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Damage Mechanics-Based Models for High-Cycle Fatigue Life Prediction of Metals

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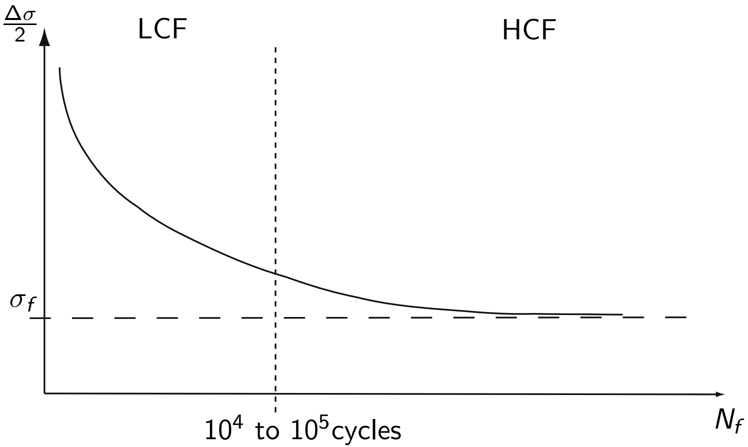
Fatigue Life Classification

- **Types of Fatigue** (form in which fatigue occurs): mechanical; thermomechanical; creep; corrosion; rolling contact; fretting;
- **Fatigue life** (duration of the fatigue life):
  - **Low-Cycle Fatigue (LCF)** ($N < 10^4 - 10^5$ cycles): stresses are generally high enough to cause appreciable plastic deformation at the mesoscale (the scale of the RVE) prior to failure;
  - **High-Cycle Fatigue (HCF)** ($N > 10^4 - 10^5$ cycles): damage is localized at the microscale as a few micro-cracks and the material deforms primarily elastically at the mesoscale up to crack initiation. The cyclic evolution of an isolated grain can be resumed by the creation of localized plastic slip bands and the nucleation of microcracks until the creation of a mesocrack.

Fatigue Life



Stages

- **Crack Initiation** (CI): is a material surface phenomenon; micro-cracks usually start on localized shear planes at the surface; once nucleation occurs and cyclic loading continues, the micro-crack tends to grow along the plane of maximum shear stress and through the grain boundary.
- **Stable Crack Growth** (SCG): is normal to the maximum principal stress; it depends on the material as a bulk property;
- **Unstable Crack Growth**: leads to ductile or brittle fracture; very short, not important from a practical point of view.

Remark: Under stress amplitudes just above the fatigue limit (HCF), the CI period may cover a large percentage of the fatigue life; for larger stress amplitudes (LCF), the SCG period can be a substantial portion of the fatigue life.

Important Features of HCF

- HCF of metals may be regarded as a form of material degradation/damage caused by cyclic loading;
- Damage is controlled by mechanisms at the grain scale (microscale) and, therefore, a description at this scale is necessary;
- At the mesoscale most of the metallic materials can be considered isotropic and homogeneous;
- Microscopic plasticity should be determined by isotropic and kinematic hardening rules;
- Mean stress effect must be taken into account: while mean normal stresses have great effects on failure, mean shear stresses do not.

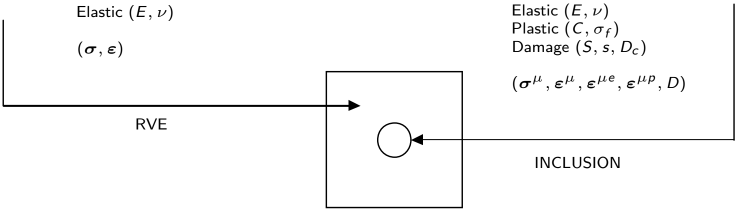
Important Features of HCF (Cont.)

- Macroscopic plasticity is for the most part negligible, and crack initiation occurs in localized plasticity spots surrounded by a material in elastic range. Damage is localized on a microscopic scale with negligible influence on the mesoscale $\Rightarrow$ **Quasi-Brittle Failures**;
- Crack initiation modeling is difficult in this fatigue regime since the scale where the mechanisms operate is not the engineering scale (mesoscale), and local plasticity and damage act simultaneously.

All these features can be well characterized by means of the Theory of Damage Mechanics.

Lemaitre's Model (1994, 1999, 2005)

- It considers a microscopic spherical inclusion with an elasto-plastic-damage behavior embedded in a macroscopic infinite elastic matrix:



Mesoscale $\longrightarrow$ Localization Law $\longrightarrow$ Microscale

Free Energy

- **Free energy of the inclusion**: $\rho\varphi^\mu = \rho\varphi_e^\mu + \rho\varphi_p^\mu$
  - **Elastic part** (affected by the damage variable to model the experimentally observed coupling between elasticity and damage)

$$\rho\varphi_e^\mu(\varepsilon^\mu - \varepsilon^{\mu p}, D) = \frac{1}{2}(\varepsilon^\mu - \varepsilon^{\mu p})\mathbf{E}(1-D)(\varepsilon^\mu - \varepsilon^{\mu p})$$

  - **Plastic part** (assumes exponential isotropic hardening and linear kinematic hardening)

$$\rho\varphi_p^\mu(\boldsymbol{\alpha}, r) = R_\infty(r^\mu + \frac{1}{b}e^{-br^\mu}) + \frac{1}{3}X_\infty\gamma\boldsymbol{\alpha}^\mu : \boldsymbol{\alpha}^\mu$$

- **Free energy of the matrix**: $\rho\varphi(\varepsilon) = \frac{1}{2}\varepsilon\mathbf{E}\varepsilon$

State Laws (Inclusion)

- **Stress Tensor**

$$\boldsymbol{\sigma}^{\mu} = \rho \frac{\partial \varphi^{\mu}}{\partial \boldsymbol{\varepsilon}^{\mu e}} = \mathbf{E}(1 - D) : \boldsymbol{\varepsilon}^{\mu e}$$

- **Isotropic Hardening** (it represents the growth in size of the yield surface)

$$R^{\mu} = \rho \frac{\partial \varphi^{\mu}}{\partial r^{\mu}} = R_{\infty}(1 - e^{-br^{\mu}})$$

- **Kinematic Hardening** (it represents the translation of the yield surface)

$$\mathbf{X}^{\mu} = \rho \frac{\partial \varphi^{\mu}}{\partial \boldsymbol{\alpha}^{\mu}} = \frac{2}{3} X_{\infty} \gamma \boldsymbol{\alpha}^{\mu}$$

- **Energy Density Release**

$$Y^{\mu} = -\rho \frac{\partial \varphi^{\mu}}{\partial D} = \frac{1}{2} \boldsymbol{\varepsilon}^{\mu e} : \mathbf{E} : \boldsymbol{\varepsilon}^{\mu e}$$

State Laws (Inclusion) (Cont.)

- The **energy density release** can be rewritten as

$$Y^{\mu} = \frac{\sigma^{\mu 2}_{eq} R^{\mu}_{\nu}}{2E(1-D)^2}$$

with

$$R^{\mu}_{\nu} = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma^{\mu}_{H}}{\sigma^{\mu}_{eq}}\right)^2$$

$$\sigma^{\mu}_{eq} = \left(\frac{3}{2}\boldsymbol{\sigma}^{\mu D} : \boldsymbol{\sigma}^{\mu D}\right)^{\frac{1}{2}}, \quad \sigma^{\mu}_{H} = \frac{1}{3} tr(\boldsymbol{\sigma}^{\mu}), \quad \boldsymbol{\sigma}^{\mu D} = \boldsymbol{\sigma}^{\mu} - \sigma^{\mu}_{H}\mathbf{I}$$

- The **mean stress effect** can be taken into account by considering a new form of the energy density release rate which includes an additional parameter, $0 \leq h \leq 1$, to model the micro-defects closure effect.

Dissipation Potential (Inclusion)

- **Dissipation Potential**

$$F^{\mu} = f^{\mu} + F_{x}^{\mu} + \frac{S}{(s+1)(1-D)}\left(\frac{-Y^{\mu}}{S}\right)^{s+1}$$

  - **Yield Potential**

$$f^{\mu} = \left(\frac{\sigma^{\mu D}}{1-D} - \mathbf{X}^{\mu}\right)_{eq} - R^{\mu} - \sigma_{f}^{\infty} \leq 0$$

  - **Kinematic Hardening Potential**

$$F_{x}^{\mu} = \frac{3}{4X_{\infty}}\mathbf{X}^{\mu} : \mathbf{X}^{\mu}$$

Complementary State Laws (Inclusion)

By means of the **normality property**, the complementary state laws are obtained as

$$
\begin{aligned}
\dot{\boldsymbol{\varepsilon}}^{\mu p} &= \dot{\lambda}^{\mu} \frac{\partial F^{\mu}}{\partial \boldsymbol{\sigma}^{\mu}} = \frac{3}{2} \frac{\boldsymbol{\sigma}^{\mu D}}{\sigma^{\mu}_{eq}} \frac{\dot{\lambda}^{\mu}}{(1-D)} \\
\dot{r}^{\mu} &= -\dot{\lambda}^{\mu} \frac{\partial F^{\mu}}{\partial R^{\mu}} = \dot{\lambda}^{\mu} \\
\dot{\boldsymbol{\alpha}}^{\mu} &= -\dot{\lambda}^{\mu} \frac{\partial F^{\mu}}{\partial \mathbf{X}^{\mu}} = \dot{\boldsymbol{\varepsilon}}^{\mu p}(1-D) - \dot{\lambda}^{\mu} \frac{3}{2X_{\infty}} \mathbf{X}^{\mu} \\
\dot{D} &= -\frac{\partial F^{\mu}}{\partial Y^{\mu}} \dot{\lambda}^{\mu} = \left(\frac{-Y^{\mu}}{S} \right)^{s} \frac{\dot{\lambda}^{\mu}}{(1-D)}
\end{aligned}
$$

Remark: The plastic multiplier $\dot{\lambda}^{\mu}$ is derived from the consistency condition $\dot{f}^{\mu} = 0$.

Damage Evolution and Meso-crack Initiation

- The accumulated plastic plastic strain p^{μ}, defined by its rate $\dot{p}^{\mu} = (\frac{2}{3}\dot{\varepsilon}^{\mu p} : \dot{\varepsilon}^{\mu p})^{\frac{1}{2}}$, comes out as $\dot{p}^{\mu} = \dot{\lambda}^{\mu}/(1-D)$;
- **Damage Evolution**: energy damage threshold w_D

$$\dot{D} = \left(\frac{-Y^{\mu}}{S}\right)^{s} \dot{p}^{\mu}, \text{ if } w_s > w_D$$

- **Meso-Crack Initiation**: critical damage D_c

$$D = D_c$$

Localization Laws

- **Lin-Taylor Localization Law** (adopted by Lemaitre et al. (1990, 1994))

$$\boldsymbol{\varepsilon}^{\mu} = \boldsymbol{\varepsilon}$$
$$\boldsymbol{\sigma}^{\mu} = \boldsymbol{\sigma}$$

- **Eshelby-Kroner Localization Law** (adopted by Lemaitre et al. (1999, 2005))

$$\boldsymbol{\varepsilon}^{\mu} = \boldsymbol{\varepsilon} + \beta \boldsymbol{\varepsilon}^{\mu p}$$
$$\boldsymbol{\sigma}^{\mu} = \boldsymbol{\sigma} - 2G(1-\beta)\boldsymbol{\varepsilon}^{\mu p}$$

with

$$\beta = \frac{2}{15}\frac{4-5\nu}{1-\nu}$$

Application of the Two-Scale Model

- **Macroscale - Uncoupled Analysis**: the meso-quantities $\boldsymbol{\sigma}(t)$, $\boldsymbol{\varepsilon}(t)$ are determined from an elastic global structural calculation (FEM);
- **Microscale - Locally Coupled Analysis**: the meso-quantities history is used as the input for the post-processing of the micro-quantities by means of the time integration of the constitutive equations;
  - **First Stage**: elasto-plasticity at the microscale with $D = 0$;
  - **Second Stage**: for $w_s = w_D \Rightarrow$ Damage initiation;
  - **Third Stage**: for $D = D_c \Rightarrow$ Meso-crack initiation.
- Jump-In Cycles Procedure

Some Limitations

- No plasticity is assumed to occur at the grain scale below the macroscopic fatigue limit, which does not correspond to the experimental observations, Cugy (2002);
- The concept of a scalar variable D used to describe damage cannot be explicitly related to a particular physical micromechanism, Monchiet et al. (2006);
- The mechanisms of plasticity and damage are assumed to be associated, which is a relatively strong limitation, Chaboche et al. (2009).

Desmorat's Model (2007)

It **extends Lemaitre's model to include thermal effects** at both micro and meso-scales, being therefore **capable of modeling thermal and/or thermo-mechanical fatigue**.

- **Free energy of the inclusion**:

$$\rho\varphi^{\mu} = \rho\varphi^{\mu}_{e} + \rho\varphi^{\mu}_{p}$$

with $\rho\varphi^{\mu}_{e} = \frac{1}{2}\varepsilon^{\mu}\mathbf{E}\varepsilon^{\mu} + \alpha(\theta - \theta_{ref})tr(\boldsymbol{\sigma}^{\mu})$

- **Free energy of the matrix**:

$$\rho\varphi(\varepsilon) = \frac{1}{2}\varepsilon\mathbf{E}\varepsilon + \alpha(\theta - \theta_{ref})tr(\boldsymbol{\sigma})$$

Eshelby-Kroner Localization Law

- **Strains Relation**

$$\varepsilon^{\mu} = \frac{1}{1-bD}\left(\varepsilon + \frac{(a-b)D}{3(1-aD)} tr(\varepsilon)\mathbf{I} + b(1-D)\varepsilon^{\mu p}\right) + \frac{a((1-D)\alpha^{\mu}-\alpha)}{1-aD}(\theta - \theta_{ref})\mathbf{I}$$

with a and b the Eshelby parameters for a spherical inclusion given by

$$a = \frac{1+\nu}{3(1-\nu)}, \quad b = \frac{2}{15}\frac{4-5\nu}{1-\nu}$$

Main Features

- uses two distinct scalar internal variables to account for mesocrack propagation (d, β);
- considers uncoupled dissipations for plasticity and damage (associative plasticity, non-associative damage);
- uses Lin-Taylor localization law to link micro and meso scales;
- failure criterion $d = d_c$.

Main Features

- **Thermo-mechanical elasto-plastic damage model** in which an **internal fatigue variable** is included to account for fatigue effects;
- **Fatigue life** predicted using $S - N$ **curves** of the material.

- **Free Energy**

$$\begin{aligned}\varphi &= \varphi_e(\mathbf{E}^e, D, \theta) + \varphi_p(\alpha^p, \theta) \\ &= (1 - D)\frac{1}{\rho_0}\mathbf{E}^e\mathbf{C}^0(\theta)\mathbf{E}^e + \varphi^p(\alpha^p, \theta) - \theta\eta\end{aligned}$$

Yield and Damage Potentials

- **Yield Potential**

$$f_y = f(\mathbf{S}) - \overline{K}(\mathbf{S}, \alpha^P) f_{red}(N, S_m, R, \theta) = 0$$

- **Damage Potential**

$$f_D = \overline{S}(\mathbf{S}) - \overline{F}^D(\mathbf{S}, D) f_{red}(N, S_m, R, \theta) = 0$$

where f is an uniaxial equivalent stress function, $\overline{K} f_{red}$ is a strength threshold, $\overline{S}$ is an equivalent stress function in the undamaged space and $\overline{F}^D f_{red}$ is a damage threshold. f_{red} represents a reduction function.

Reduction Function

- The scalar function f_{red}, regarded as an internal variable, is defined by

$$f_{red}(N, S_m, R, \theta) = f_N(N, S_m, R) f_\theta(\theta)$$

  where $f_N(N, S_{max}, R)$ is the mechanical reduction function, influenced by the number of cycles N, and $f_\theta(\theta)$ is the thermal reduction function. S_m represents the mean stress;
- The elastic domain can be modified independently either by a thermo-mechanical load or by a reduction factor related to the cumulative degradation of the fatigue strength;
- The reduction function is determined from $S - N$ curves obtained for different stress ratios.

Main Features

- **Endurance Surface** (in the stress space)

$$\beta = \frac{1}{\sigma_{oe}}(\sigma_{eq} + Atr(\boldsymbol{\sigma}) - \sigma_{oe}) = 0$$

with $\sigma_{eq} = \left(\frac{3}{2}(\boldsymbol{\sigma}^D - \boldsymbol{\alpha}) : (\boldsymbol{\sigma}^D - \boldsymbol{\alpha})\right)^{\frac{1}{2}}$

  - σ_{oe} represents the endurance limit for zero mean stress; A determines the influence of the mean stress in σ_{oe};
  - stress states inside the surface ($\beta < 0$) do not lead to fatigue damage development;
  - fatigue damage may develop for stress states outside the surface ($\beta > 0$);
  - the surface may move in the stress space as function of the load history.

Evolutions

- **Back-Stress Tensor Evolution**

$$d\boldsymbol{\alpha} = d\beta C(\boldsymbol{\sigma}^D - \boldsymbol{\alpha})$$

- **Damage Evolution**

$$dD = d\beta\ g(\beta, D), \quad \text{with} \quad g(\beta \geq 0, D) \geq 0$$

$$g(\beta) = Ke^{L\beta}$$

- **Failure Criterion**: $D = 1$

Some Conclusions Regarding HCF Modeling of Metals

- Damage and plasticity take place on a scale lower than the scale of the RVE;
- Failure can be well characterized by the Theory of Damage Mechanics;
- Multi-scale models based on the Theory of Damage Mechanics are suitable;
- A single internal scalar variable might not be enough for mesocrack modeling;
- Microplasticity should be determined assuming both isotropic and kinematic hardening effects;
- Mean stress effect must be considered.

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