ **then** (Newton loop)
8: Solve the linearized problem
9: Update the unknowns $\mathbf{S}$
go to line 6 for next iteration
10: **else**
11: store $\bar{\boldsymbol{\sigma}}^j$ and the history variable $\mathbf{\Pi}$ and $\boldsymbol{\alpha}$
12: **end if**
13: update $\mathbf{C}_0$ and $\bar{\mathbf{\Gamma}}^j$ via the secant modulus approach (optional)
14: **end if**

## Plasticity with isotropic hardening

- Average stress in $\Omega^j$

$$\bar{\boldsymbol{\sigma}}^j = \mathbf{C}^j \left( \mathbf{E} + \bar{\hat{\varepsilon}} - \boldsymbol{\pi}^j \right)$$

- Activation function

$$f = q - \sigma_y - K\alpha.$$

- Accumulated plastic strain

$$\alpha = \int_0^t \|\dot{\pi}\| dt.$$

- Evolution of $\pi$, $\alpha$

$$\dot{\pi} = \dot{\gamma}\frac{\partial f}{\partial \sigma}, \qquad \dot{\alpha} = \dot{\gamma}$$

# PWUHS numerical procedure

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- **Online stage at the typical time step $t$:**

Backward Euler implicit scheme time integration

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Predictor-Corrector approach

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# PWUHS numerical procedure

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Trial state

$$\mathbf{\Pi} = \mathbf{\Pi}_n, \quad \boldsymbol{\alpha} = \boldsymbol{\alpha}_n$$

$$\begin{aligned}
\bar{\boldsymbol{\tau}}^j &= \bar{\boldsymbol{\sigma}}^j - \mathbf{C}_0 \left(\mathbf{E} + \bar{\tilde{\varepsilon}}^j\right) \\
\bar{\tilde{\varepsilon}}^j &= \bar{\mathbf{\Gamma}}^j \mathbf{T} \\
\bar{\boldsymbol{\sigma}}^j &= \mathbf{C}^j \left(\mathbf{E} + \bar{\tilde{\varepsilon}} - \boldsymbol{\pi}^j\right) \\
f^j &= q - \sigma_y - K\alpha^j \\
q^j &= \sqrt{\frac{3}{2}\bar{\boldsymbol{\sigma}}^T \mathbf{M} \bar{\boldsymbol{\sigma}}}
\end{aligned}$$

# PWUHS numerical procedure

**Algorithm 1** PWUHS Homogenization Scheme

- **Offline stage (Precomputations):**

Perform $3 \times m$ elastic analyses, get localization tensors $\bar{\boldsymbol{\Gamma}}^j$

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4: `exit` (elastic step)

Elastic Step

5: **else**

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# PWUHS numerical procedure

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Correction

System of 13xm nonlinear equations solved via Newton Method

# PWUHS numerical procedure

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- **Online stage at the typical time step $t$:**

```
1:  Assign E
2:  With the history variables Π_n and α_n at t_n, a trial state is
        evaluated in all the subsets                          (prediction)
3:  if f^j ≤ 0 for j = 1, ..., m then
4:      exit                                                  (elastic step)
5:  else
6:      Get residual R                                        (correction)
7:      if |R| > tol then                                     (Newton loop)
8:          Solve the linearized problem
9:          Update the unknowns S
            go to line 6 for next iteration
10:     else
11:         store σ̄^j and the history variable Π and α
12:     end if
13: update C_0 and Γ̄^j via the secant modulus approach       (optional)
14: end if
```

Variables storing at convergence

# PWUHS numerical procedure

**Algorithm 1** PWUHS Homogenization Scheme

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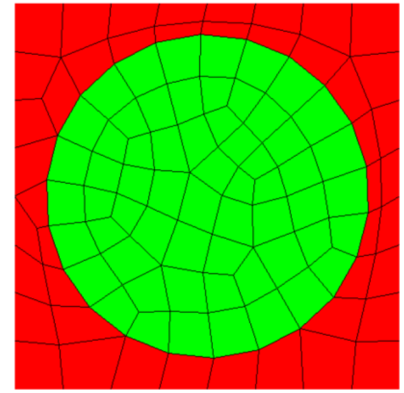
Reference and localization matrix updates

# PWUHS numerical applications

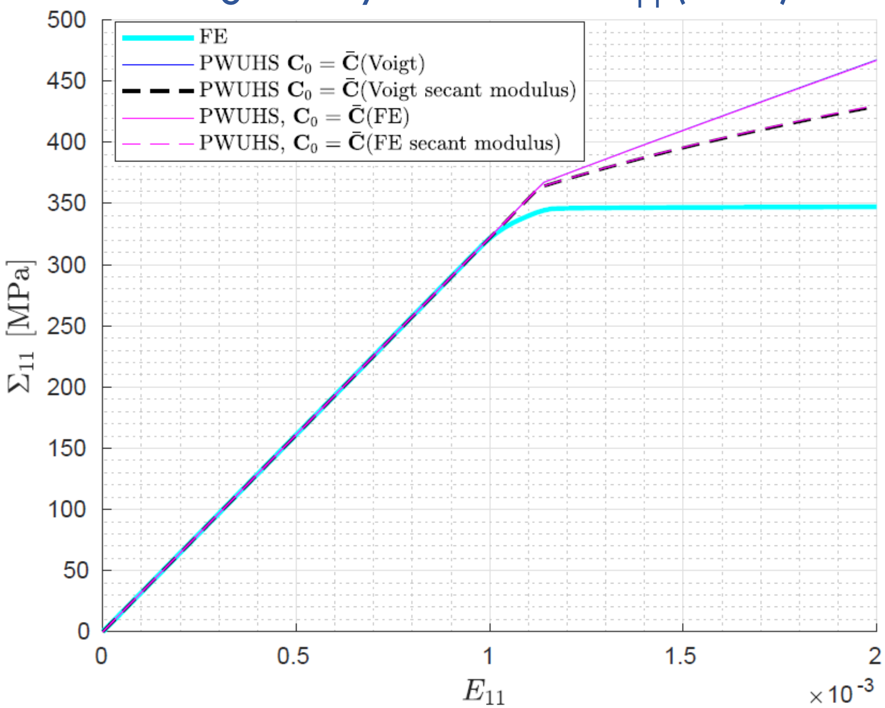
## Ceramic inclusion in a metal matrix

| $E^1$ [GPa] | $\nu^1$ | $k$ [MPa] | $\sigma_y$ [MPa] | $E^2$ [GPa] | $\nu^2$ |
|---|---|---|---|---|---|
| 210 | 0.3 | 100 | 300 | 300 | 0.25 |

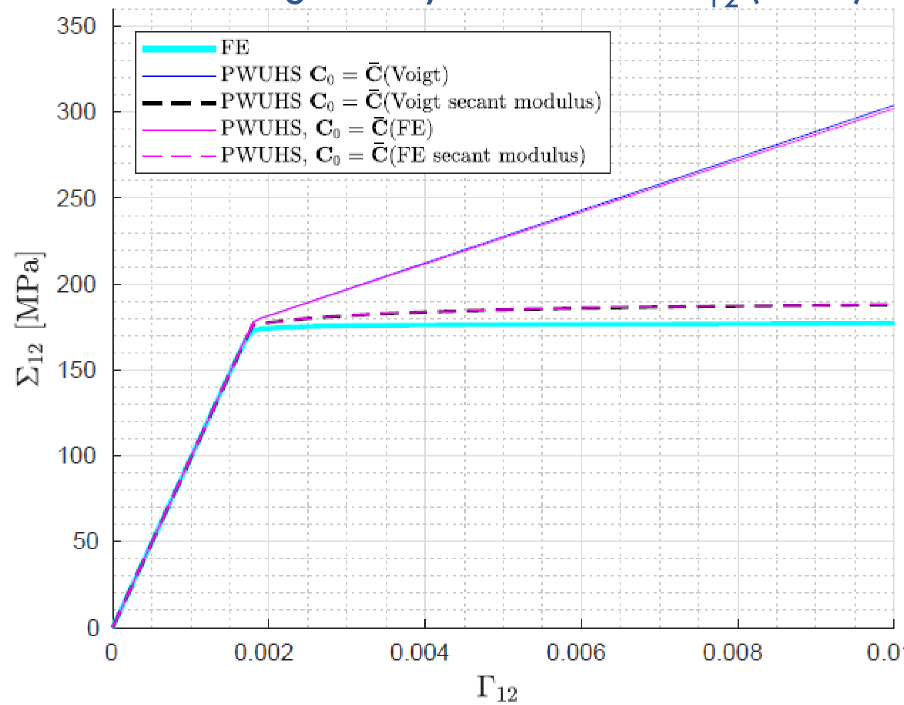


- Single inclusion UC (UC-1), 10x10mm, $c^2 = 0.54$
- Number of history variables: FE = 420, PWUHS = 8

- Loading history in terms of $E_{11}$ (LC-1)




- Loading history in terms of $\Gamma_{12}$ (LC-2)



# PWUHS numerical applications

## Ceramic inclusion in a metal matrix

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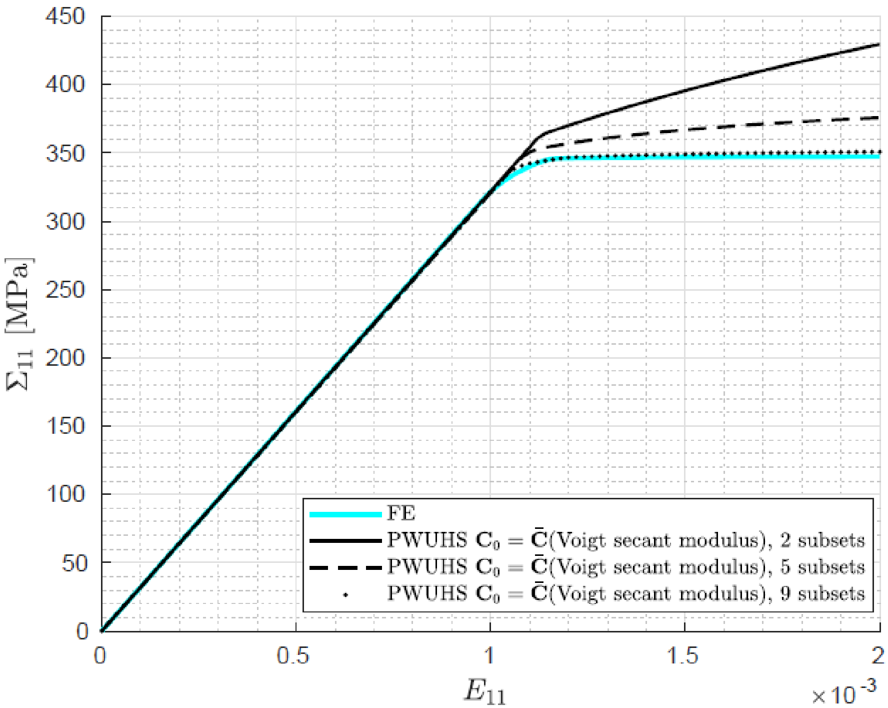
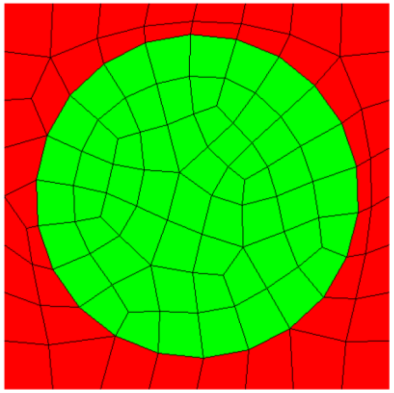
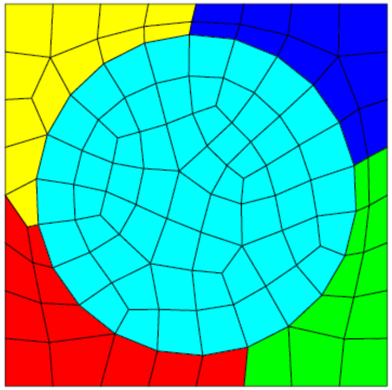
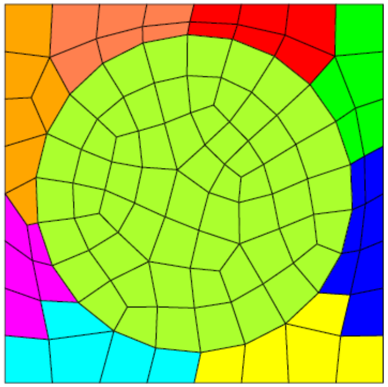
- ❑ Increasing number of subsets (2, 5, 9) in UC-1
- ❑ 8, 20, 36 history variables

- LC-1







# PWUHS numerical applications

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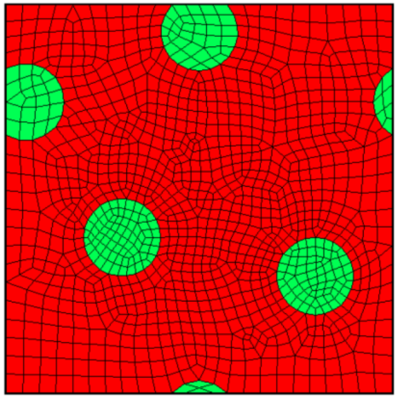
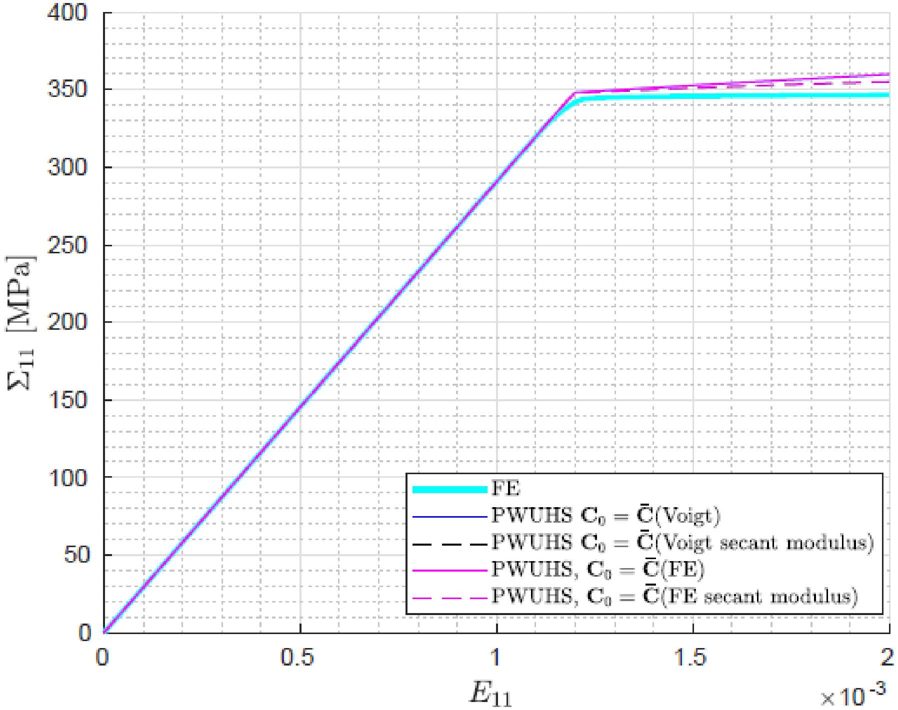
|---|---|---|---|---|---|
| 210 | 0.3 | 100 | 300 | 300 | 0.25 |

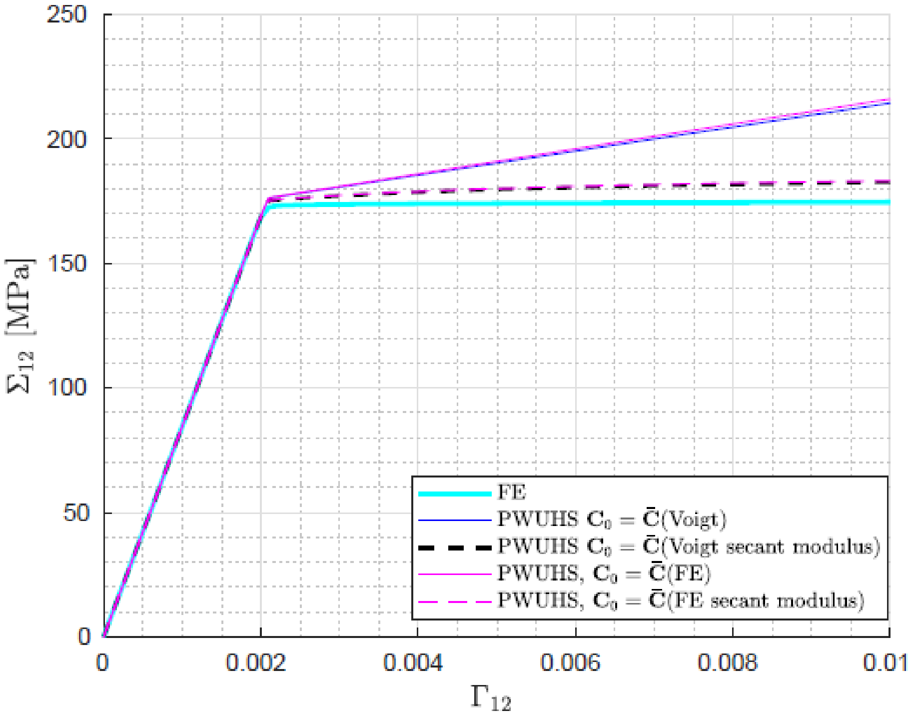


- Complex UC, $c^2 = 0.12$ (UC-2)
- Number of history variables: FE = 4856, PWUHS = 8

- LC-1



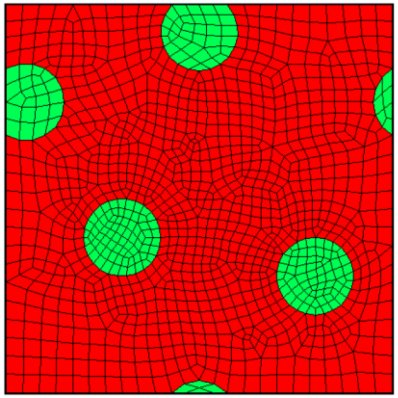

- LC-2



# PWUHS numerical applications

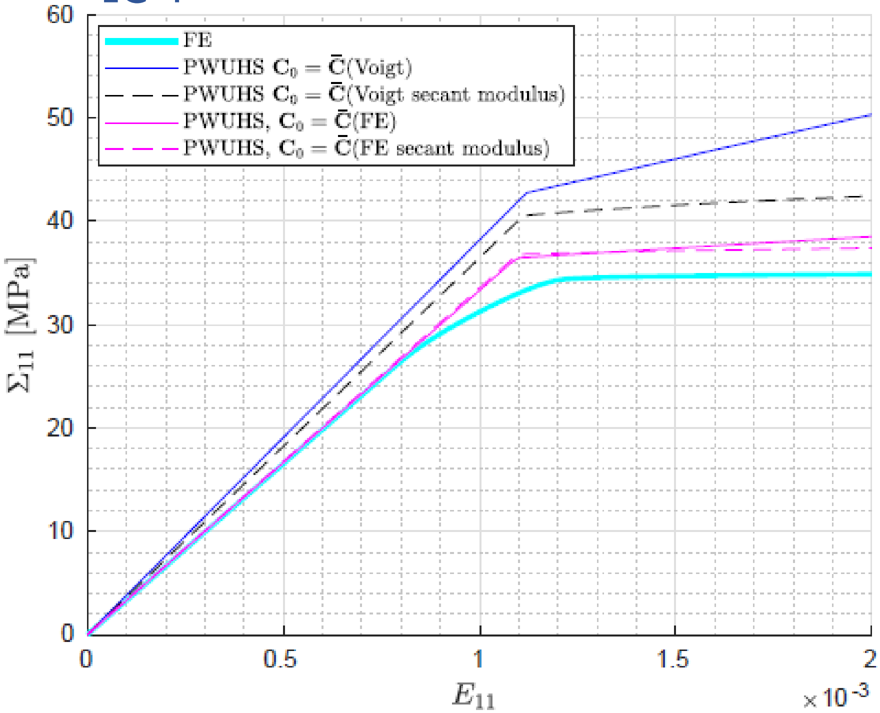
## Fiber reinforced epoxy resin

| $E^1$ [GPa] | $\nu^1$ | $k$ [MPa] | $\sigma_y$ [MPa] | $E^2$ [GPa] | $\nu^2$ |
|---|---|---|---|---|---|
| 21 | 0.3 | 100 | 30 | 210 | 0.25 |

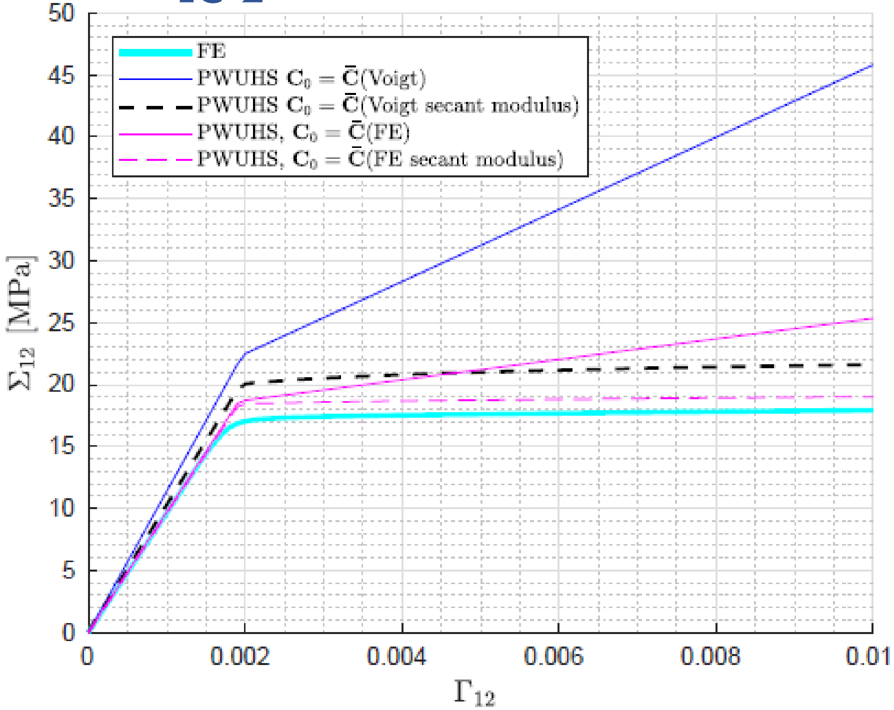


- UC-2, $c^2 = 0.12$
- Number of history variables: FE = 4856, PWUHS = 8

### LC-1

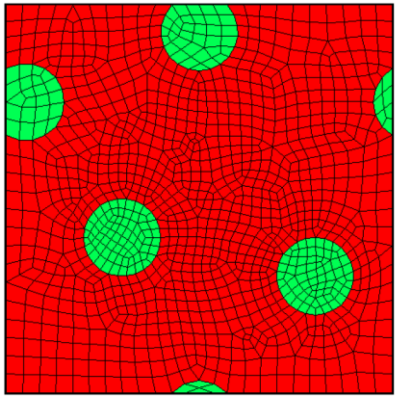



### LC-2



# PWUHS numerical applications

## Fiber reinforced epoxy resin



- History Variables (elastoplasticity with isotropic hardening)
  - FE = 4856
  - PWUHS = 8

| Load Case | $\mathbf{C}_0$ | CPU time FE [s] | PWUHS speed-up |
|---|---|---|---|
| LC-1 | $\bar{\mathbf{C}}$ (Voigt) | 711.07 | 4182.76 |
| | $\bar{\mathbf{C}}$ (FE) | | 582.84 |
| | $\bar{\mathbf{C}}$ (Voigt), secant | | 1341.64 |
| | $\bar{\mathbf{C}}$ (FE), secant | | 470.90 |
| LC-2 | $\bar{\mathbf{C}}$ (Voigt) | 855.31 | 4072.90 |
| | $\bar{\mathbf{C}}$ (FE) | | 562.70 |
| | $\bar{\mathbf{C}}$ (Voigt), secant | | 425.52 |
| | $\bar{\mathbf{C}}$ (FE), secant | | 409.23 |

# PWUHS

## REMARKS

- PWUHS is an efficient numerical tool for the analysis of composites, results are in agreement with the nonlinear FE analyses
- A remarkable reduction of the number of history variables and computational effort, with respect to FE analysis, is achieved
- Deriving the overall elasticity matrix by FE homogenization $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{FE})$ increases the method accuracy
- The **updated secant modulus approach** increases the method accuracy slightly affecting the computational efficiency

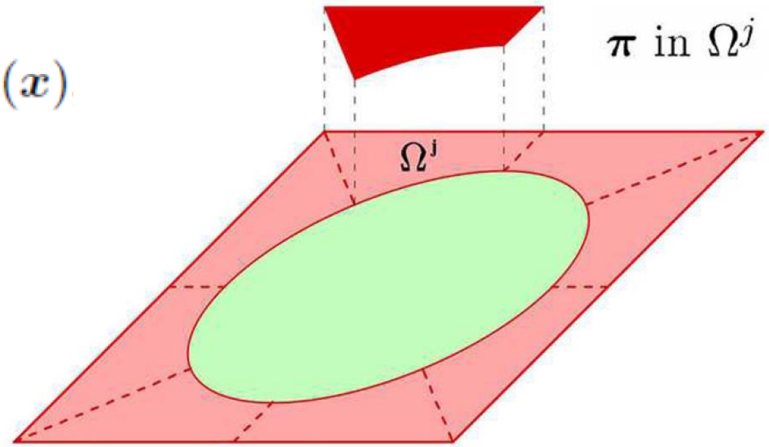
# OUTLINE

- The PWUHS Reduced Order Model
  - Homogenization of Nonlinear Composites
  - Numerical Procedure
  - Numerical applications
  - Remarks
- PWUHS comparison to PWUTFA
  - Equivalence between PWUHS and PWUTFA
  - Numerical applications
  - Convergence study
  - Remarks
- Multiscale Analysis using PWUHS
  - Auxetic composites
  - Implementing the Multiscale Procedure
  - Multiscale Analysis of Auxetic Honeycombs
  - Experimental validation
- Concluding remarks

# Transformation Field Analysis

- Eigenstrain based Reduced Order Model
  - Uniform inelastic strain distribution (UTFA)
  - Piecewise Uniform Inelastic Strain distribution (**PWUTFA**)
  - Nonuniform inelastic strain distribution (NUTFA)

- Constant inelastic strain $\boldsymbol{\pi}^j$ in each subset $\Omega^j$ with $j = 1, .., m$.
  - Periodic strain fluctuation $\hat{\boldsymbol{\varepsilon}}^j(\boldsymbol{x}) = \boldsymbol{e}^j(\boldsymbol{x}) + \boldsymbol{p}^j(\boldsymbol{x})$
  - Macro strain localization $\boldsymbol{e}^j(\boldsymbol{x}) = \mathbf{L}_{\mathbf{E}}^j(\boldsymbol{x})\mathbf{E}$
  - Inelastic strain $\boldsymbol{\Pi} = \{\boldsymbol{\pi}^1, ..., \boldsymbol{\pi}^m\}^T$
  - Inelastic strain localization $\boldsymbol{p}^j(\boldsymbol{x}) = \mathbf{L}_{\pi}^j(\boldsymbol{x})\boldsymbol{\Pi}$
  - Average strain fluctuation $\bar{\hat{\boldsymbol{\varepsilon}}}^j = \boxed{\bar{\mathbf{L}}_{\mathbf{E}}^j}\mathbf{E} + \boxed{\bar{\mathbf{L}}_{\pi}^j}\boldsymbol{\Pi}$




Average Localization Matrices

# PWUTFA Reduced Order Model

## Precomputations on the real composite

- Construction of $m$ average Localization Matrices $\bar{\mathbf{L}}_{\mathbf{E}}^{j}$

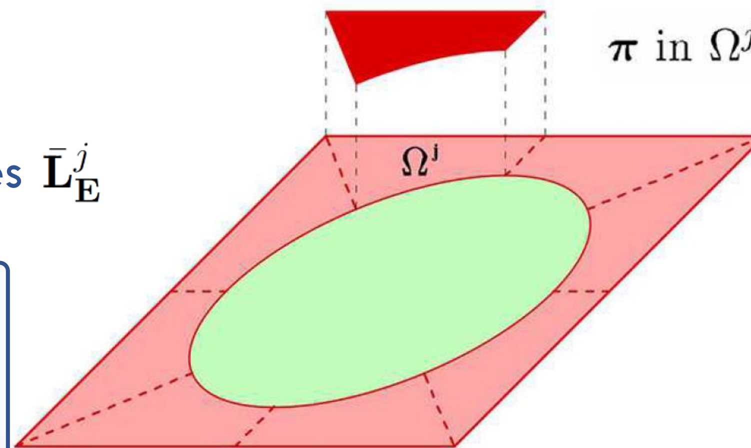
  - 3 micromechanical elastic analyses
  - unit value is assigned to one of the three macrostrain $\mathbf{E}$ components

  NOT REQUIRED IN PWUHS HOMOGENIZATION

- Construction of $m$ average Localization Matrices $\bar{\mathbf{L}}_{\mathbf{E}}^{j}$

  - 3x $m$ micromechanical nonlinear analyses
  - a unit value is assigned to only one of the 3x $m$ inelastic strain components in $\boldsymbol{\Pi}$

  ELASTIC IN PWUHS HOMOGENIZATION



# PWUTFA numerical procedure

## Backward Euler implicit scheme time integration

- Predictor-corrector approach

- History variables

$$\mathbf{\Pi} = \mathbf{\Pi}_n, \quad \boldsymbol{\alpha} = \boldsymbol{\alpha}_n$$

- Trial state

$$\bar{\hat{\varepsilon}}^j = \bar{\mathbf{L}}^j_{\mathbf{E}}\mathbf{E} + \bar{\mathbf{L}}^j_{\pi}\mathbf{\Pi}$$

$$\bar{\boldsymbol{\sigma}}^j = \mathbf{C}^j\left(\mathbf{E} + \bar{\hat{\varepsilon}} - \boldsymbol{\pi}^j\right)$$

$$f^j = \sqrt{\frac{3}{2}}\|\bar{\boldsymbol{\sigma}}'^j\| - \sigma_y - K\alpha^j$$

IF $f^j \leq 0$ in all the subsets $\Omega^j$ → Elastic Step

else → Correction via Newton-Raphson method

### Plasticity with isotropic hardening

- Average stress in each subset

$$\bar{\boldsymbol{\sigma}}^j = \mathbf{C}^j\left(\mathbf{E} + \bar{\hat{\varepsilon}} - \boldsymbol{\pi}^j\right)$$

- Activation function

$$f = q - \sigma_y - K\alpha,$$

# PWUHS comparison to PWUTFA

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## Equivalence between PWUHS and PWUTFA

- Composite made by two materials is divided in two subsets $\Omega^1$ and $\Omega^2$
- Assuming that:
  - **Material 1** in subset $\Omega^1$ is **elastic,** $\pi^1 = 0$;
  - **Material 2** in subset $\Omega^2$ is **elastoplastic,** $\pi^2 \neq 0$;
  - Same elastic properties $E^1 = E^2, \quad \nu^1 = \nu^2 \longrightarrow \mathbf{C}^1 = \mathbf{C}^2 = \mathbf{C}$

**PWUHS**

$$\mathbf{C}^1 = \mathbf{C}^2 = \mathbf{C} = \mathbf{C}_0$$

$$\boxed{\begin{aligned} \bar{\tau}^1 &= 0, \\ \bar{\tau}^2 &= -\mathbf{C}\pi^2 \end{aligned}} \longrightarrow \mathbf{T} = \{0, \bar{\tau}^2\}^T,$$

$$\bar{\hat{\varepsilon}}^1 = \bar{\mathbf{\Gamma}}^1\mathbf{T}$$

$$\bar{\hat{\varepsilon}}^2 = \bar{\mathbf{\Gamma}}^2\mathbf{T}$$

$$\bar{\sigma}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{\Gamma}}^1\mathbf{T}\right),$$

$$\bar{\sigma}^2 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{\Gamma}}^2\mathbf{T} - \pi^2\right)$$

**PWUTFA**

$$\bar{\hat{\varepsilon}}^1 = \bar{\mathbf{L}}^1_\pi\mathbf{\Pi}$$

$$\bar{\hat{\varepsilon}}^2 = \bar{\mathbf{L}}^2_\pi\mathbf{\Pi}$$

$$\bar{\sigma}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{L}}^1_\pi\mathbf{\Pi}\right)$$

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**PWUHS**

$$\mathbf{C}^1 = \mathbf{C}^2 = \mathbf{C} = \mathbf{C}_0$$

$$\boxed{\begin{aligned} \bar{\boldsymbol{\tau}}^1 &= \mathbf{0}, \\ \bar{\boldsymbol{\tau}}^2 &= -\,\mathbf{C}\boldsymbol{\pi}^2 \\ \bar{\hat{\boldsymbol{\varepsilon}}}^1 &= \bar{\boldsymbol{\Gamma}}^1\mathbf{T} \\ \bar{\hat{\boldsymbol{\varepsilon}}}^2 &= \bar{\boldsymbol{\Gamma}}^2\mathbf{T} \end{aligned}} \longrightarrow \boxed{\mathbf{T} = \{\mathbf{0}, \bar{\boldsymbol{\tau}}^2\}^T}$$

$$\bar{\hat{\boldsymbol{\varepsilon}}}^1 = \bar{\boldsymbol{\Gamma}}^1\mathbf{T} = -\bar{\boldsymbol{\Gamma}}^1\mathbb{C}\boldsymbol{\Pi}$$

$$\bar{\hat{\boldsymbol{\varepsilon}}}^2 = \bar{\boldsymbol{\Gamma}}^2\mathbf{T} = -\bar{\boldsymbol{\Gamma}}^2\mathbb{C}\boldsymbol{\Pi}$$

$$\bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\boldsymbol{\Gamma}}^1\mathbf{T}\right),$$

$$\bar{\boldsymbol{\sigma}}^2 = \mathbf{C}\left(\mathbf{E} + \bar{\boldsymbol{\Gamma}}^2\mathbf{T} - \boldsymbol{\pi}^2\right)$$

**PWUTFA**

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$$\bar{\hat{\boldsymbol{\varepsilon}}}^2 = \bar{\mathbf{L}}^2_\pi\boldsymbol{\Pi}$$

$$\bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{L}}^1_\pi\boldsymbol{\Pi}\right)$$

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$$\bar{\hat{\varepsilon}}^1 = \bar{\mathbf{\Gamma}}^1\mathbf{T}$$

$$\bar{\hat{\varepsilon}}^2 = \bar{\mathbf{\Gamma}}^2\mathbf{T}$$

$$\bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{\Gamma}}^1\mathbf{T}\right),$$

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$$\longrightarrow \boxed{\begin{aligned} \bar{\hat{\varepsilon}}^1 &= \bar{\mathbf{\Gamma}}^1\mathbf{T} = -\bar{\mathbf{\Gamma}}^1\mathbb{C}\mathbf{\Pi} \\ \bar{\hat{\varepsilon}}^2 &= \bar{\mathbf{\Gamma}}^2\mathbf{T} = -\bar{\mathbf{\Gamma}}^2\mathbb{C}\mathbf{\Pi} \end{aligned}} \longrightarrow \begin{aligned} \bar{\mathbf{L}}_\pi^1 &= -\bar{\mathbf{\Gamma}}^1\mathbb{C} \\ \bar{\mathbf{L}}_\pi^2 &= -\bar{\mathbf{\Gamma}}^2\mathbb{C} \end{aligned} \longleftarrow$$

**PWUTFA**

$$\boxed{\begin{aligned} \bar{\hat{\varepsilon}}^1 &= \bar{\mathbf{L}}_\pi^1\mathbf{\Pi} \\ \bar{\hat{\varepsilon}}^2 &= \bar{\mathbf{L}}_\pi^2\mathbf{\Pi} \end{aligned}}$$

$$\bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{L}}_\pi^1\mathbf{\Pi}\right)$$

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UNIVERSITÀ DI PAVIA

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PWUHS

PWUTFA

$$\mathbf{C}^1 = \mathbf{C}^2 = \mathbf{C} = \mathbf{C}_0$$

$$\bar{\boldsymbol{\tau}}^1 = \mathbf{0}, \qquad \mathbf{T} = \{\mathbf{0}, \bar{\boldsymbol{\tau}}^2\}^T,$$

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$$\bar{\hat{\varepsilon}}^1 = \bar{\mathbf{\Gamma}}^1\mathbf{T} \rightarrow \bar{\hat{\varepsilon}}^1 = \bar{\mathbf{\Gamma}}^1\mathbf{T} = -\bar{\mathbf{\Gamma}}^1\mathbb{C}\mathbf{\Pi} \rightarrow \boxed{\bar{\mathbf{L}}_\pi^1 = -\bar{\mathbf{\Gamma}}^1\mathbb{C}} \leftarrow \bar{\hat{\varepsilon}}^1 = \bar{\mathbf{L}}_\pi^1\mathbf{\Pi}$$

$$\bar{\hat{\varepsilon}}^2 = \bar{\mathbf{\Gamma}}^2\mathbf{T} \qquad \bar{\hat{\varepsilon}}^2 = \bar{\mathbf{\Gamma}}^2\mathbf{T} = -\bar{\mathbf{\Gamma}}^2\mathbb{C}\mathbf{\Pi} \qquad \boxed{\bar{\mathbf{L}}_\pi^2 = -\bar{\mathbf{\Gamma}}^2\mathbb{C}} \qquad \bar{\hat{\varepsilon}}^2 = \bar{\mathbf{L}}_\pi^2\mathbf{\Pi}$$

$$\bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{\Gamma}}^1\mathbf{T}\right), \longleftrightarrow \bar{\boldsymbol{\sigma}}^1 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{L}}_\pi^1\mathbf{\Pi}\right)$$

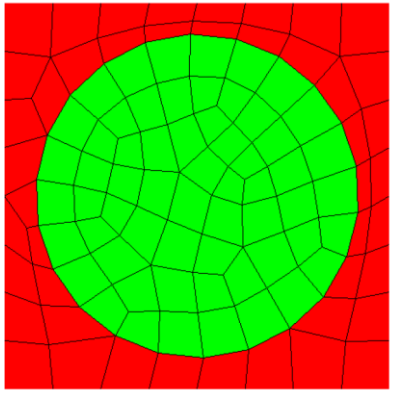
$$\bar{\boldsymbol{\sigma}}^2 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{\Gamma}}^2\mathbf{T} - \boldsymbol{\pi}^2\right) \longleftrightarrow \bar{\boldsymbol{\sigma}}^2 = \mathbf{C}\left(\mathbf{E} + \bar{\mathbf{L}}_\pi^2\mathbf{\Pi} - \boldsymbol{\pi}^2\right)$$

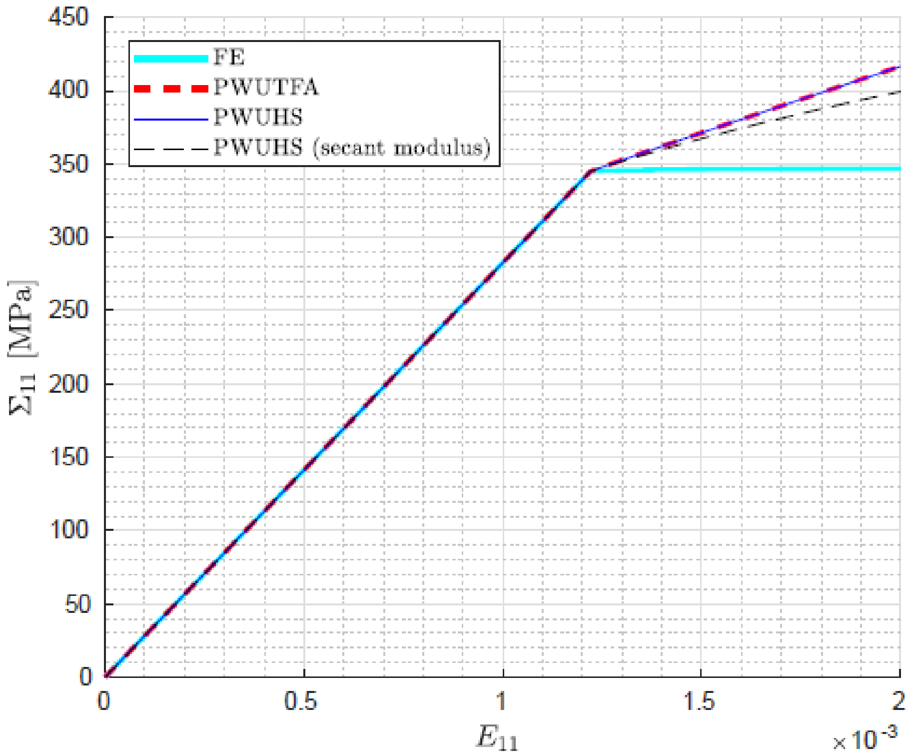
$$\mathbf{\Sigma} = c^1\bar{\boldsymbol{\sigma}}^1 + c^2\bar{\boldsymbol{\sigma}}^2$$

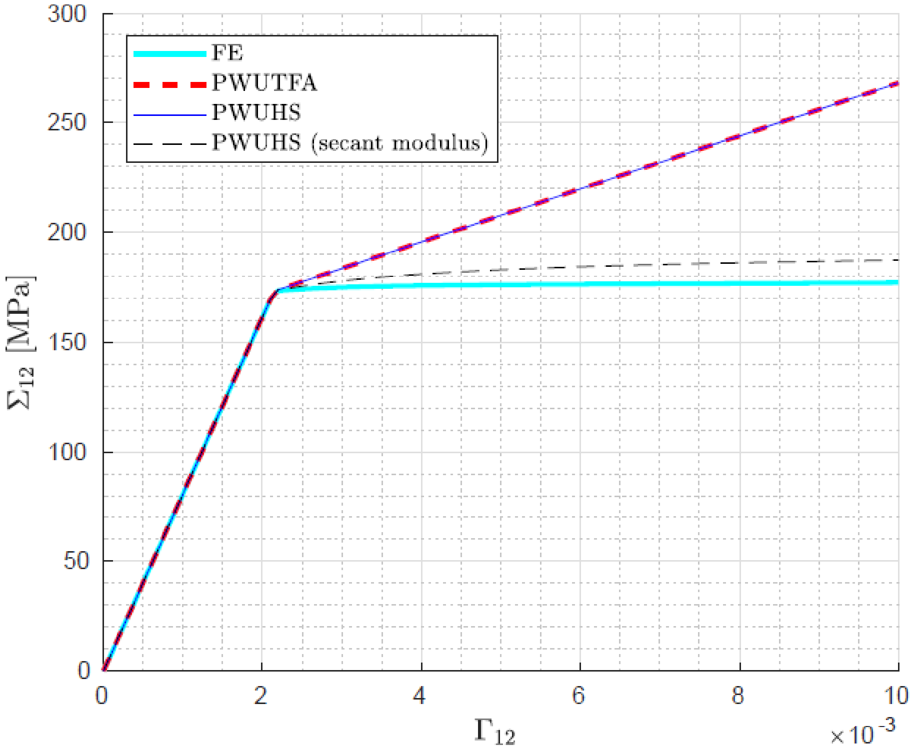
# PWUHS comparison to PWUTFA

- Numerical applications:
  - Homogeneous composite material

| $E^1$ [GPa] | $\nu^1$ | $k$ [GPa] | $\sigma_y$ [MPa] | $E^2$ [GPa] | $\nu^2$ |
|---|---|---|---|---|---|
| 210 | 0.3 | 100 | 300 | 210 | 0.3 |



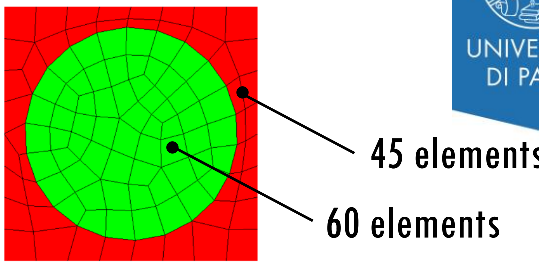


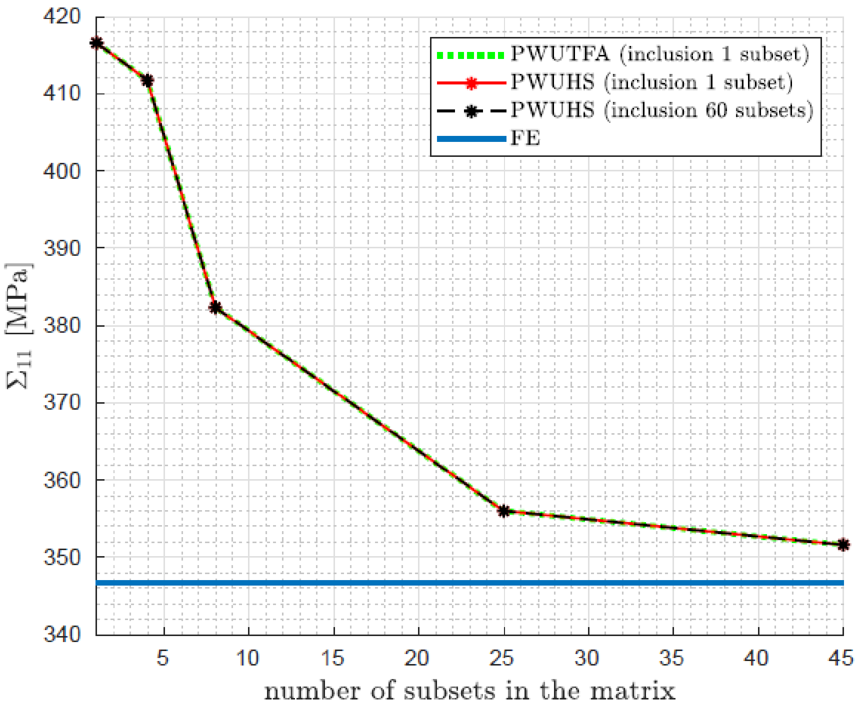
# PWUHS comparison to PWUTFA

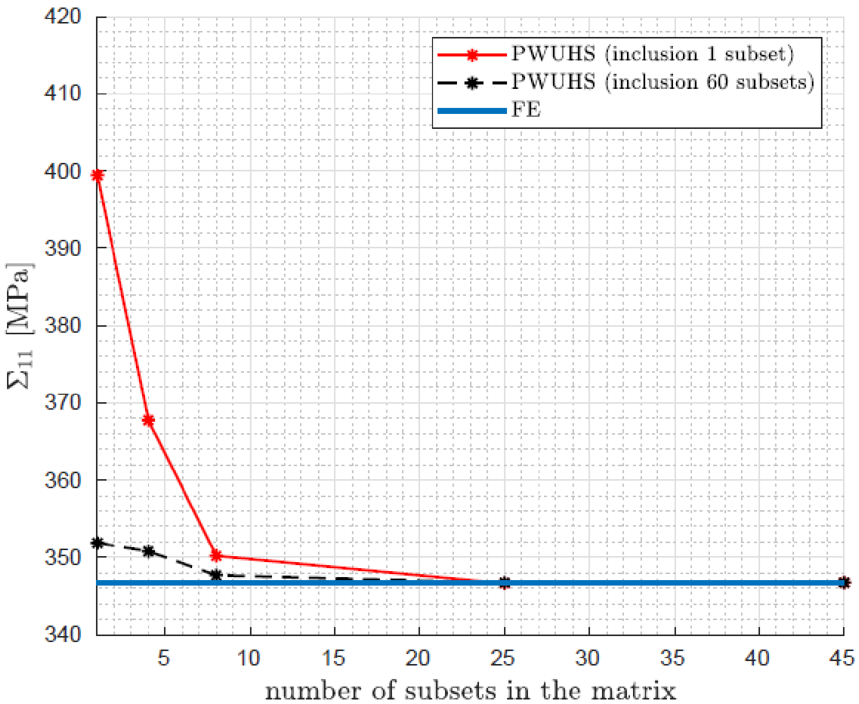
❑ Convergence study:

- **PWUTFA:** 1 subset in the inclusion, subsets refinement in the matrix (up to 45); **PWUHS:** 1 or 60 subsets in the inclusion and subsets refinement in the matrix considering fixed $\mathbf{C}_0$




- **PWUHS:** 1 or 60 subsets in the inclusion, subsets refinement in the matrix (up to 45) using the **secant modulus approach**

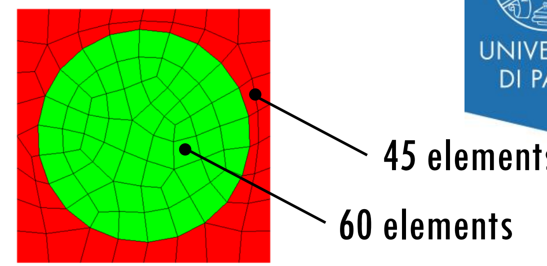


# PWUHS comparison to PWUTFA

## Convergence study:

- **PWUTFA**: 1 subset in the inclusion, subsets refinement in the matrix (up to 45); **PWUHS**: 1 or 60 subsets in the inclusion and subsets refinement in the matrix considering fixed $\mathbf{C}_0$

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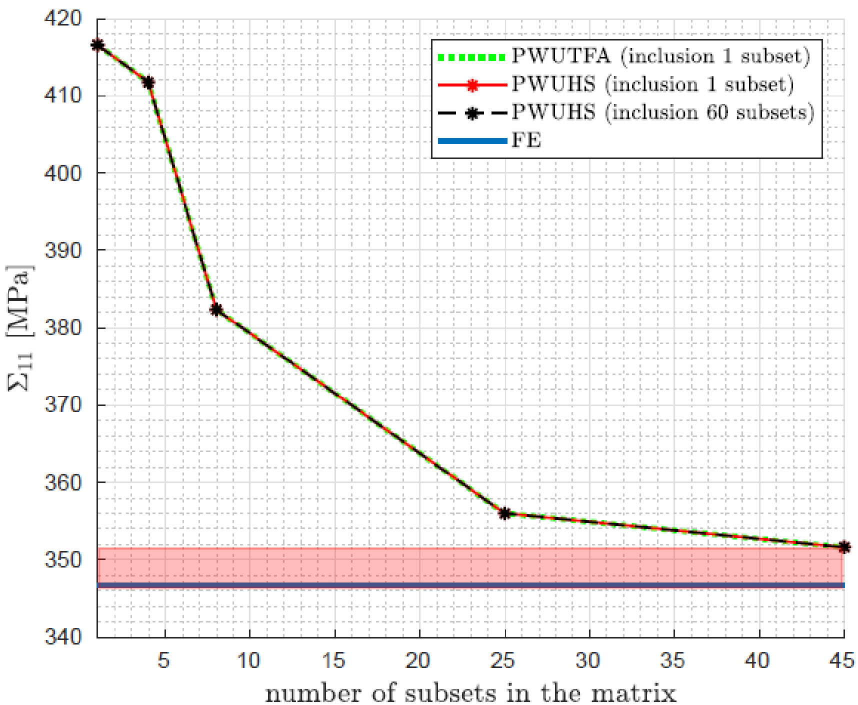
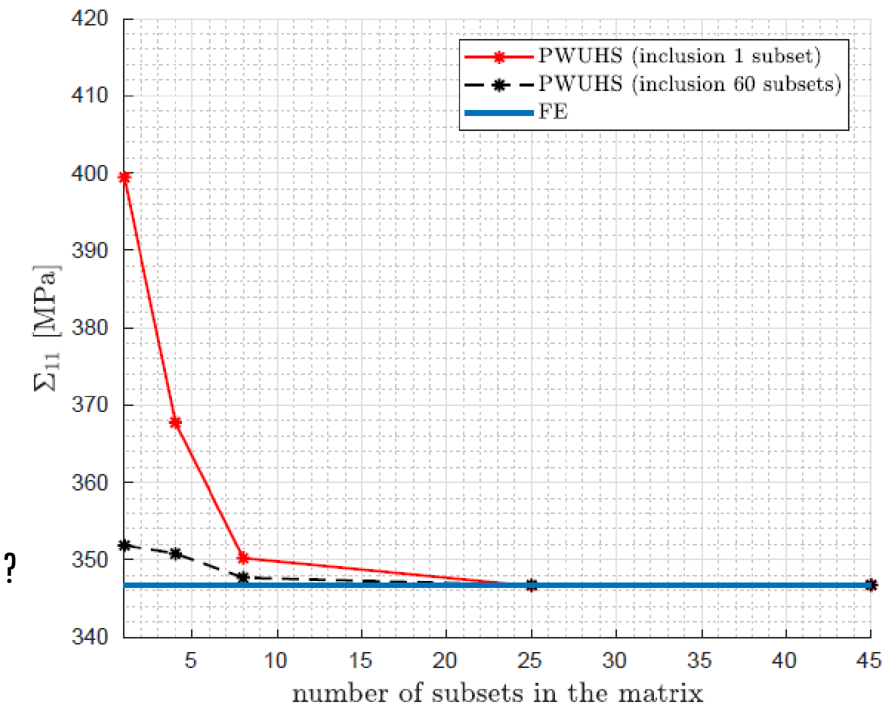
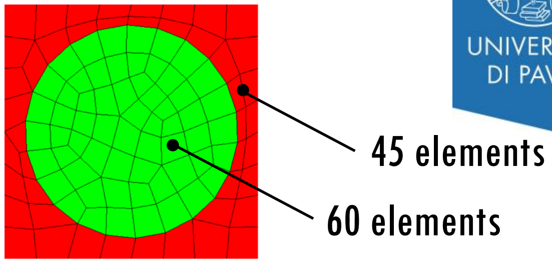


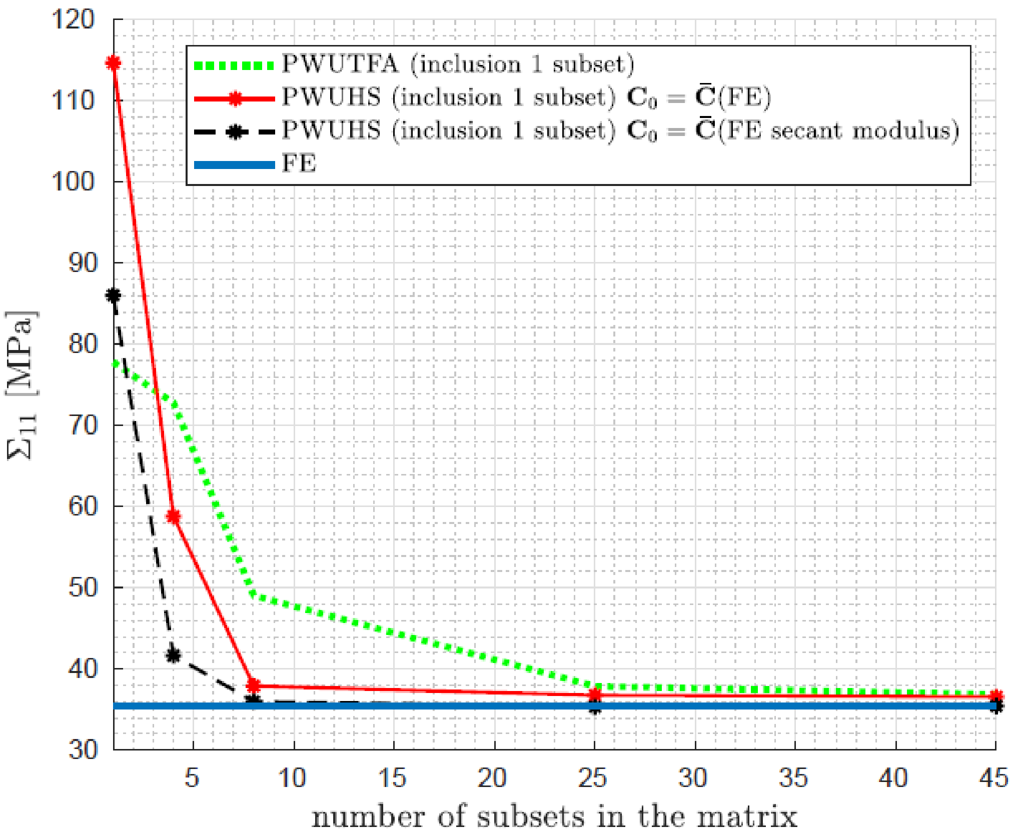

LOCKING ?



# PWUHS comparison to PWUTFA

- Convergence study:
  - 1 subset in the inclusion, subsets refinement in the matrix (up to 45)




- Fiber reinforced epoxy resin

| $E^1$ [GPa] | $\nu^1$ | $k$ [MPa] | $\sigma_y$ [MPa] | $E^2$ [GPa] | $\nu^2$ |
|---|---|---|---|---|---|
| 21 | 0.3 | 100 | 30 | 210 | 0.25 |

# PWUHS comparison to PWUTFA

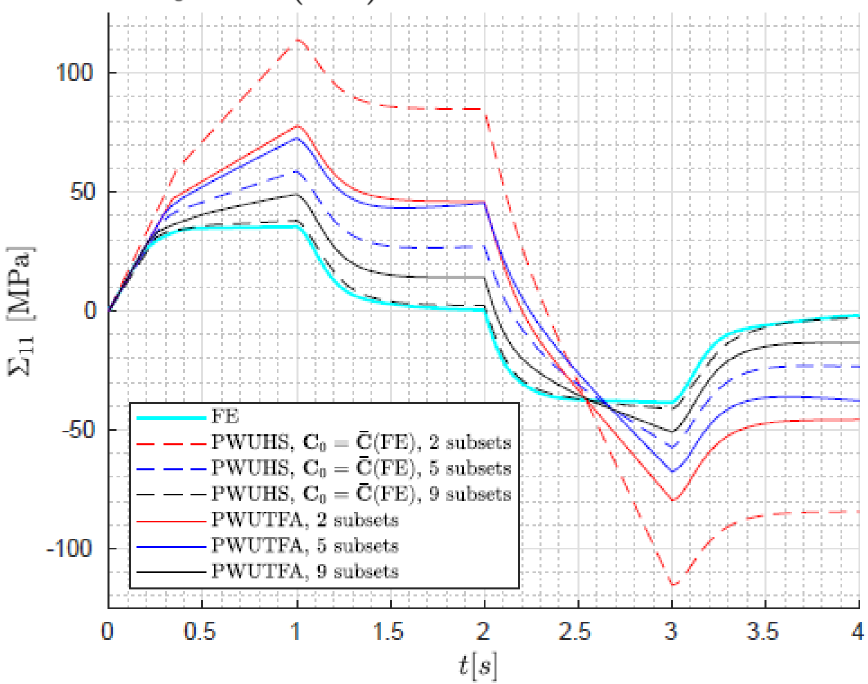
- Numerical application:
  - Heterogeneous composite material
  - Increasing number of subsets

| $E^1$ [GPa] | $\nu^1$ | $k$ [MPa] | $\sigma_y$ [MPa] | $E^2$ [GPa] | $\nu^2$ |
|---|---|---|---|---|---|
| 21 | 0.3 | 100 | 30 | 210 | 0.25 |

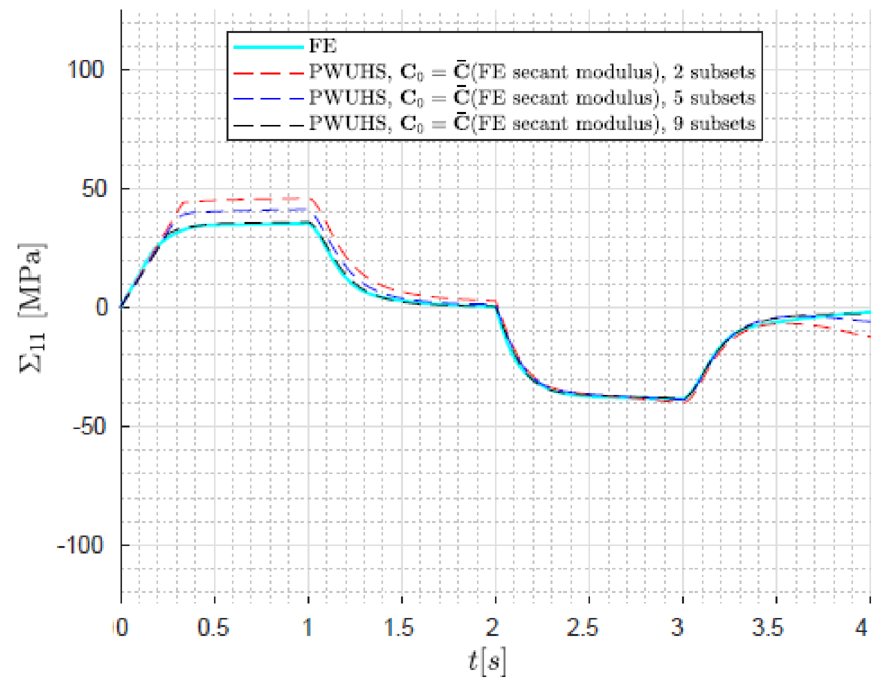
- LC-3

| t [s] | $E_{11}$ | $E_{12}$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.002 | 0 |
| 2 | 0.002 | 0.01 |
| 3 | -0.002 | 0.01 |
| 4 | -0.002 | 0 |

- $\mathbf{C}_0 = \bar{\mathbf{C}}(\mathrm{FE})$




- Updated secant modulus approach



# PWUHS comparison to PWUTFA



- Relative error in numerical applications

$$\texttt{err} = \frac{\|\Sigma_t - \Sigma_t^{\mathrm{FE}}\|}{\|\Sigma_t^{\mathrm{FE}}\|}$$

- $\Sigma_{11}$

| | $t=1$ | | | $t=2$ | | | $t=3$ | | | $t=4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| subsets | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$ | 2.216 | 0.659 | 0.069 | 199.099 | 63.095 | 4.288 | 1.997 | 0.491 | 0.075 | 47.951 | 12.532 | 0.665 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$, secant | 0.301 | 0.167 | $\boxed{0.017}$ | 5.790 | 2.251 | $\boxed{0.250}$ | 0.038 | 0.006 | $\boxed{0.009}$ | 6.066 | 2.363 | $\boxed{0.586}$ |
| PWUTFA | 1.194 | 1.053 | 0.383 | 107.205 | 105.600 | 32.084 | 1.067 | 0.758 | 0.324 | 25.458 | 20.865 | 6.600 |

- $\Sigma_{22}$

| | $t=1$ | | | $t=2$ | | | $t=3$ | | | $t=4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| subsets | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$ | 0.486 | 0.120 | 0.176 | 134.447 | 219.465 | 16.600 | 0.409 | 0.271 | 0.119 | 316.208 | 114.960 | 92.263 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$, secant | 0.188 | 0.206 | 0.213 | 24.106 | 4.502 | $\boxed{2.906}$ | 0.134 | 0.151 | 0.194 | 163.867 | 67.057 | $\boxed{42.529}$ |
| PWUTFA | 0.082 | 0.087 | $\boxed{0.080}$ | 241.957 | 449.935 | 42.721 | 0.063 | 0.302 | $\boxed{0.050}$ | 447.873 | 301.884 | 79.570 |

- $\Sigma_{12}$

| | $t=1$ | | | $t=2$ | | | $t=3$ | | | $t=4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| subsets | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 | 2 | 5 | 9 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$ | – | – | – | 1.824 | 0.388 | 0.079 | 160.854 | 23.243 | 4.378 | 0.079 | 0.053 | 0.001 |
| PWUHS, $\mathbf{C}_0 = \bar{\mathbf{C}}$, secant | – | – | – | 0.074 | 0.002 | $\boxed{0.018}$ | 8.944 | $\boxed{0.717}$ | 0.278 | 0.048 | 0.005 | $\boxed{0.024}$ |
| PWUTFA | – | – | – | 1.034 | 0.825 | 0.670 | 91.878 | 56.333 | 54.449 | 0.059 | 0.078 | 0.027 |

# PWUHS comparison to PWUTFA

## REMARKS

- Numerical evidence of the **equivalence** between the two reduced order models, **PWUTFA and PWUHS**, for composites with the **same elastic properties,** is given

- Convergence study proves that both ROMs results tend toward to FE solution increasing the number of subsets

- **PWUHS** using the **updated secant modulus approach gives an accurate prediction of the macroscopic stresses** $\Sigma$ also for complex loading-unloading history, even if a discretization in a low number of subsets in considered

- The updated secant modulus approach gives a faster convergence to the FE solution in comparison to PWUTFA and PWUHS with fixed $\mathbf{C}_0 = \bar{\mathbf{C}}$

Publications:
A. Castrogiovanni, S. Marfia, F. Auricchio, E. Sacco, "TFA and HS based homogenization techniques for nonlinear composites", International Journal of Solids and Structures, Volume 225, 2021

# OUTLINE

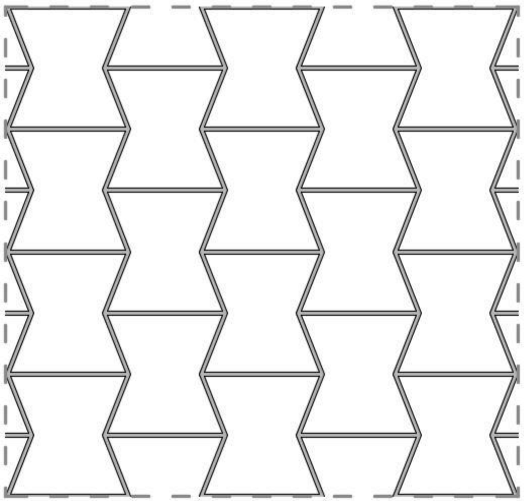
- The PWUHS Reduced Order Model
  - Homogenization of Nonlinear Composites
  - Numerical Procedure
  - Numerical applications
  - Remarks
- PWUHS comparison to PWUTFA
  - Equivalence between PWUHS and PWUTFA
  - Numerical applications
  - Convergence study
  - Remarks
- Multiscale Analysis using PWUHS
  - Experimental investigation
  - Auxetic composites
  - Implementing the Multiscale Procedure
  - Multiscale Analysis of Auxetic Honeycombs
- Concluding remarks

# Multiscale analysis using PWUHS

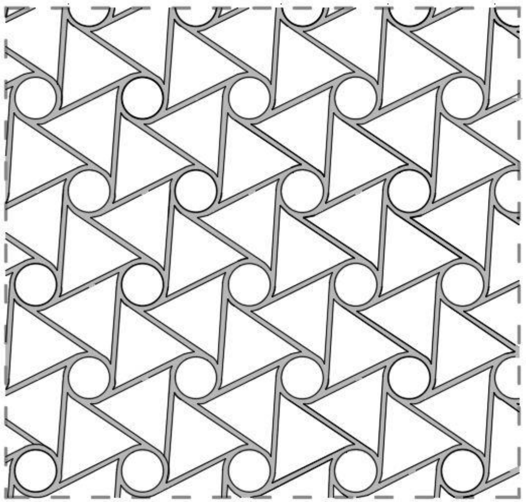
## Auxetic materials

- **Negative Poisson ratio**
- Enhance the mechanical properties of crash absorbers
- Auxetic foams, auxetic laminates , **auxetic honeycombs**

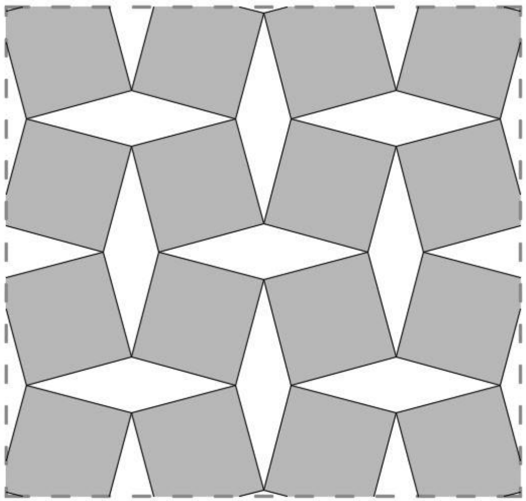
- RE-ENTRANT TYPE



- CHIRAL TYPE



- ROTATING POLYGONS



# Multiscale analysis using PWUHS

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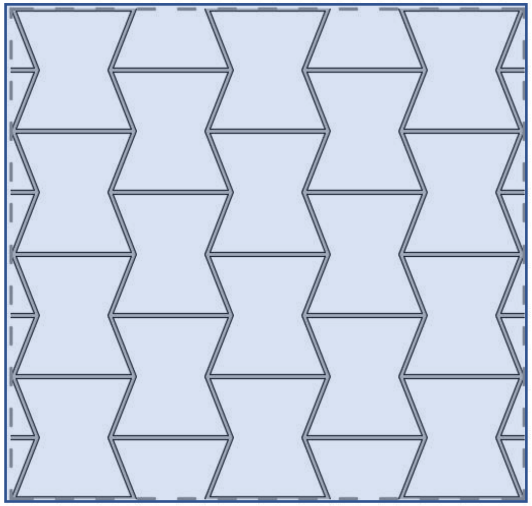
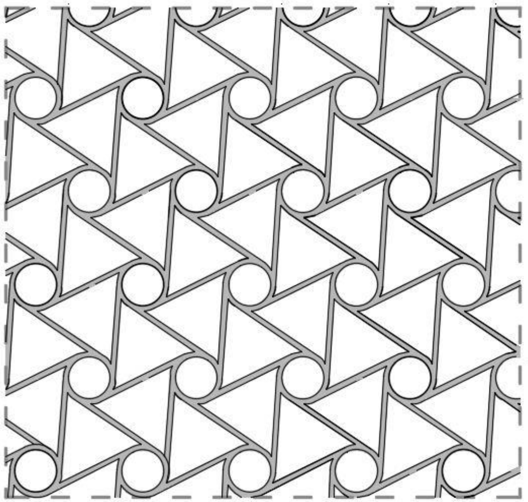
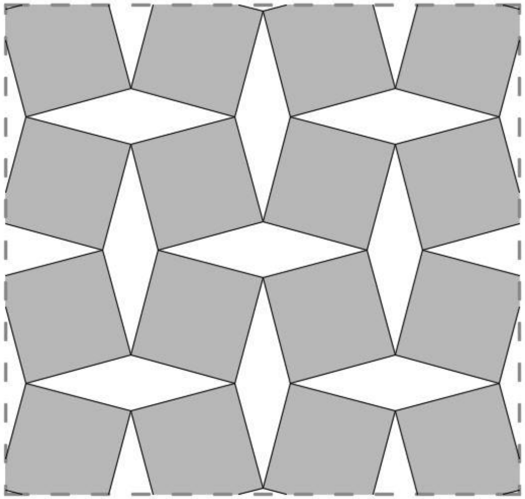


- CHIRAL TYPE



- ROTATING POLYGONS



# Multiscale analysis using PWUHS

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## 3D Printed Foam-Filled Auxetic Honeycomb

- 3D Printed polymeric (Polyamide, PA12) **auxetic frame**
  - Re-entrant auxetic honeycomb geometry
  - "3D High Reusability PA 12" produced by HP is referred for the material properties
  - The **Elastic-perfectly plastic material** model is considered

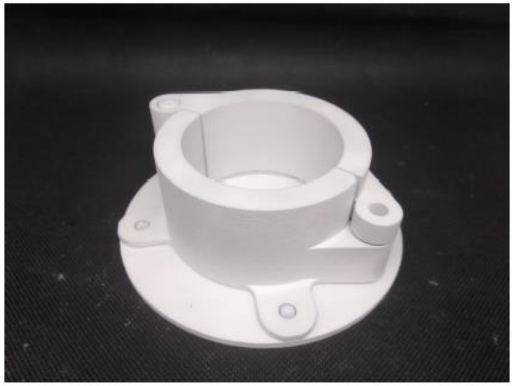
| $E$ [MPa] | $\nu$ | $\sigma_y$ [MPa] |
|---|---|---|
| 1700 | 0.41 | 48 |

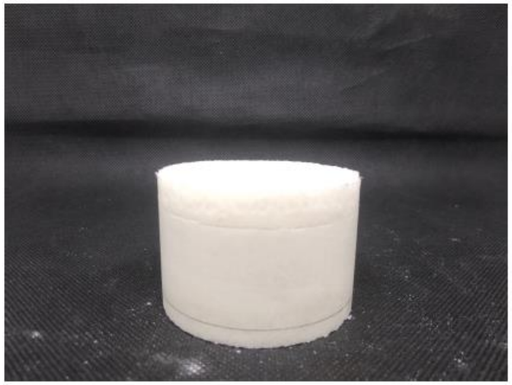
- Rigid Polyurethane **foam filler**
  - Polyol + Isocyanate + Water = Polyurethane (PU)
  - Mechanical properties not known *a priori*

# Multiscale analysis using PWUHS

## PU foam material characterization

- PU foams provided by BCI Polyurethane Europe S.R.L.
  - Isocyanate Isotem® P200
  - Polyols: Promol® DP 25/10B1 (low density), Promol® VA 50/6A3 (high density)
- Experimental setup
  - *“Standard Test Method for Compressive Properties of Rigid Cellular Plastics”, ASTM*
  - MTS Insight Testing System equipped with a 10KN load cell and two steel compression plates
  - Specimens dimensions: ϕ = 60 mm, h = 40 mm





# Multiscale analysis using PWUHS

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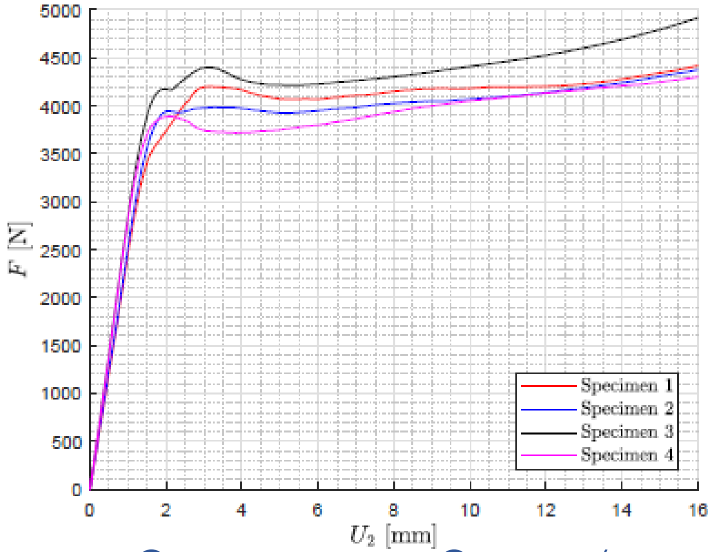
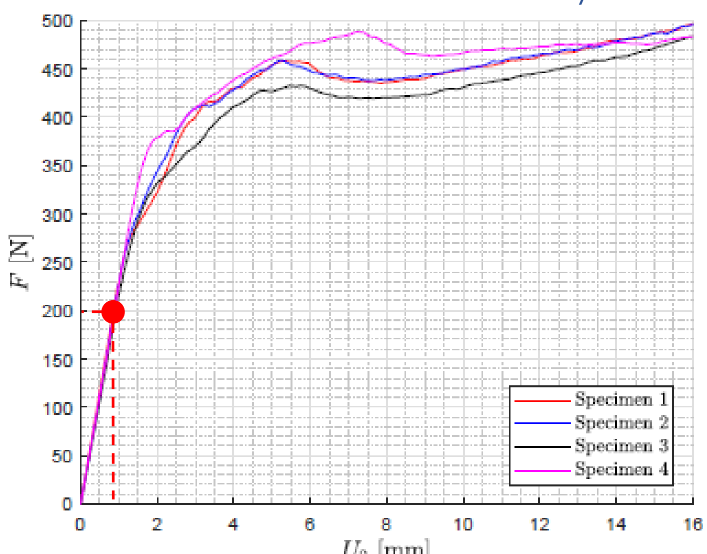
## Isotem® P200 + Promol® VA 50/6A3




## Isotem® P200 + Promol® DP 25/10B1




**DP 25/10B1 - Isotem P200**

| | Diameter $\phi$ [mm] | | | |
|---|---|---|---|---|
| Measurement n | 1 | 2 | 3 | Average |
| Specimen 1 | 59.48 | 60.07 | 59.89 | 59.81 |
| Specimen 2 | 59.55 | 59.58 | 59.99 | 59.71 |
| Specimen 3 | 59.84 | 59.4 | 59.44 | 59.56 |
| Specimen 4 | 59.59 | 59.71 | 59.78 | 59.69 |

| | Height $h$ [mm] | | | |
|---|---|---|---|---|
| Measurement n | 1 | 2 | 3 | Average |
| Specimen 1 | 40.09 | 40.09 | 40.22 | 40.13 |
| Specimen 2 | 39.88 | 39.99 | 40.10 | 39.99 |
| Specimen 3 | 40.11 | 40.30 | 40.56 | 40.32 |
| Specimen 4 | 41.05 | 40.99 | 40.76 | 40.93 |

| | Base Area A [$mm^2$] | Elastic Modulus $E$ [MPa] |
|---|---|---|
| Specimen 1 | 2809.87 | 3.34 |
| Specimen 2 | 2799.85 | 3.12 |
| Specimen 3 | 2786.12 | 3.09 |
| Specimen 4 | 2798.60 | 3.32 |

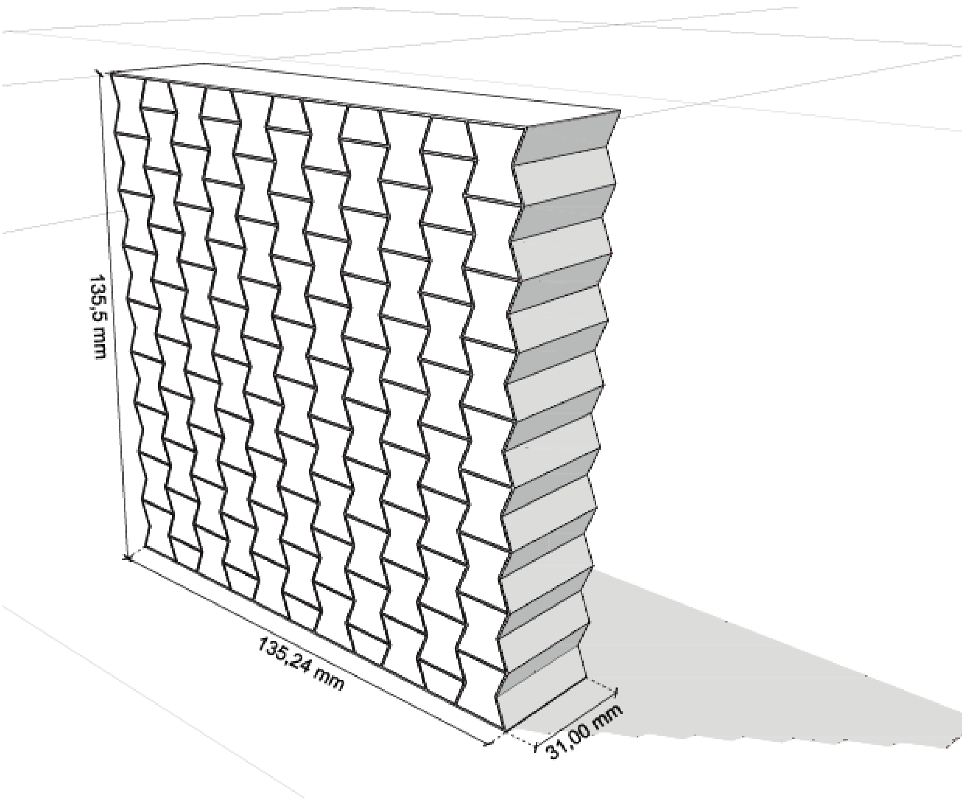
| $E$ [MPa] | $\nu$ | $\sigma_y$ [MPa] | $k$ [MPa] |
|---|---|---|---|
| 3.22 | 0.27 | 0.13 | 0.1 |

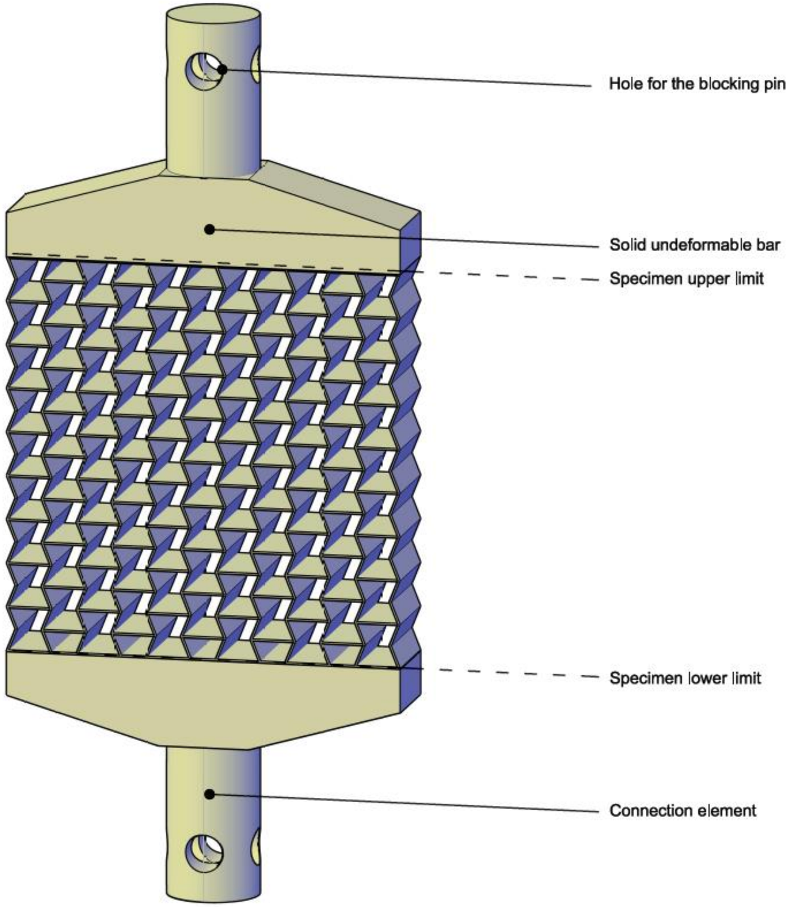
Elastoplasticity with isotropic hardening

# Multiscale analysis using PWUHS

## Experimental investigation

- Polymeric PA12 frame 3D printed using “HP Multi Jet Fusion” (MJF) technology
- DP 25/10B1 PU foam filler



# Multiscale analysis using PWUHS

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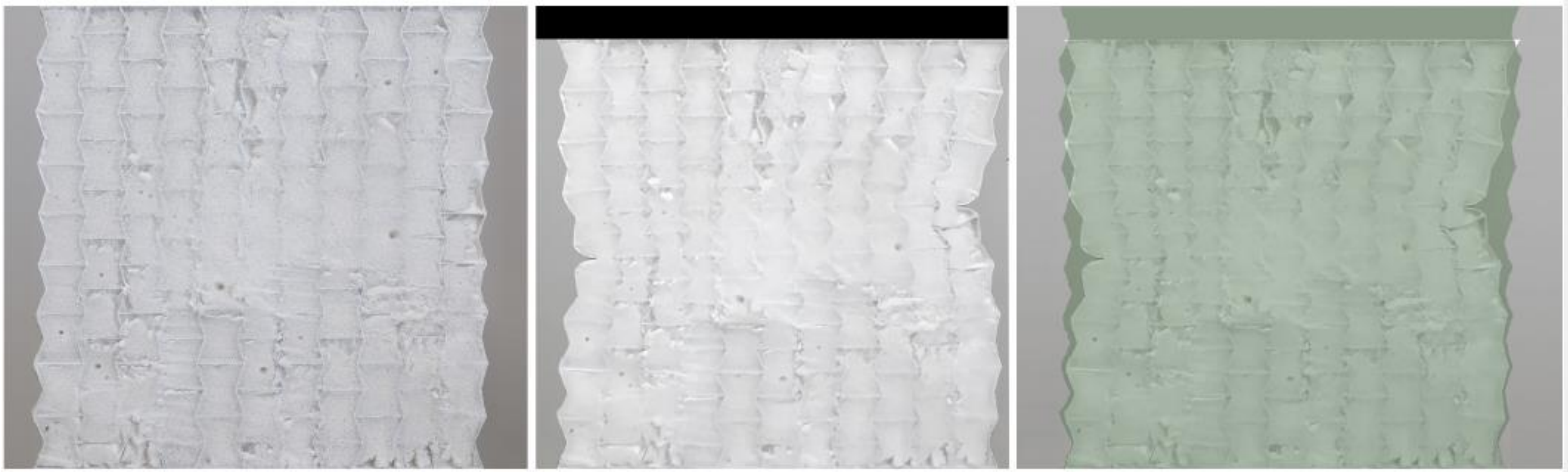
## Experimental investigation

- Test in compression $U_0 = -10 \text{ mm}$



- Arising of buckling phenomena
- **Comparison** with the numerical counterpart (small strain regime) is **not possible**

# Multiscale analysis using PWUHS

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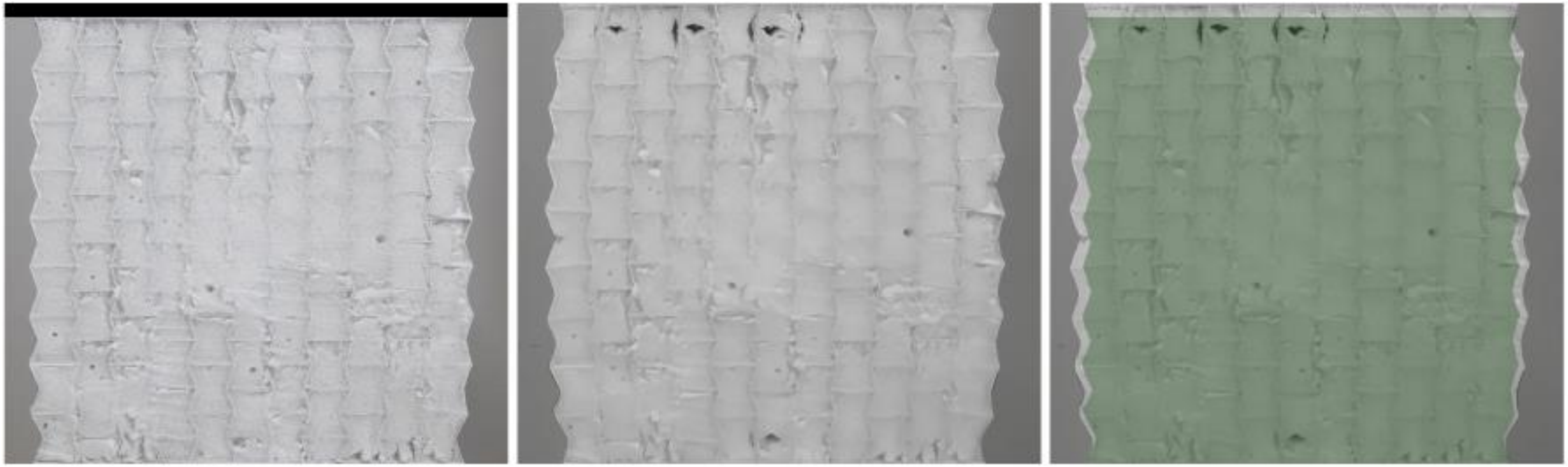
## Experimental investigation

- Test in tension $U_0 = 10\,\mathrm{mm}$
- Failure of the foam-filled composite at $U_0 = 4.6\,\mathrm{mm}$ due to decohesion



- **Considered for comparison** with the numerical counterpart

# Multiscale analysis using PWUHS

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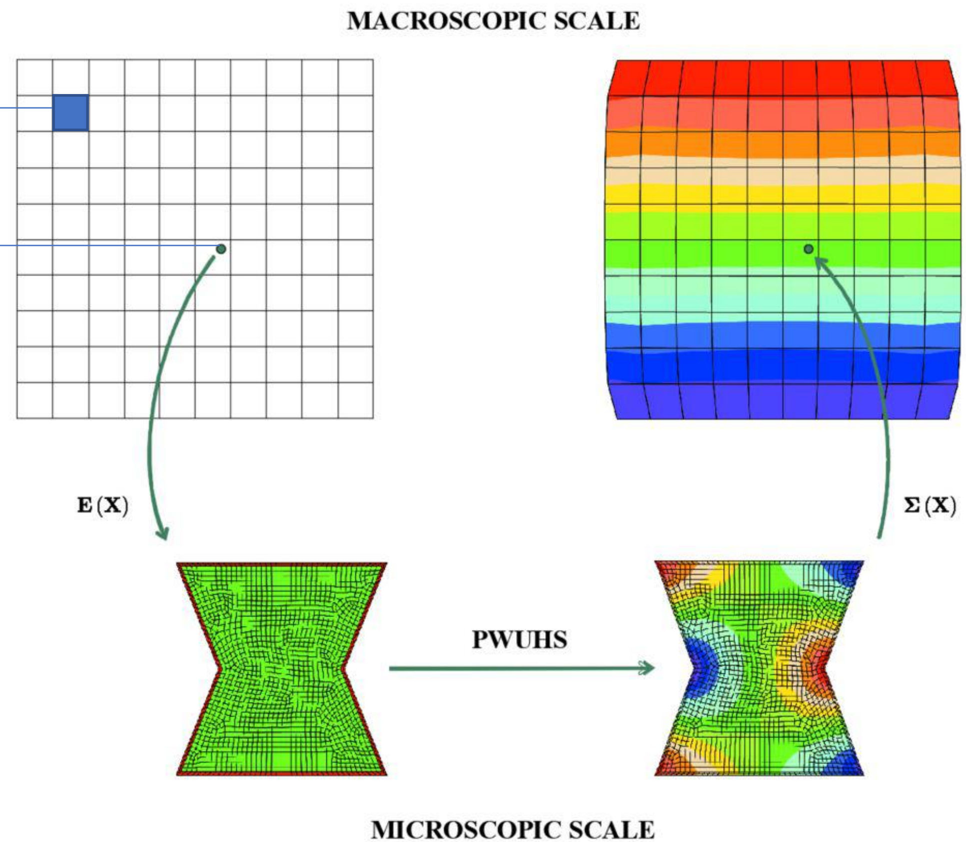
## Numerical procedure

- Precomputations on UC
- Solution of the structural problem (Online stage)
- Multiscale analysis and PWUHS implemented in a finite element code (**FEAP**)

$N_e$ Macroscopic elements

GP



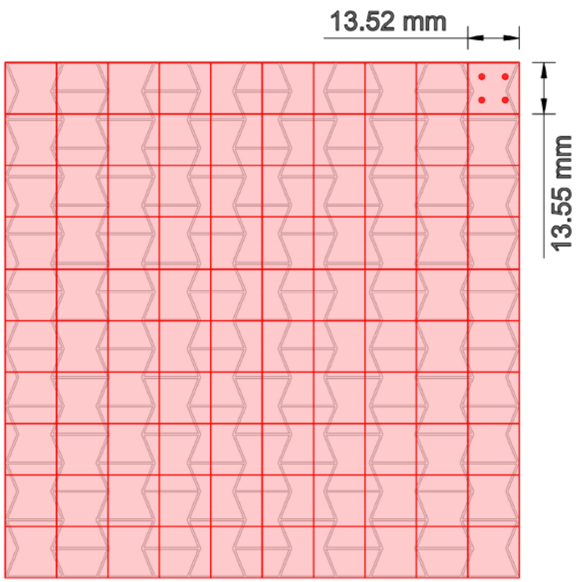

Global equilibrium at the macroscopic scale

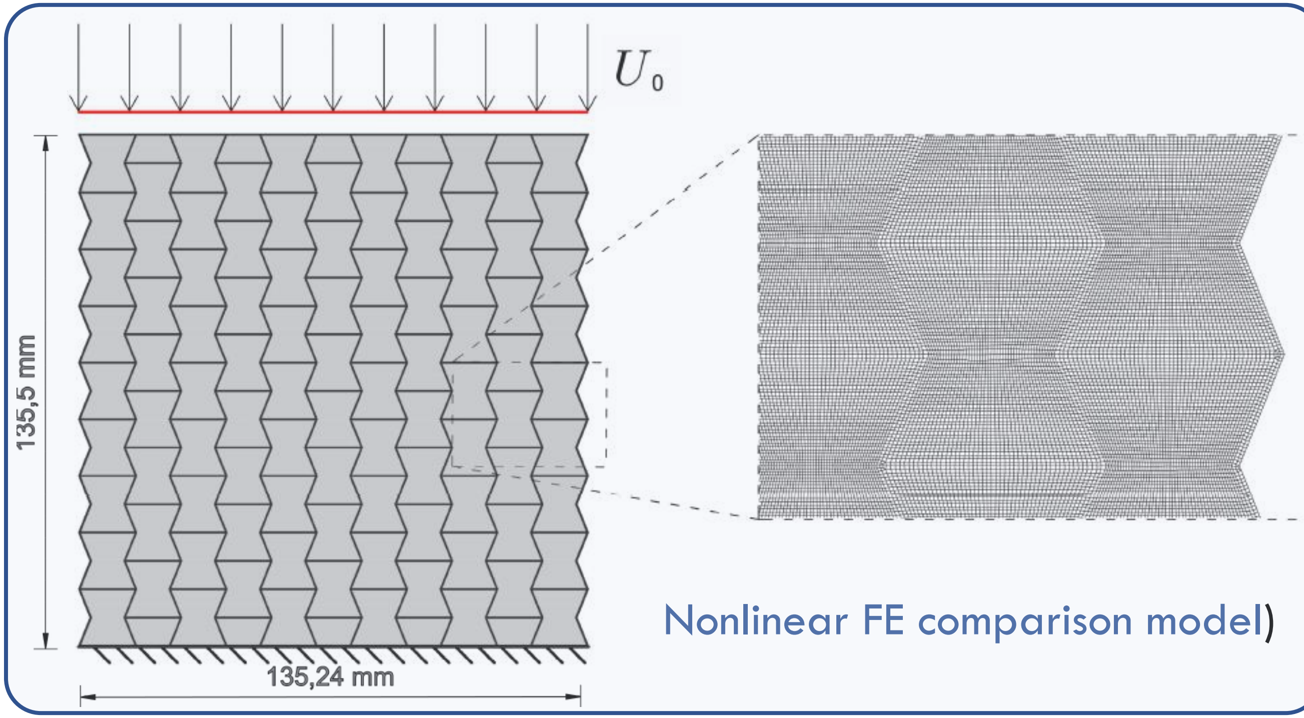
$$\mathbf{KU} = \mathbf{F}$$

$$\mathbf{F} = -\sum_{e=1}^{N_e} \int_{\Omega_e} \mathbf{B}^T \boldsymbol{\Sigma} \, dA$$

# Multiscale analysis using PWUHS

## Analysis of Auxetic honeycomb structure




Nonlinear FE comparison model)

### Equivalent homogeneous macroscopic model

- 100 elements (4GP)
- 11200 history variables

### Nonlinear FE

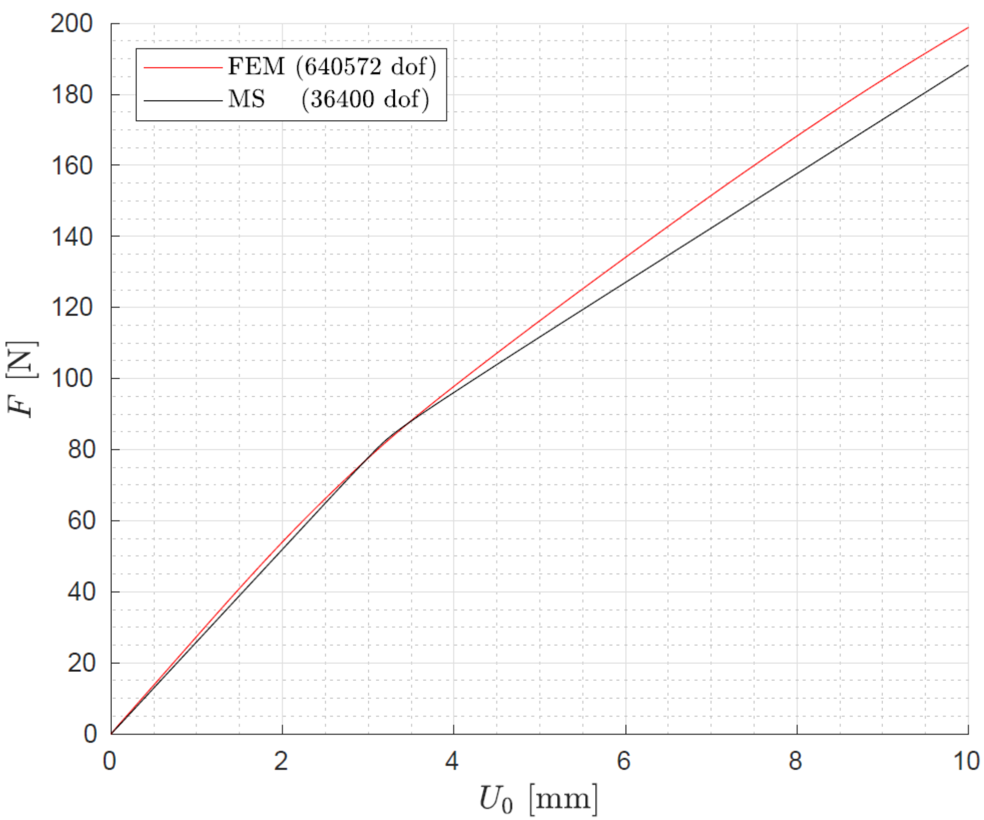
- 320286 quad elements
- 5124576 history variables

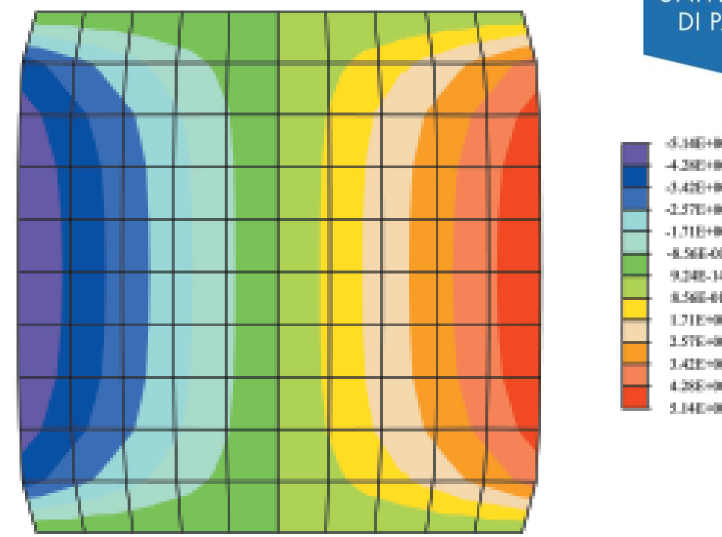
# Multiscale analysis using PWUHS

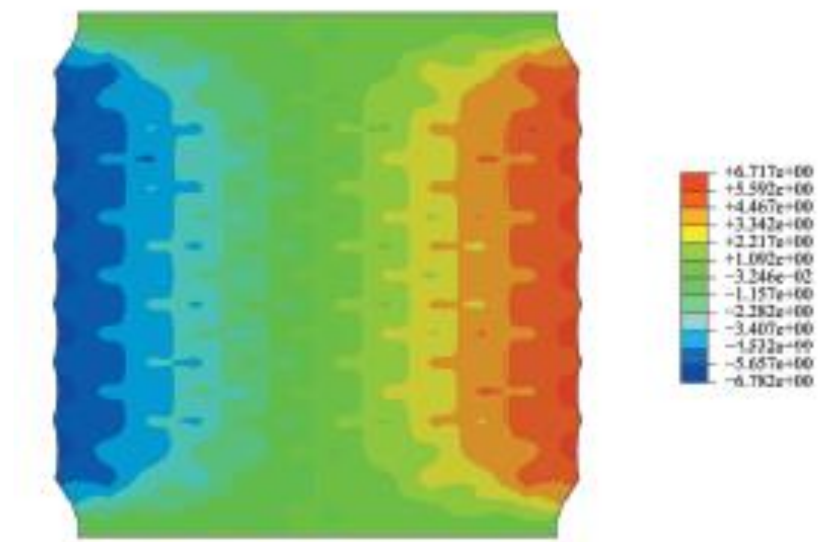


## Analysis of Auxetic honeycomb structure

- Multiscale analysis in **plane strain** condition
- Tension loading history $U_0 = 10 \text{ mm}$



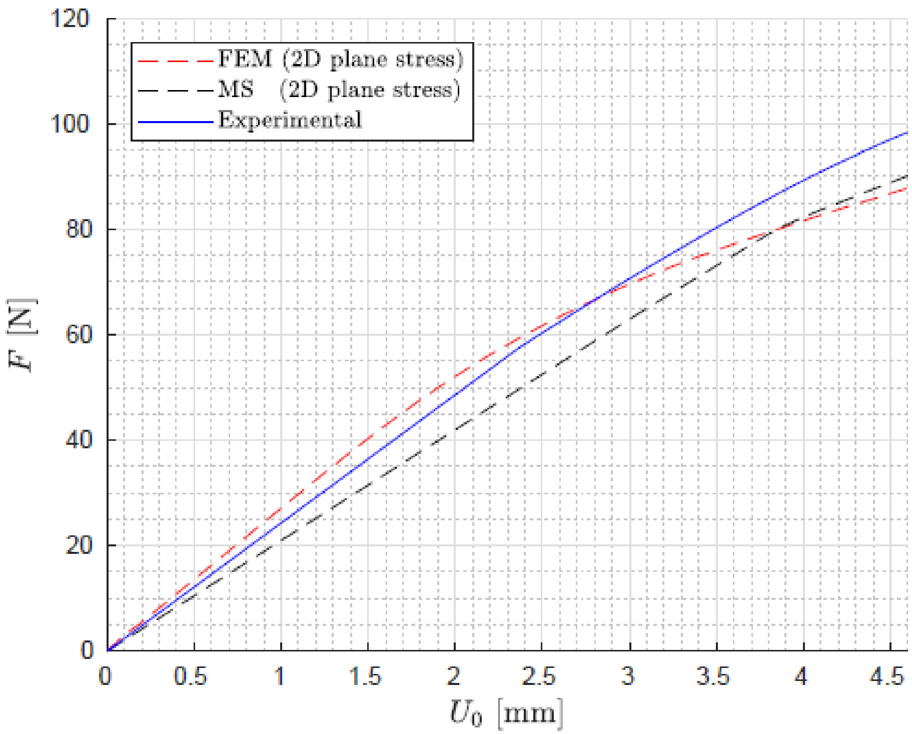



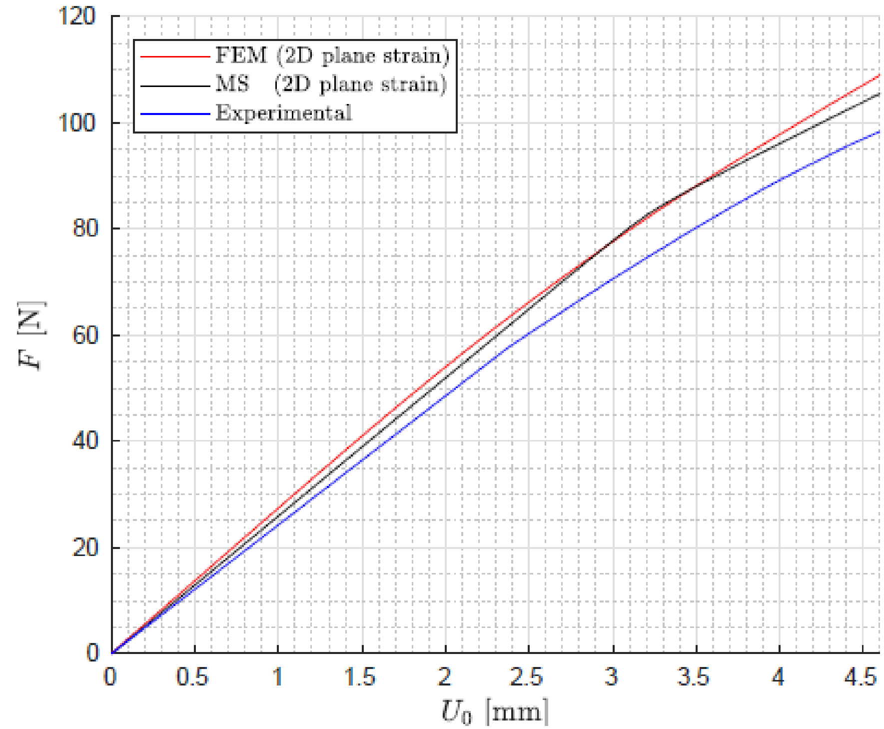


# Multiscale analysis using PWUHS

## Comparison with Experimental investigations

- numerical and experimental results in terms of the resultant force F before failure at $U_0 = 4.6\ \mathrm{mm}$



# Concluding remarks

- PWUHS is introduced for the homogenization of nonlinear composites.
- A novel approach for the calculation of the elastic reference matrix using the FE homogenization of the composite is proposed.
- A comparative study between PWUTFA and PWUHS shows the similarities and differences between the two ROMs.
- PWUHS ROM is an efficient numerical tool for reducing the computational cost in Multiscale Analysis of nonlinear composites.
- Multiscale Analysis predicts the behaviour of 3D-Printed Auxetic Composites structures with a drastically reduced number of history variables in comparison to the FE solution.
- Results of the Multiscale scheme are in line with experimental tests on foam-filled auxetic honeycomb.

# Concluding remarks

## What's next?

- Introduction of a clustering technique during the precomputations for the division of the RVE in optimized subsets.
- Investigation of the influence of UC's dimensions in Multiscale Analysis, with respect to the structural size, on the model capability to naturally account for size effects.
- Extension of the FE code with implemented PWUHS homogenization to parallel computing

# THANK YOU ALL !

# Multiscale analysis using PWUHS

## UC homogenization

- UC identification by mean of periodicity directions (UC-A)
  - 7 subsets
  - 4 x 7 =28 history variables (7160 in FE)

1790 elements