Form and structural optimization: from beam modeling to 3D printing of reinforced concrete members

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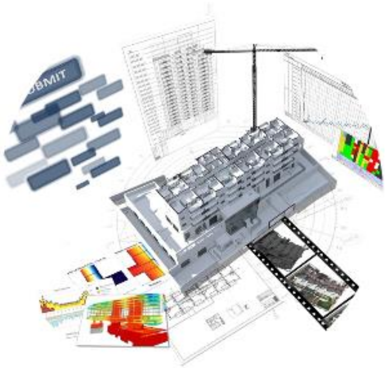
ARCHITECTURE & ENGINEERING FIELDS

Growing demand for designing complex and ambitious buildings

OBJECTIVES



RESEARCH

Problem and motivations

OPEN CHALLENGE:

Design, optimization and manufacturing of **structural elements with curved/ non-prismatic shapes**

Usage motivations:

- ⋆ aesthetic
- ⋆ functional
- ⋆ structural

} ⇒ economical

PROBLEMS

- DESIGN: non-prismatic elements behave differently from prismatic ones
  ⟶ correct modelling strategy

  COMMERCIAL SOFTWARE/ NUMERICAL TOOLS capabilities

- MANUFACTURING: to enable freedom in shape reducing costs and time (material, labour equipment, ..)

  INNOVATIVE TECHNOLOGIES VS TRADITIONAL CONSTRUCTION METHODS (FORMWORK SYSTEMS..)

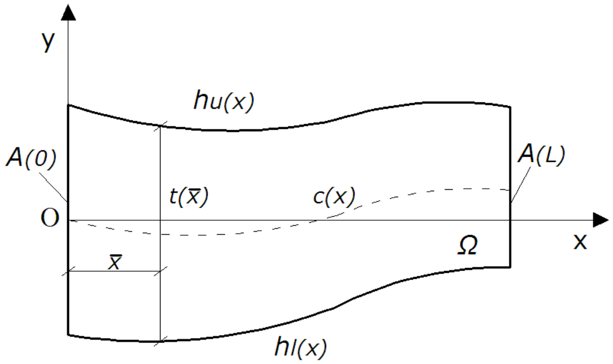
RESEARCH

MODELING OF NON-PRISMATIC ELEMENTS

- Implementation of an accurate non-prismatic beam model (NP-Model) and its comparison with conventional building software in real modeling problems

MANUFACTURING AND OPTIMIZATION METHODS

- To propose an innovative 3D printing method for the production of Reinforced Concrete (RC) non-prismatic elements and possible compatible topology optimization tools



The governing differential equations of non-prismatic beams are characterized by variable coefficients ⟶ difficulties in the exact integration of the solution

NON-PRISMATIC BEAM BEHAVIOR:

- Strong coupling between internal forces
- Modification of boundary equilibrium
- Non-trivial stress distribution
- Complex constitutive relations

⇨ Conventional Euler-Bernoulli and Timoshenko beam theories are NO longer valid!

BASIC (POOR) MODELING APPROACHES:

- Timoshenko beam + variable coefficients (area, inertia)
- Stepped FE
- Methods starting from prismatic beam theories

Adopted in advanced and recent literature, in design manuals/codes and in FE commercial software (e.g., SAP2000, R-STAB, STRAUS7)

GOAL= To evaluate **accuracy** of **commercial software** compared to an **accurate literature model** in real design problems

Software SAP2000 VS NP-Model

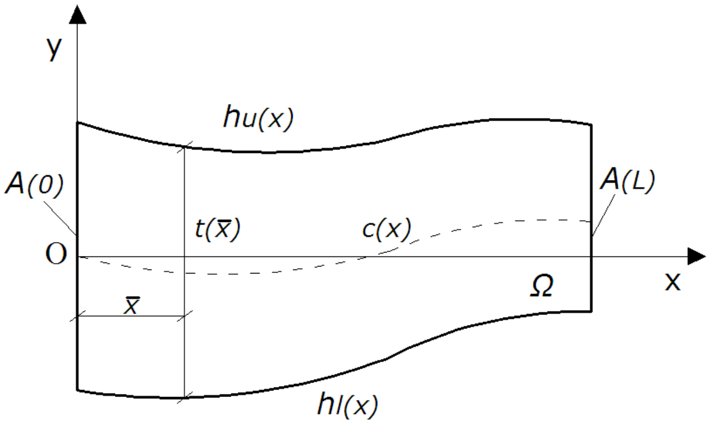
2D Non prismatic beam model - NP-Model

References: Auricchio et al. [2010], Balduzzi [2013] and Beltempo et al. [2015]

The approach adopted for the model derivation is the so-called **dimensional reduction** starting from the **Hellinger–Reissner functional**

Strenghts of the NP-Model:

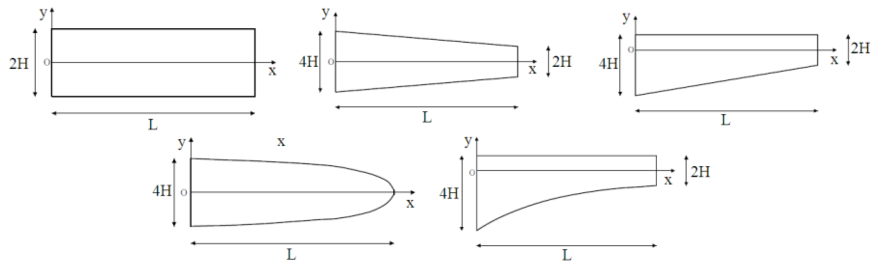
- Respect of the coupling effect
- Respect of the boundary equilibrium at the surfaces
- Generic non-prismatic geometry
- Ease of implementation



Steps of the work

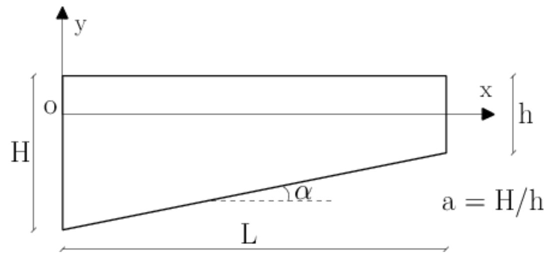
➢ Implementation of the NP-Model and validation

Several geometries tested

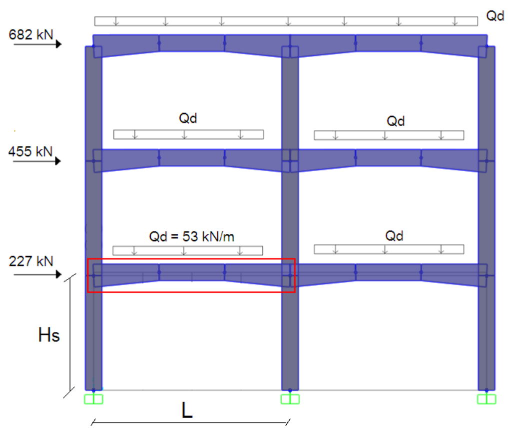


➢ Numerical examples: comparison between the NP-Model and SAP2000

Problem at the *element-scale*

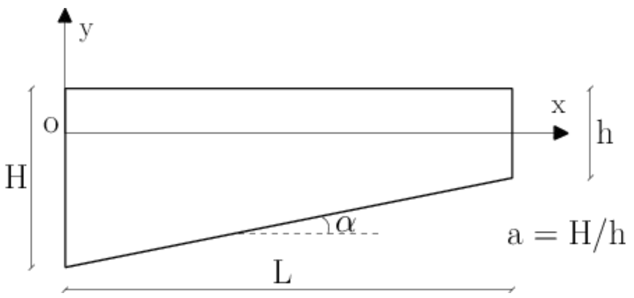


Problem at the *frame-scale*



NUMERICAL EXAMPLES

Problem at the *element-scale*



Stiffness matrix rel. error

$$(k_e r)_{i,j} = \frac{\left| K_{Ref_{i,j}} - K_{ABQ_{i,j}} \right|}{\left| K_{ABQ_{i,j}} \right|}$$

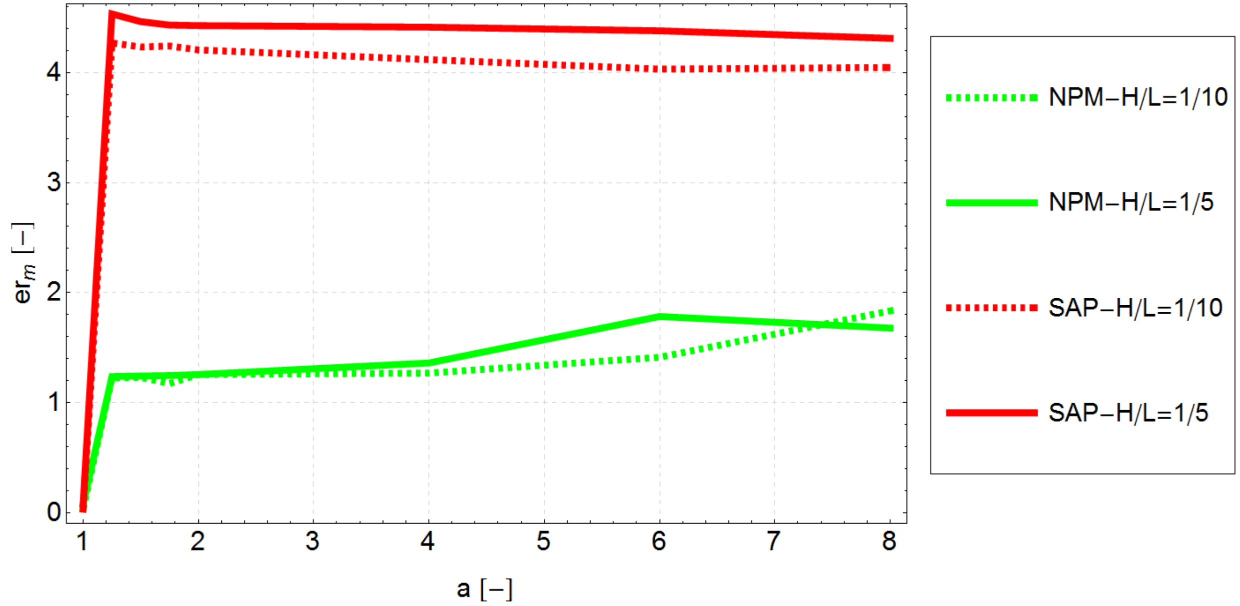
Stiffness matrix avg error

$$e_{rm} = \sum_{i,j=1...N} (k_{er})_{i,j}$$

Parametric study on stiffness matrix average error

- SAP2000 model approximation: Stepped FE + variable coefficients
- ABAQUS overkilled FEA = reference solution

SAP2000 VS NP-Model

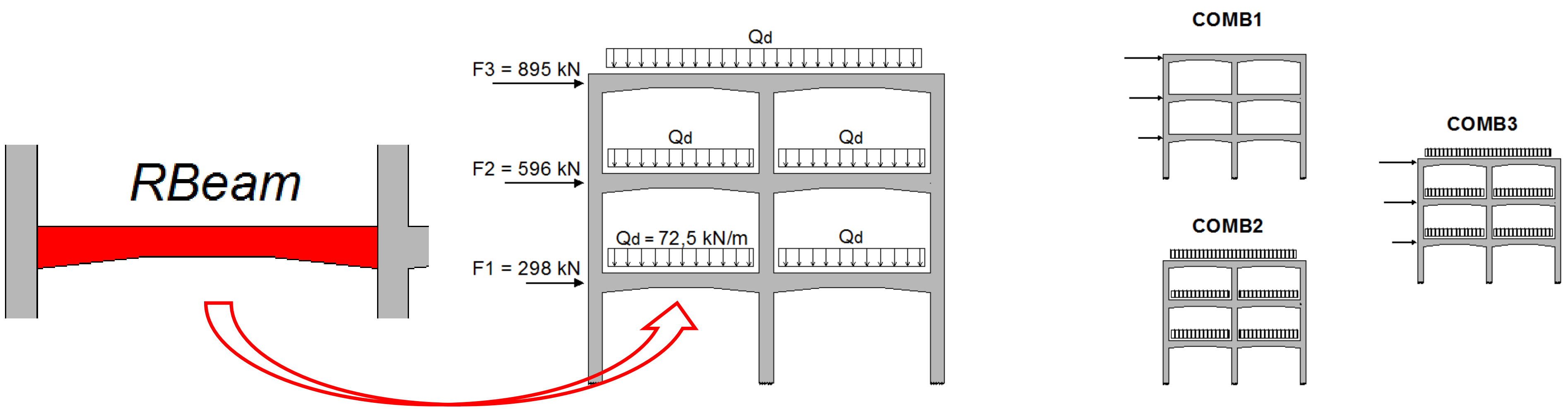


SAP2000 error is about four times greater

NUMERICAL EXAMPLES

Problem at the *frame-scale*

2D FRAME WITH RC HAUNCHED BEAMS



- Comparison of SAP2000 and NP-Model results:
  - Internal forces
  - Displacements and rotations
  - Stress
- ABAQUS overkilled FEA = reference solution

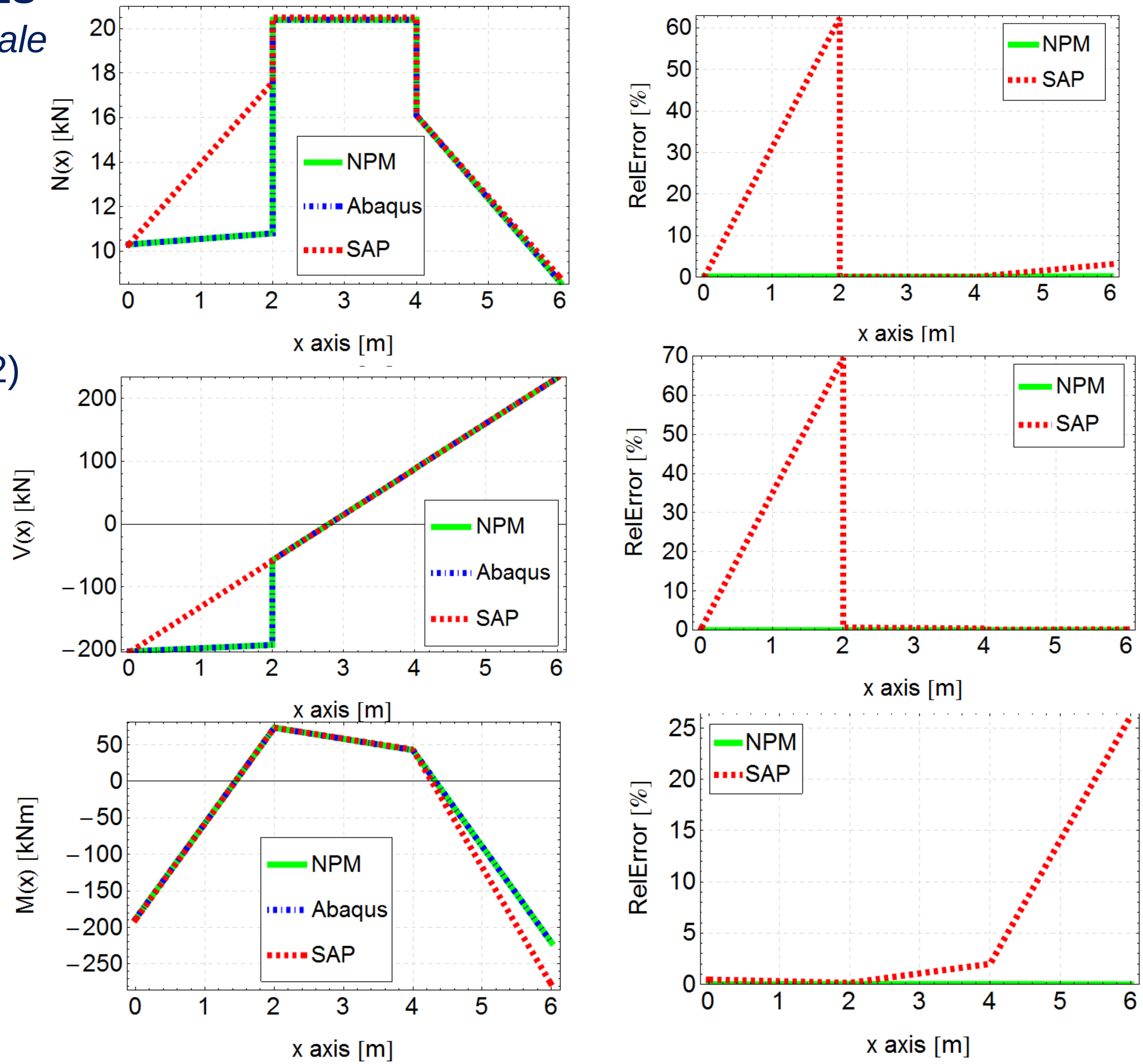
NUMERICAL EXAMPLES

Problem at the *frame-scale*

$$RelError = \frac{|q - q_{ABQ}|}{|q_{ABQ}|}$$

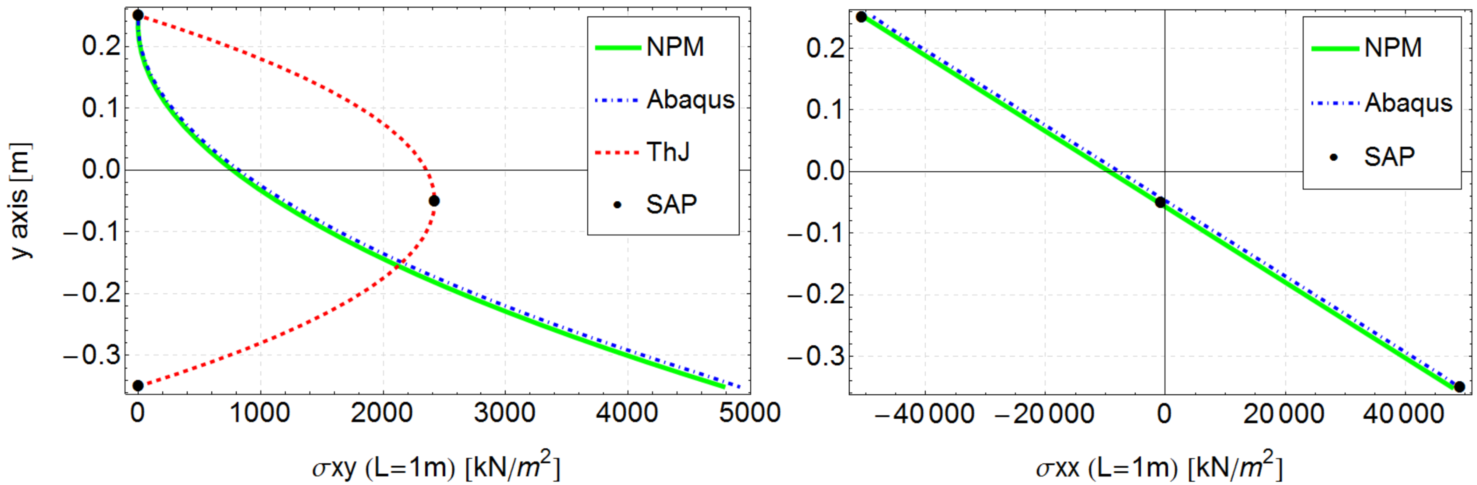
Internal forces (COMB2)

- very good correlation between NP-Model and Abaqus results
- Considerable errors in SAP2000 results (25%-70%)



NUMERICAL EXAMPLES
Problem at the *frame-scale*

-Stress (COMB3)



- The σ_{xy} recovered for NP-Model agrees very well with Abaqus, while SAP2000 traces the conventional Jouransky parabolic distribution valid for prismatic cross-sections.

-Displacements and rotations

- Good correlation between output obtained with the NP-Model and SAP2000
- SAP2000 errors more significant for rotations

SAP2000 present greater errors of approximation compared to the NP-Model

Accuracy of the **modeling approach** is of **crucial importance** especially when **non-trivial problems** have to be handled!!

3D PRINTING of RC MEMBERS

Goal of the activity

Novel approach for the fabrication of **reinforced concrete** (RC) **members** based on **3D printing** technology of concrete

Project Partners

3D PRINTING PROCESS

SHAPE/TOPOLOGY OPTIMIZATION

DESIGN CONCEPT

MATERIAL OPTIMIZATION

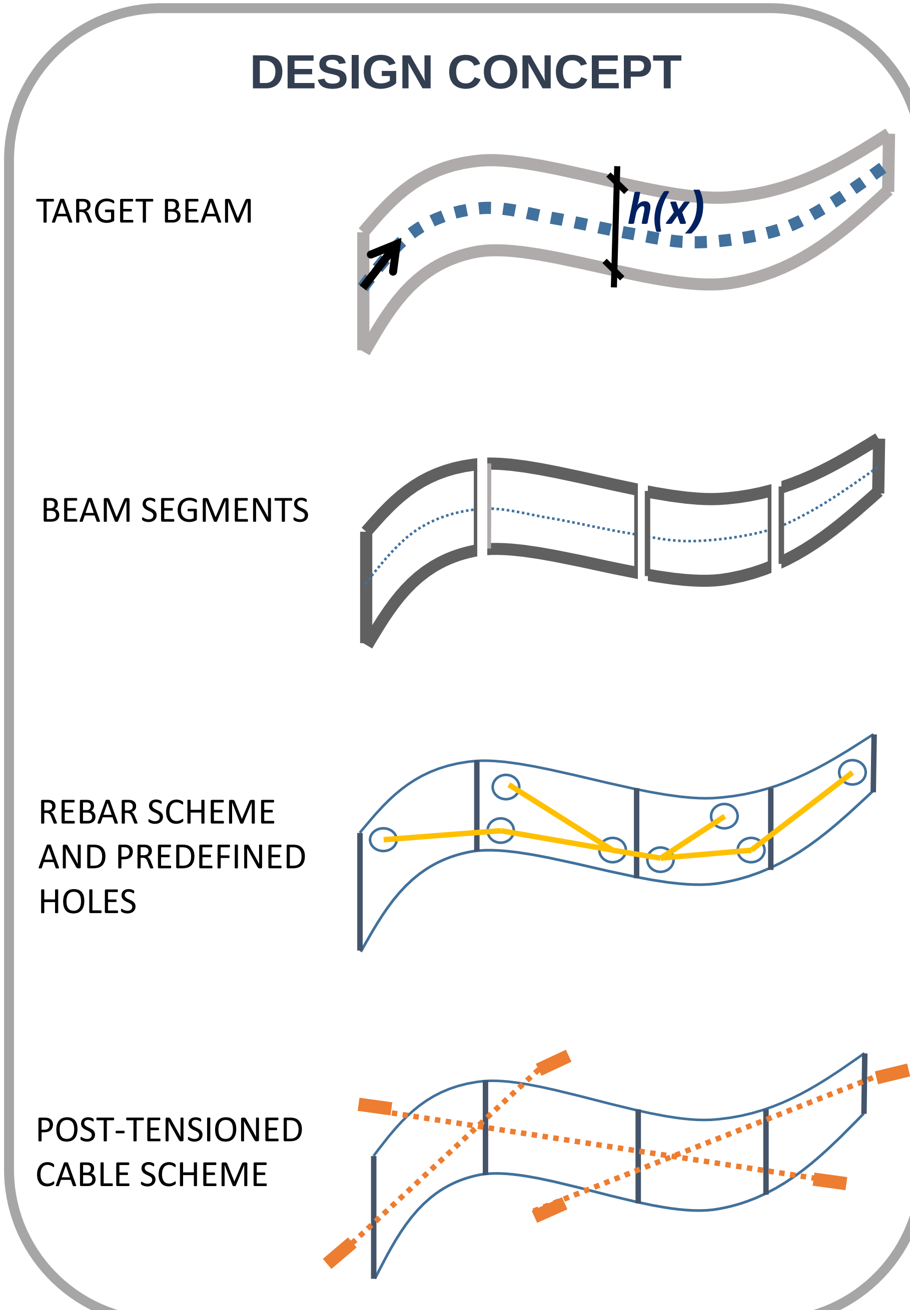
CONCRETE MODULUS

EXTERNALLY/POST APPLIED REBAR SYSTEM

3D PRINTING of RC MEMBERS

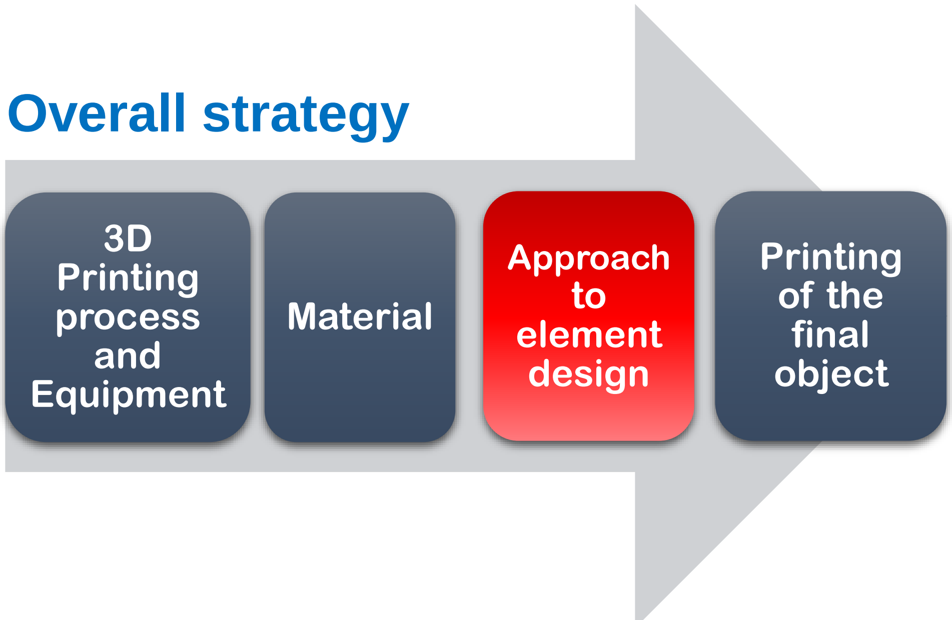
Overall strategy

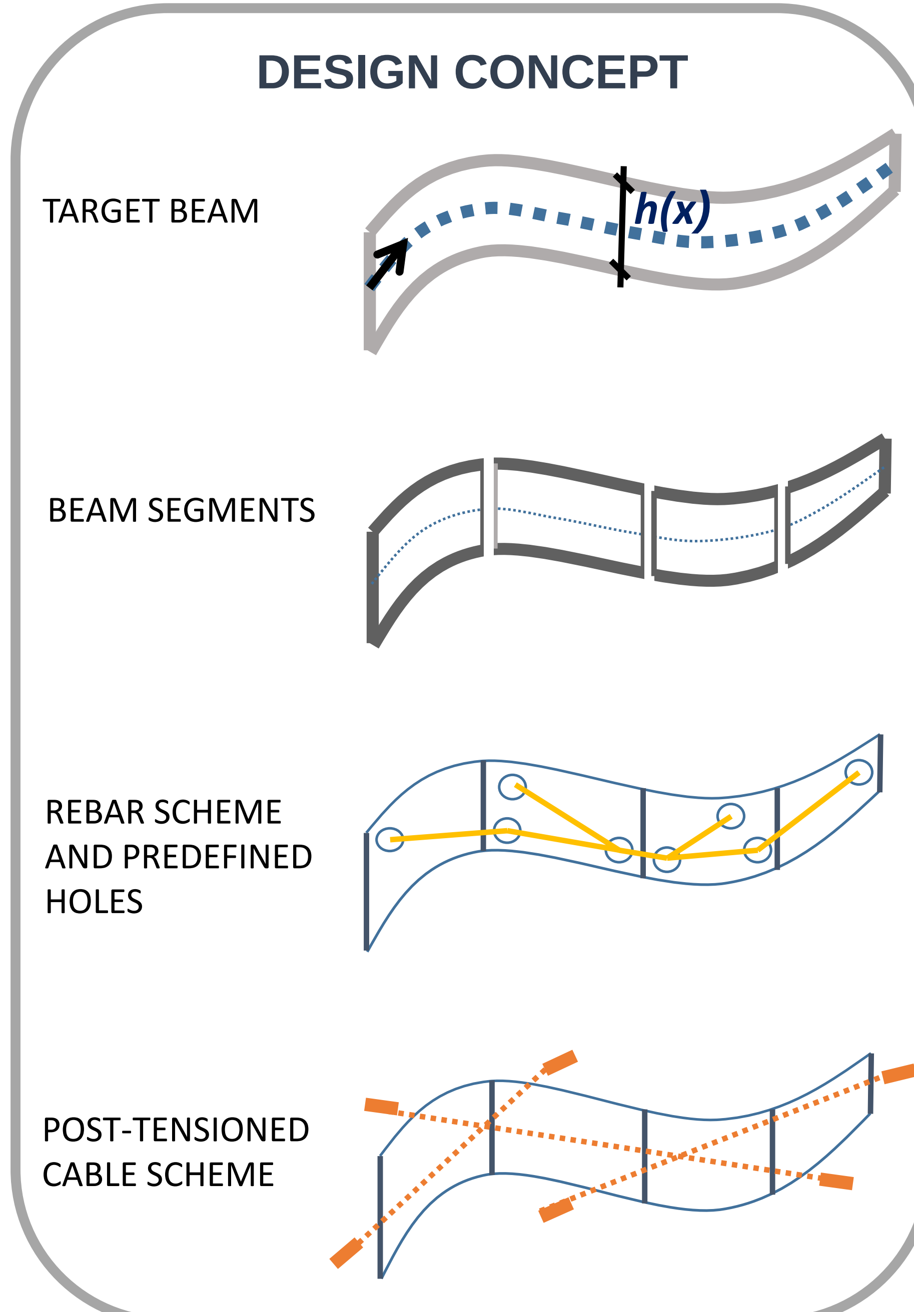
- 3D Printing process and Equipment
- Material
- Approach to element design
- Printing of the final object



3D PRINTING of RC MEMBERS

Overall strategy



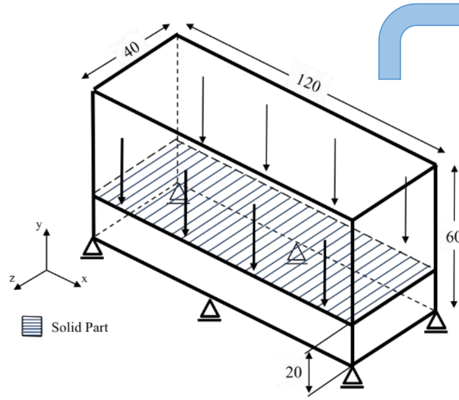


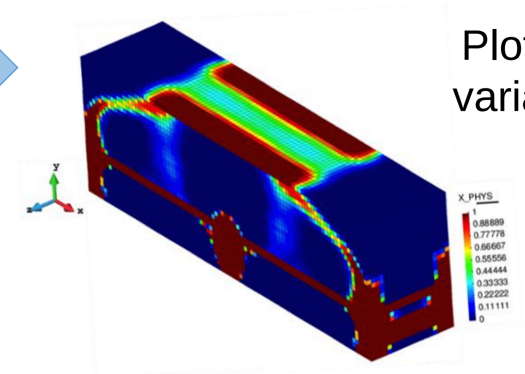
Topology optimization

Topology optimization represents a fundamental step for the development of a complete 3D-printing reinforced concrete framework

General problem

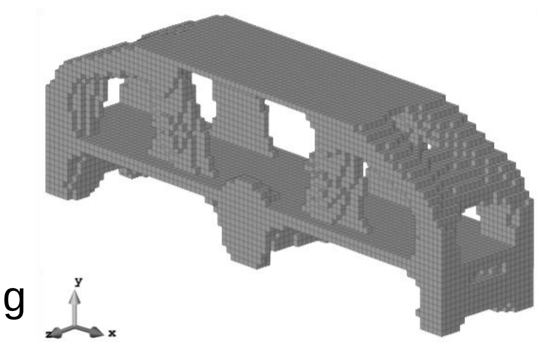
$$\begin{cases} min\, f(\boldsymbol{x}) \\ s.t. \quad g(\boldsymbol{x}) \leq 0 \\ \boldsymbol{x}_{min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{max} \end{cases}$$



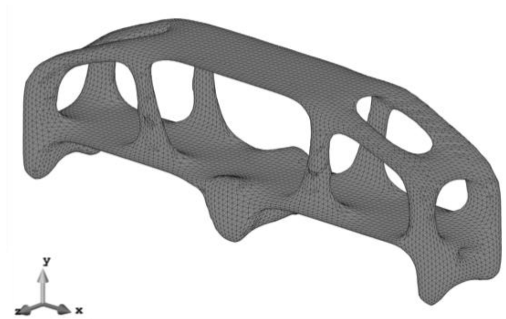


Plot of the design variable (example: density)

Topology Optimization flow



Post-processing



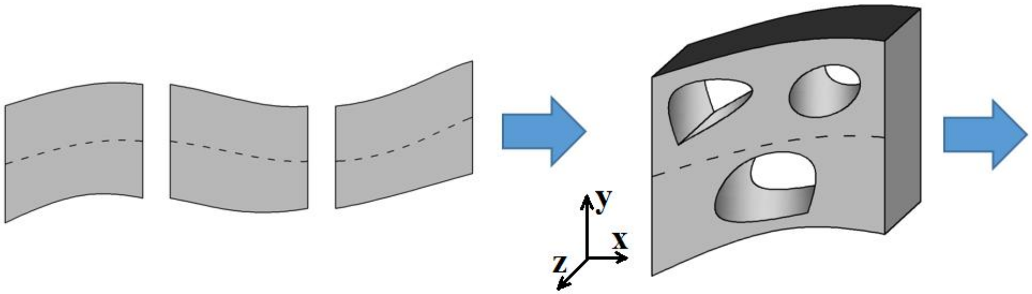
Stl file

3D PRINTING

Ref.: Octaviano Malfavon Farìas, Master Thesis

NEVERTHELESS…

The application of classical optimization strategies to concrete 3D Printing is not straightforward!



IMPORTANT ASPECTS

- ✓ *Topology optimization problem*
  - Stress-constraint problem
- ✓ *Stages of the design process*
  - Pre-post processing
- ✓ *Printing material*
  - *Concrete – No Von Mises stress*
- ✓ *Technology peculiarities*
  - *Extrusion constraints*

To find optimization strategies aligned with the proposed 3D printing approach

OPEN-SOURCE SOFTWARE

KRATOS MULTI-PHYSICS

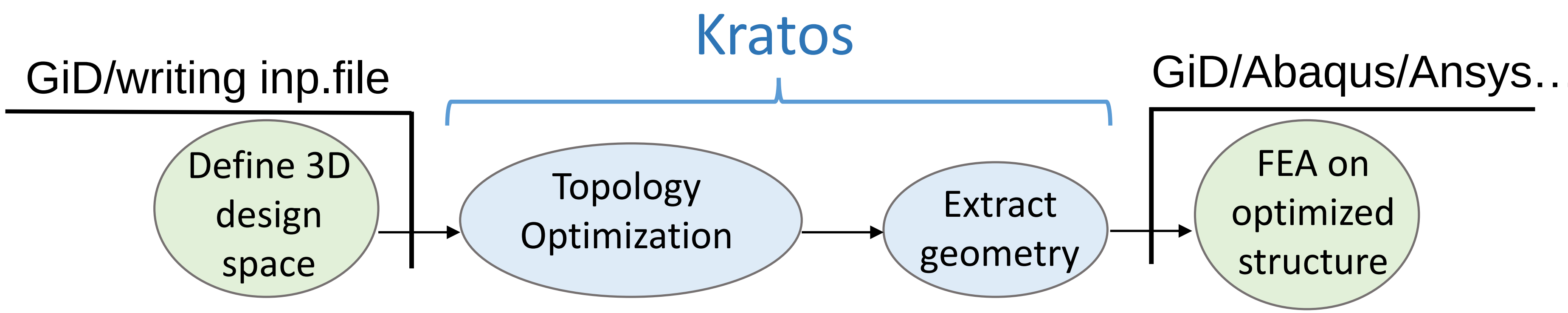
NEW OPTIMIZATION ALGORITHM

MATLAB CODE

TECHNISCHE UNIVERSITÄT MÜNCHEN

KRATOS TopOpt application

Reference: O. Sigmund [2001]



COMPLIANCE minimization problem

$$\min x_e : c(x_e) = F^T u = u^T K u = \sum_{e=1}^{N} E_e(x_e) u_e^T k_0 u_e$$

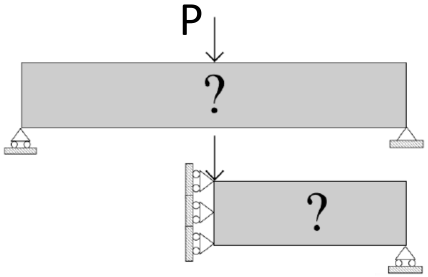
Subject to: $\begin{cases} \dfrac{V(x)}{V_0} = f \\ Ku = F \qquad 0 \le x_e \le 1 \end{cases}$

Design variable
x = density vector **ρ**

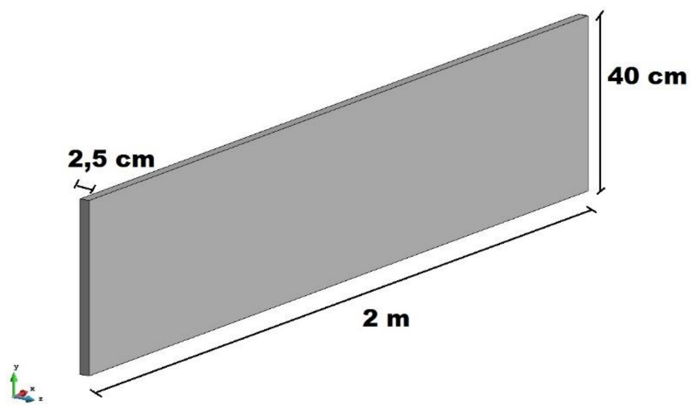
- **SIMP** approach: $E_e(x_e) = E_{min} + x_e^P (E_0 - E_{min}), \quad x_e \in [0,1]$ ➡ “VOID-SOLID” SOLUTION (P=3÷4)
- Gradient based approach: Adjoint method for sensitivity analysis
- Optimization algorithm: Optimality Criteria (OC) method

KRATOS TopOpt application

TEST 1: PRISMATIC (MBB) BEAM



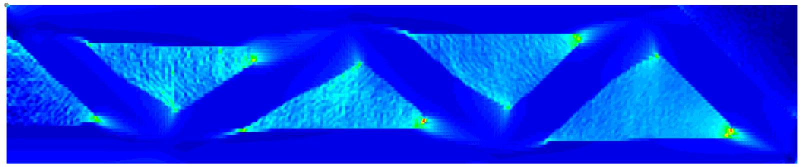
- Material properties: CONCRETE
- P_layer = 60 N



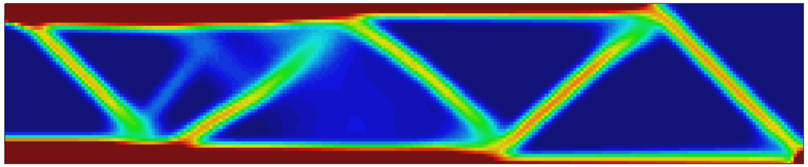
“Slice” modeling of the initial beam in order to have constant cross section along z-axis (≈ 2D)
Udispl_z = 0 for all nodes

V fraction = 0,3
Abs.Obj $c(x)$= -79,24 %
T tot = 2624 s

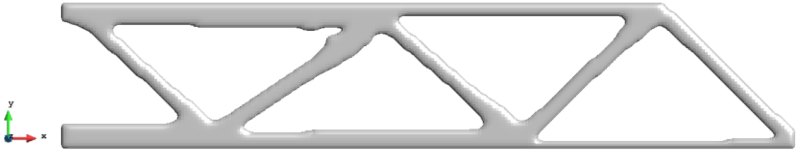
Contour plot sigma Von Mises: σmax = 2,51 MPa



Contour plot density: from 0 (blue) to 1 (red)

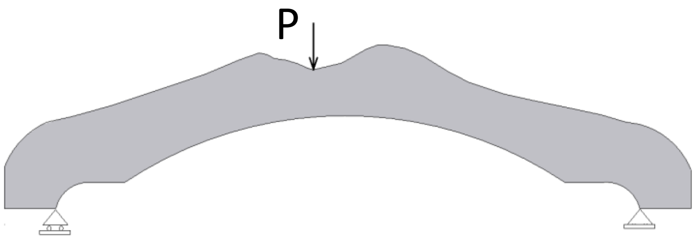


Extracted geometry
X_PHYS threshold = 0,3

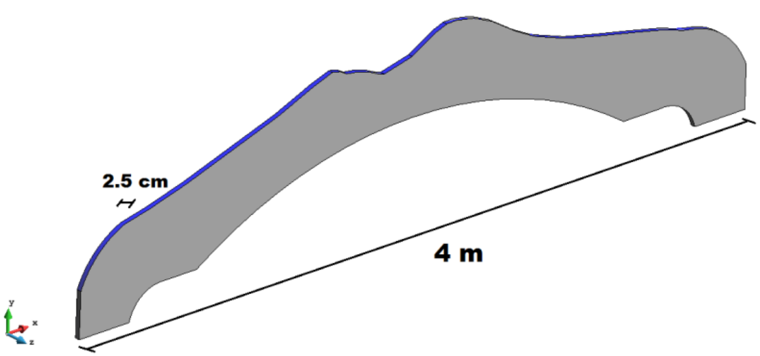


KRATOS TopOpt application

TEST 2: VESUVIO BEAM

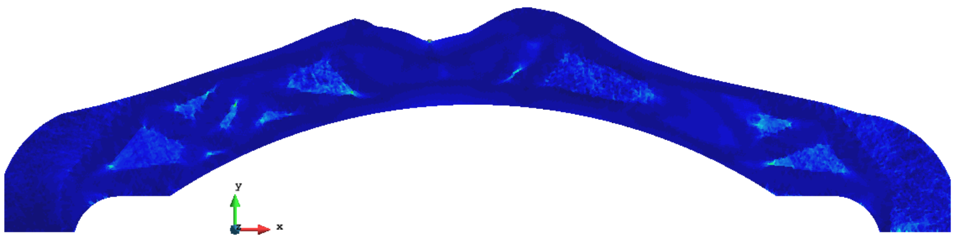


- Material properties: CONCRETE
- P_layer = 60 N
- “Slice” modeling

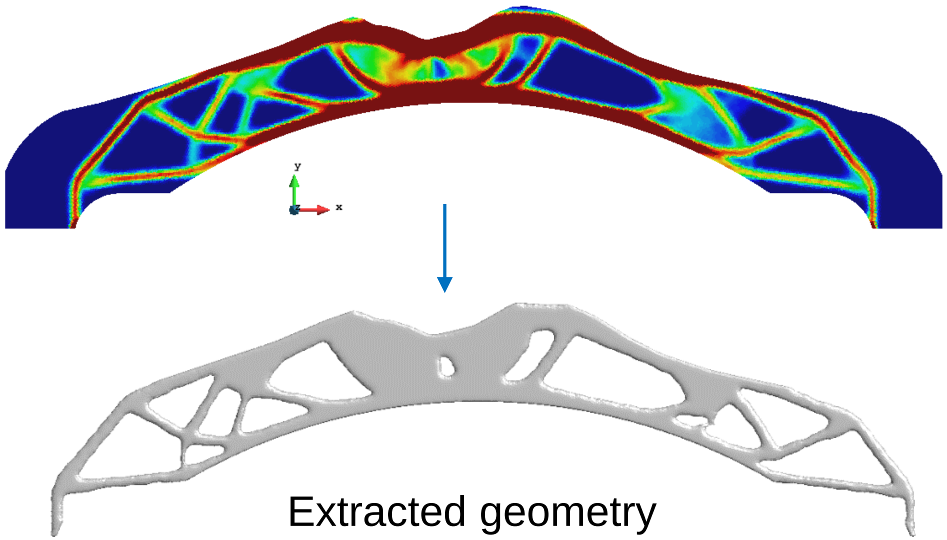


V fraction = 0,4
Abs.Obj $c(x)$= -37,78 %
T tot =15478 s

Contour plot sigma Von Mises: σmax = 1,41 MPa



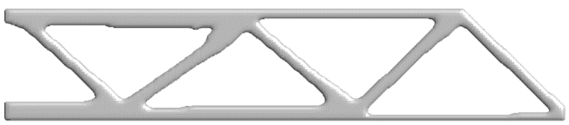
Contour plot density: from 0 (blue) to 1 (red)



Extracted geometry
X_PHYS threshold = 0,4

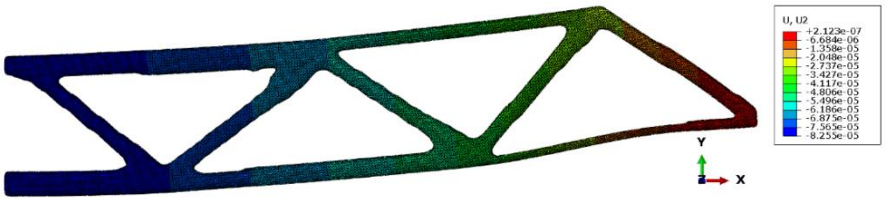
KRATOS TopOpt application

Extracted geometry



→ **STL File** →

Post-processing



ABAQUS

PROS

✓ *Stages of the design process* OK!

- Capability in handling complex 3D problems
- Open source software

CONS

✓ *Topology optimization problem* X

✓ *Printing material* X

✓ *Technology peculiarities* X

Matlab code: PSTOpt algorithm

Reference: Biyikli et al. [2015]
(Proportional Topology Optimization approach)

MASS minimization problem

$$\min \sum_{i=1}^{N} x_e$$

Design variable
x = density vector **ρ**

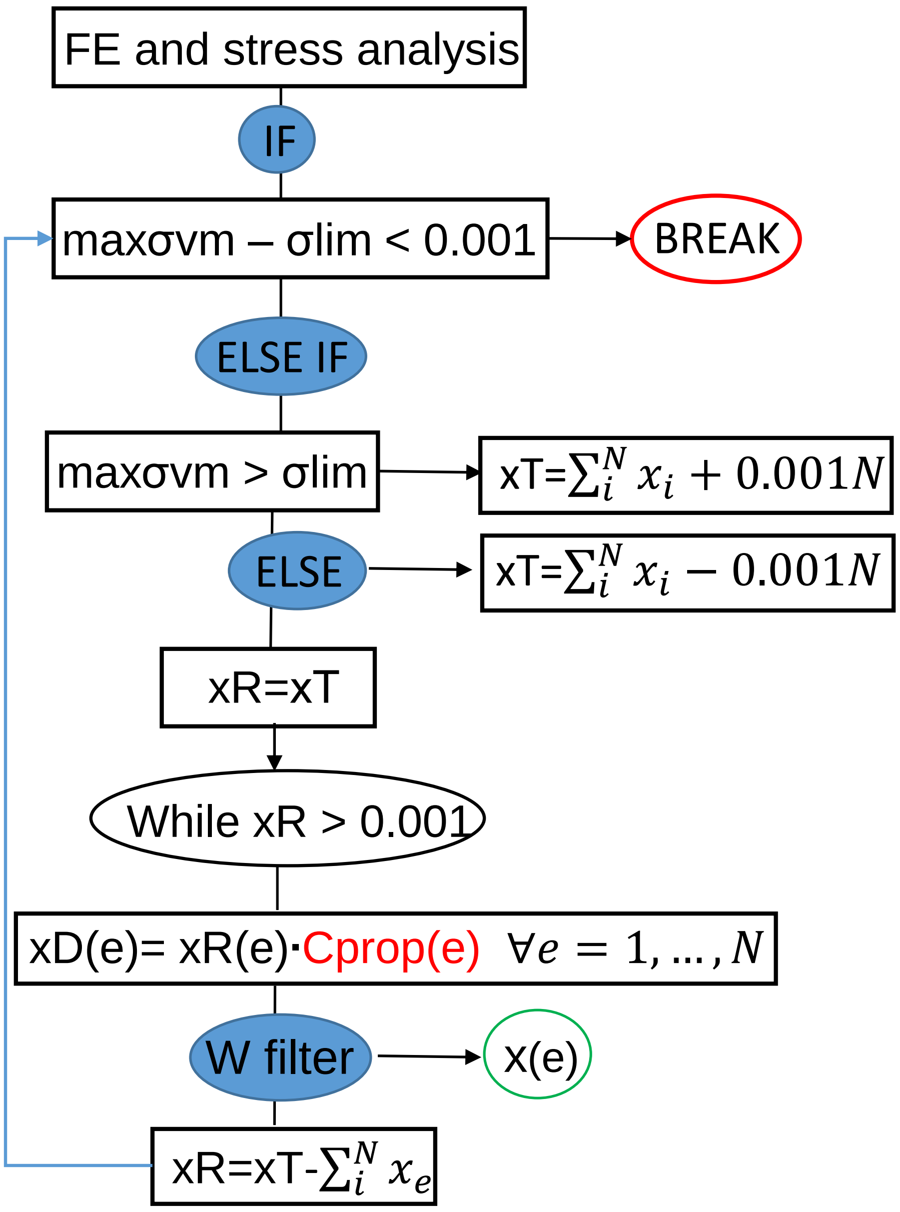
Subject to:

$$\begin{cases} \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \\ \sigma_e < \sigma_{lim} \\ 0 < x_{min} \leq x \leq 1 \end{cases}$$

- **SIMP** approach: $E_e(x_e) = E_{min} + x_e^P(E_0 - E_{min}), \quad x_e \in [0,1]$
- Heuristic approach
- Optimization algorithm: Proportional distribution

Design variables assigned to elements proportionally to the value of stress

Original algorithm



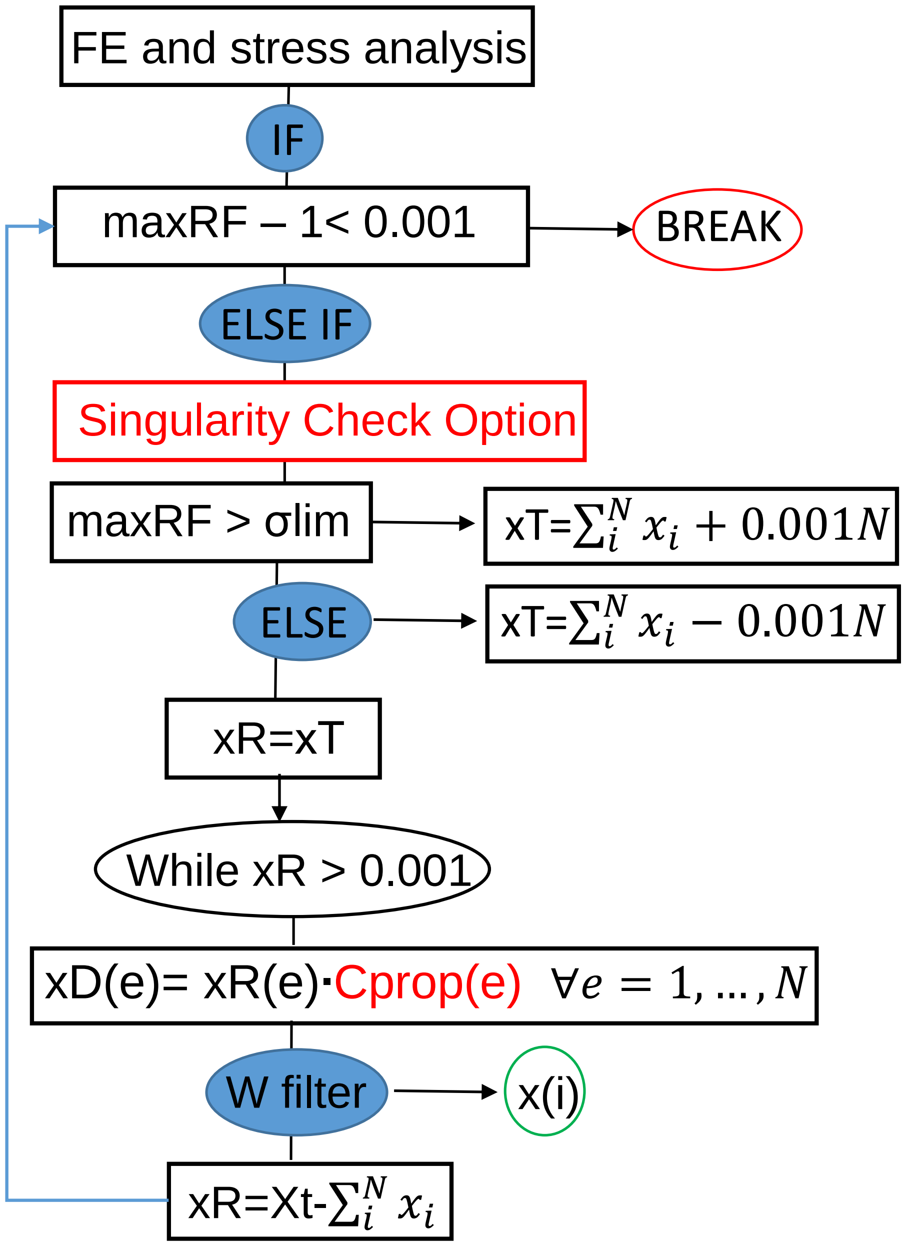
Reference: Biyikli et al. [2015]
(Proportional Topology Optimization approach)

Constraint on the
VON MISES STRESSES

$$Cprop_e = \frac{\sigma vm_e^q}{\sum_e^N \sigma vm_e^q}$$

q = proportion exponent

New PSTOpt algorithm



Constraint on the PRINCIPAL STRESSES

$$\sigma_- \leq \sigma_x \leq \sigma_+$$

$$\sigma_- \leq \sigma_y \leq \sigma_+$$

RISK FACTORS

$$RF_1 = \left(\frac{\sigma_x}{\sigma_+}, \frac{\sigma_x}{\sigma_-}\right)$$

$$RF_2 = \left(\frac{\sigma_y}{\sigma_+}, \frac{\sigma_y}{\sigma_-}\right)$$

$$Cprop(e) = \frac{1}{2}\left[\frac{(RF_1^q)_{(e)}}{\sum_1^N RF_{(1)}^q} + \frac{(RF_2^q)_{(e)}}{\sum_1^N RF_{(2)}^q}\right]$$

q = proportion exponent

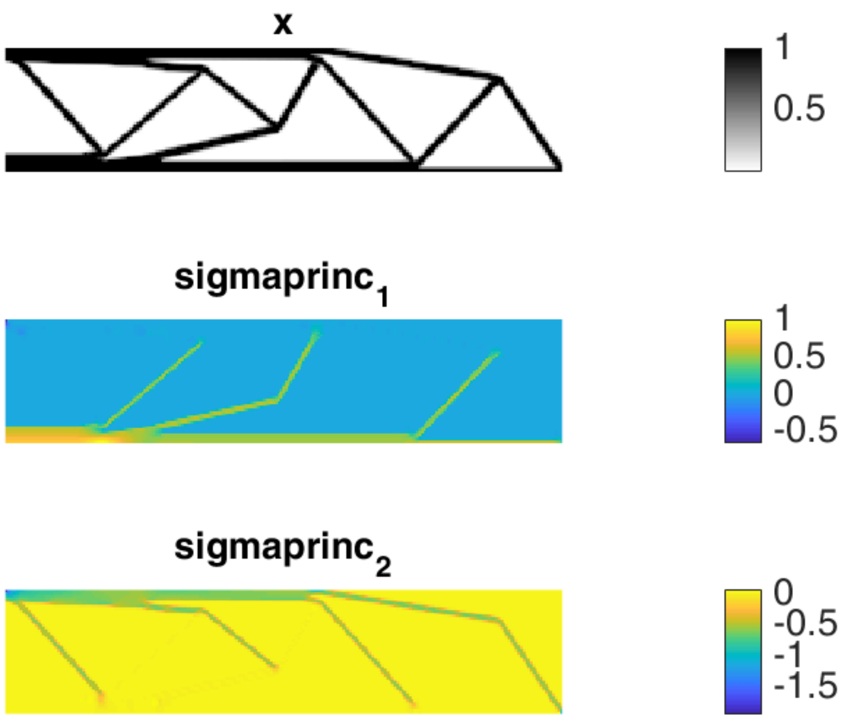
New PSTOpt algorithm

TEST 1: CONCRETE MBB BEAM

- Mesh = 225x50

- Material properties: CONCRETE
- Limit values for the principal stress:

$$\sigma_{-} = -20\ N/mm^2$$
$$\sigma_{+} = 1\ N/mm^2$$



RESULTS:

N iterations = 616, Time = 324 s, Avg_ρ = 0,26

u_y	$max\sigma_{11}$	$max\sigma_{12}$	$max\sigma_{22}$
-0,30 mm	1,00 Mpa	0,42 Mpa	0,50 Mpa

$max\sigma_{princ}$	$min\sigma_{princ}$
1,00 Mpa	-1,93 Mpa

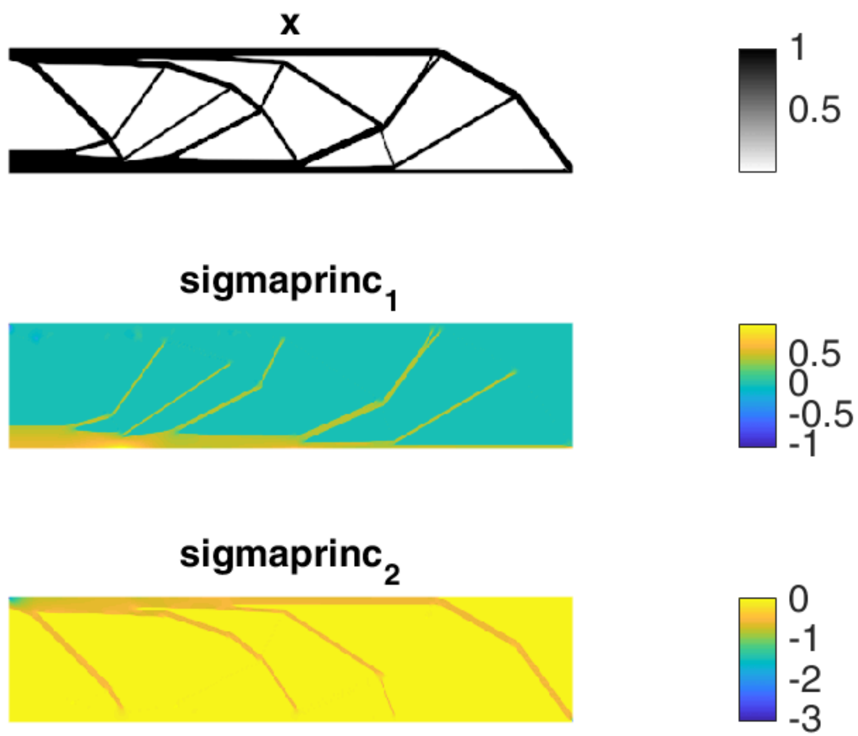
New PSTOpt algorithm

TEST 2: CONCRETE MBB BEAM

- Mesh = 450x100

- Material properties: CONCRETE
- Limit values for the principal stress:

$$\sigma_{-} = -20\ N/mm^2$$
$$\sigma_{+} = 1\ N/mm^2$$



RESULTS:

N iterations = 601, Time = 602 s, Avg_ρ = 0,26

u_y	$max\sigma_{11}$	$max\sigma_{12}$	$max\sigma_{22}$
-0,27 mm	1,00 Mpa	0,42 Mpa	0,98 Mpa

$max\sigma_{princ}$	$min\sigma_{princ}$
1,00 Mpa	-3,05 Mpa

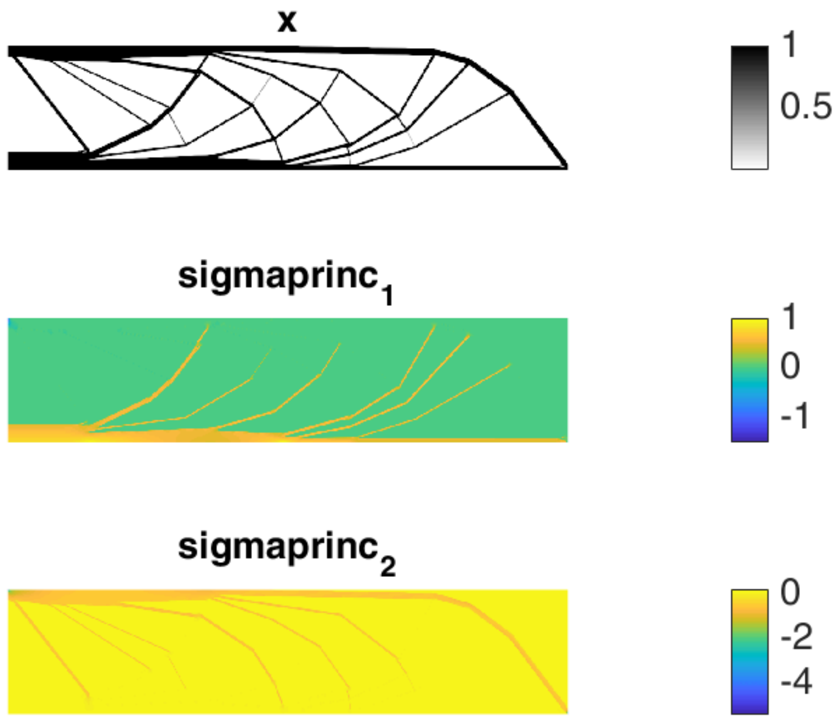
New PSTOpt algorithm

TEST 3: CONCRETE MBB BEAM

- Mesh = 900x200

- Material properties: CONCRETE
- Limit values for the principal stress:

$$\sigma_- = -20\ N/mm^2$$
$$\sigma_+ = 1\ N/mm^2$$



RESULTS:

N iterations = 639, Time = 2239 s, Avg_ρ = 0,22

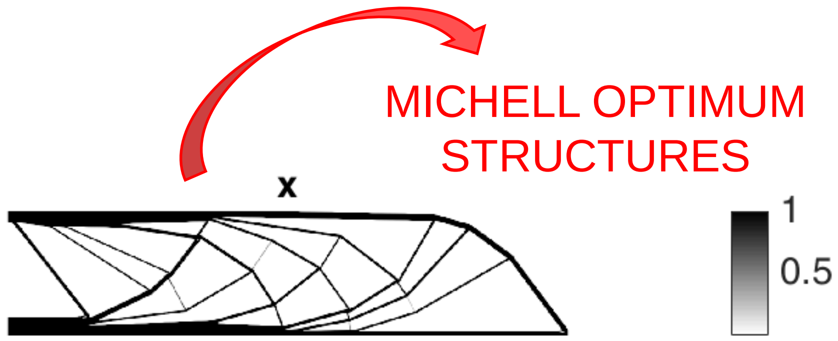
u_y	$max\sigma_{11}$	$max\sigma_{12}$	$max\sigma_{22}$
-0,32 mm	1,35 Mpa	0,56 Mpa	1,95 Mpa

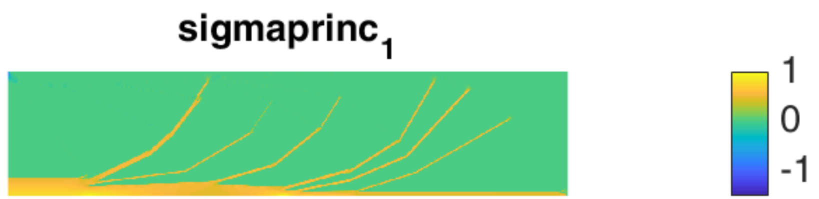
$max\sigma_{princ}$	$min\sigma_{princ}$
1,00 Mpa	-5,42 Mpa

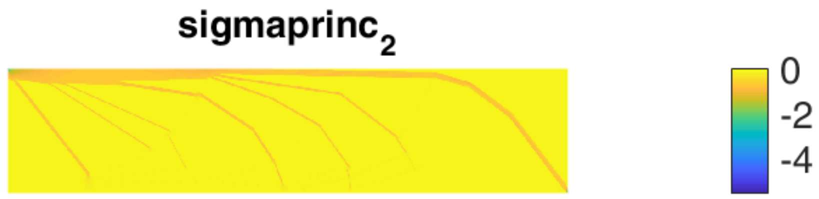
New PSTOpt algorithm

TEST 3: CONCRETE MBB BEAM

- Mesh = 900x200







- Material properties: CONCRETE
- Limit values for the principal stress:

$$\sigma_{-} = -20\ N/mm^2$$
$$\sigma_{+} = 1\ N/mm^2$$

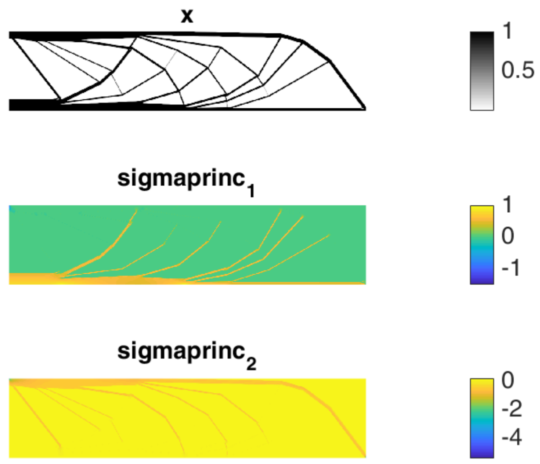
RESULTS:

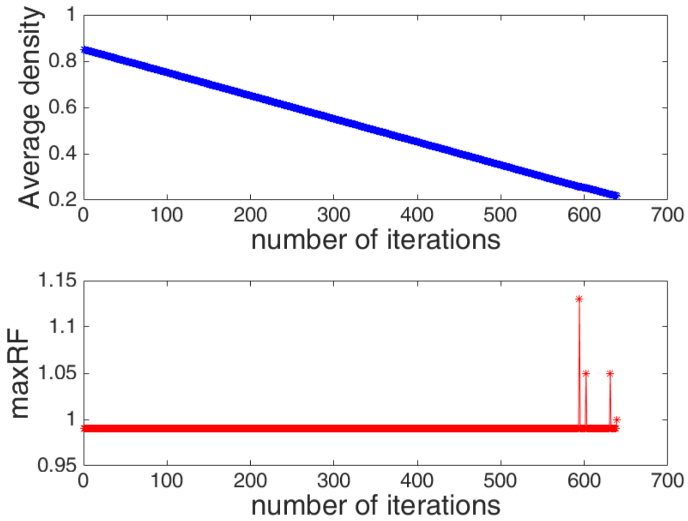
N iterations = 639, Time = 2239 s, Avg_ρ = 0,22

u_y	$max\sigma_{11}$	$max\sigma_{12}$	$max\sigma_{22}$
-0,32 mm	1,35 Mpa	0,56 Mpa	1,95 Mpa

$max\sigma_{princ}$	$min\sigma_{princ}$
1,00 Mpa	-5,42 Mpa

New PSTOpt algorithm





PROS

✓ *Topology optimization problem* OK!

✓ *Printing material* OK!

- Simplicity

CONS

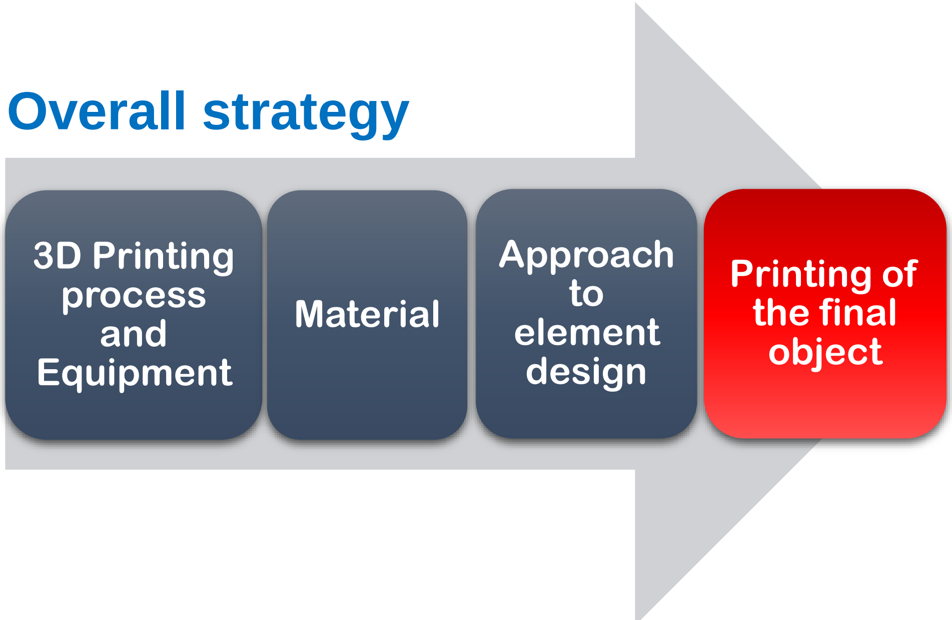
✓ *Stages of the design process* ✗

✓ *Technology peculiarities* ✗

- Only 2D simple problems
- Improvements in the optimal criteria needed

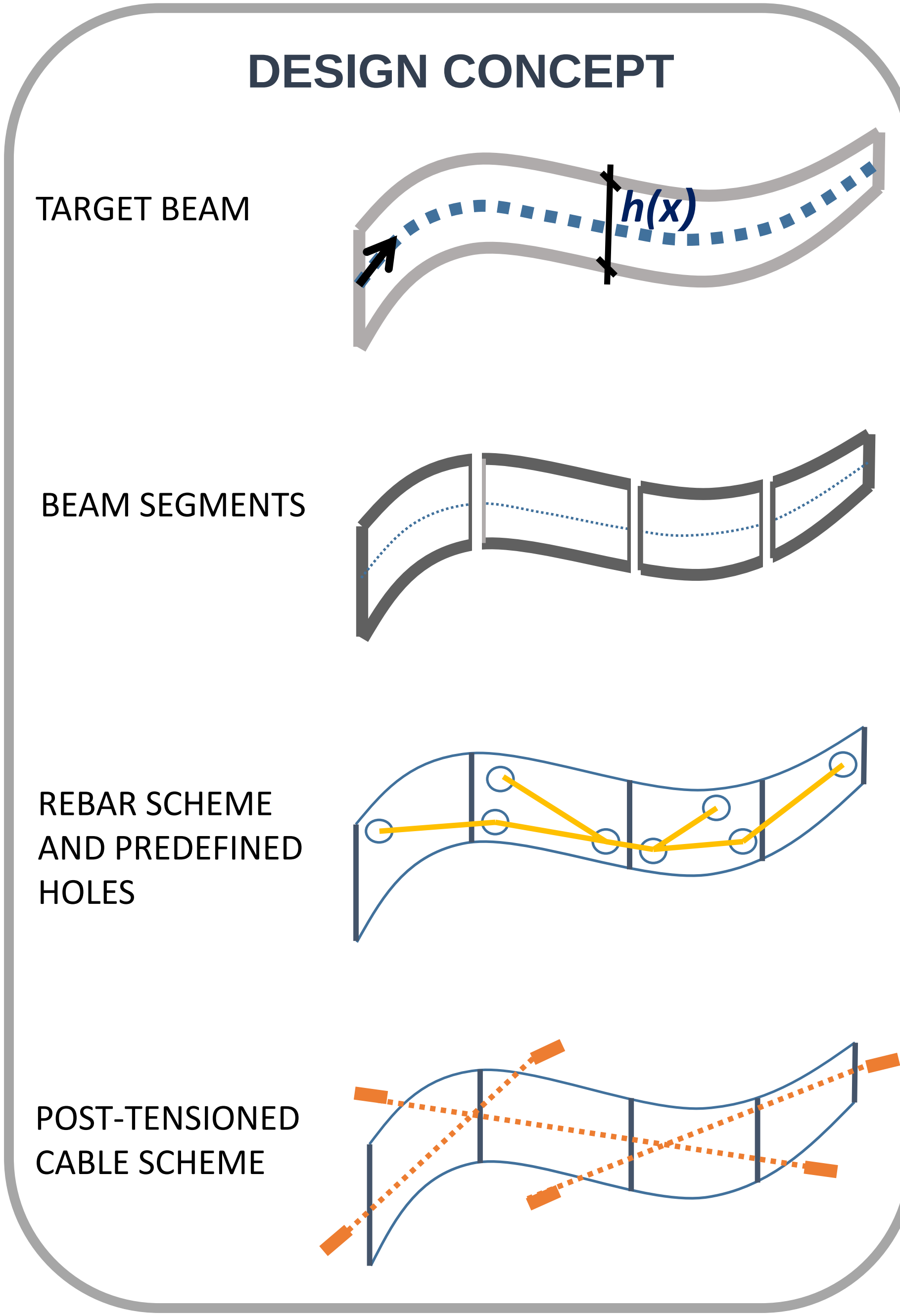
3D PRINTING of RC MEMBERS

Overall strategy



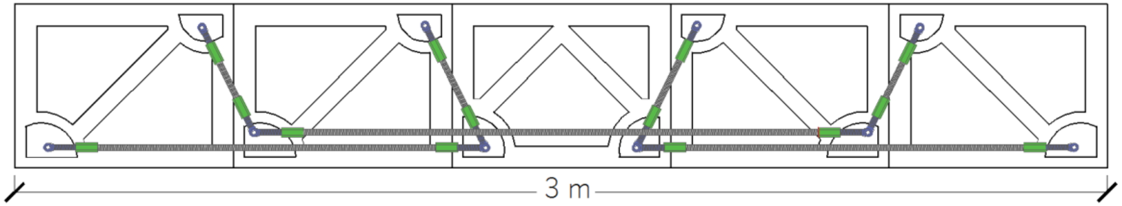
- Topology optimization

Preliminary results:
- Curved RC beam – “Vesuvio Beam”
- **Straight RC beam**
  - Experimental test
  - Numerical Analysis

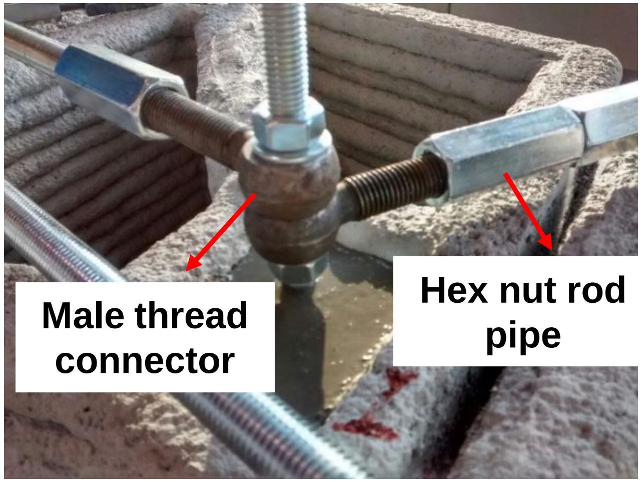


SIMPLY SUPPORTED STRAIGHT BEAM UNDER CONCENTRATED LOAD

Simple Bending : *Strut and tie* model
(no topology optimization)



→ Possibility to easily compare preliminary outcomes with classical beam theory results



Bars connector system

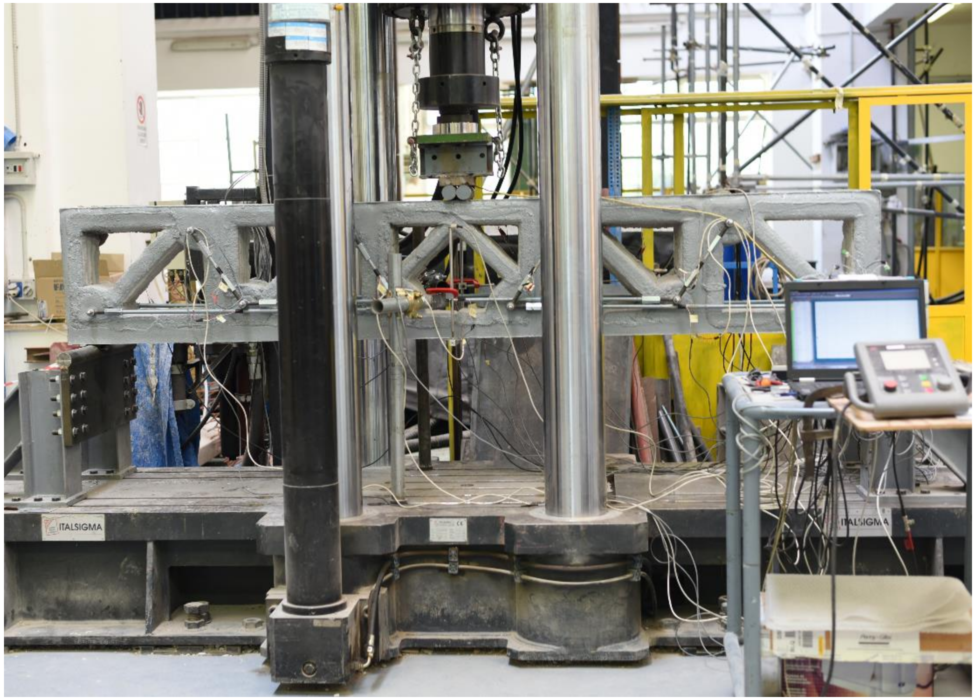
- **DISPLACEMENT CONTROL THREE-POINT BENDING EXPERIMENTAL TEST**
- **NUMERICAL ANALYSIS (SAP2000)**

To understand the mechanism of resistance of a 3D printed RC beam

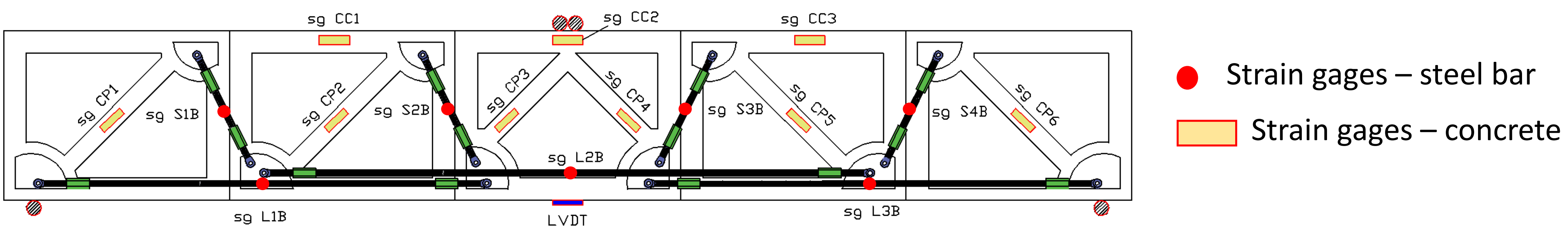
EXPERIMENTAL SET UP

- Universal servo-hydraulic testing machine
- Load scheme is set to ensure tensile or compression primary failure
- Displacement control test
- Load and displacement measurements made by strain gages

- **Velocity = 0,5 mm/min**
- **Cell load = 500 kN**



Side B (Back View-Smooth)



NUMERICAL ANALYSIS

- Abaqus
- SAP2000

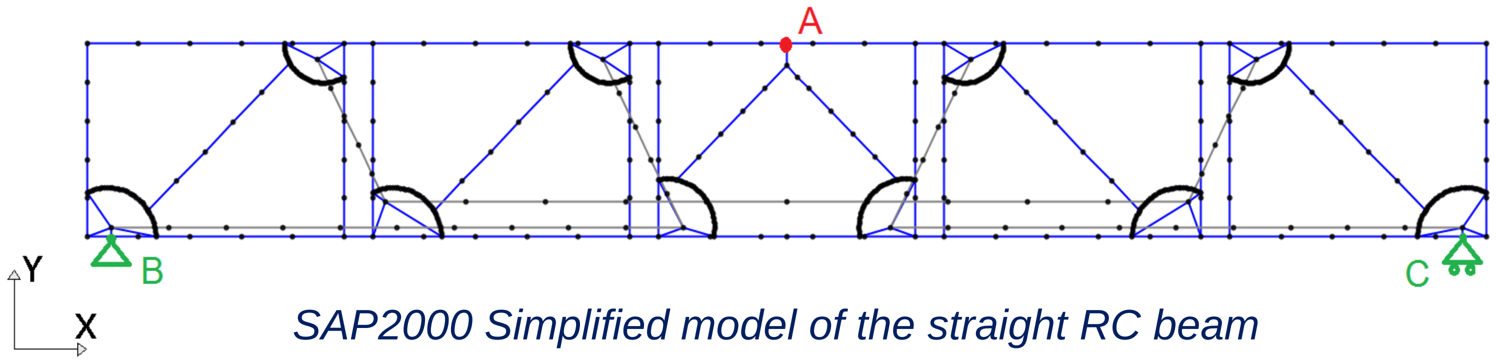
SIMPLIFIED **2D MODEL** ADOPTING **1D-FRAME ELEMENTS**

- **Load:** concentrated force at point A + gravity load
- **BCs :** simply supported beam (points B and C; Uz = 0 at each node of the mesh

- **LINEAR STATIC ANALYSIS**

Geometric configurations were modified in accordance with the development of the cracking process

→ 3 different analysis to model the 3 main steps of the cracking formation



SAP2000 Simplified model of the straight RC beam

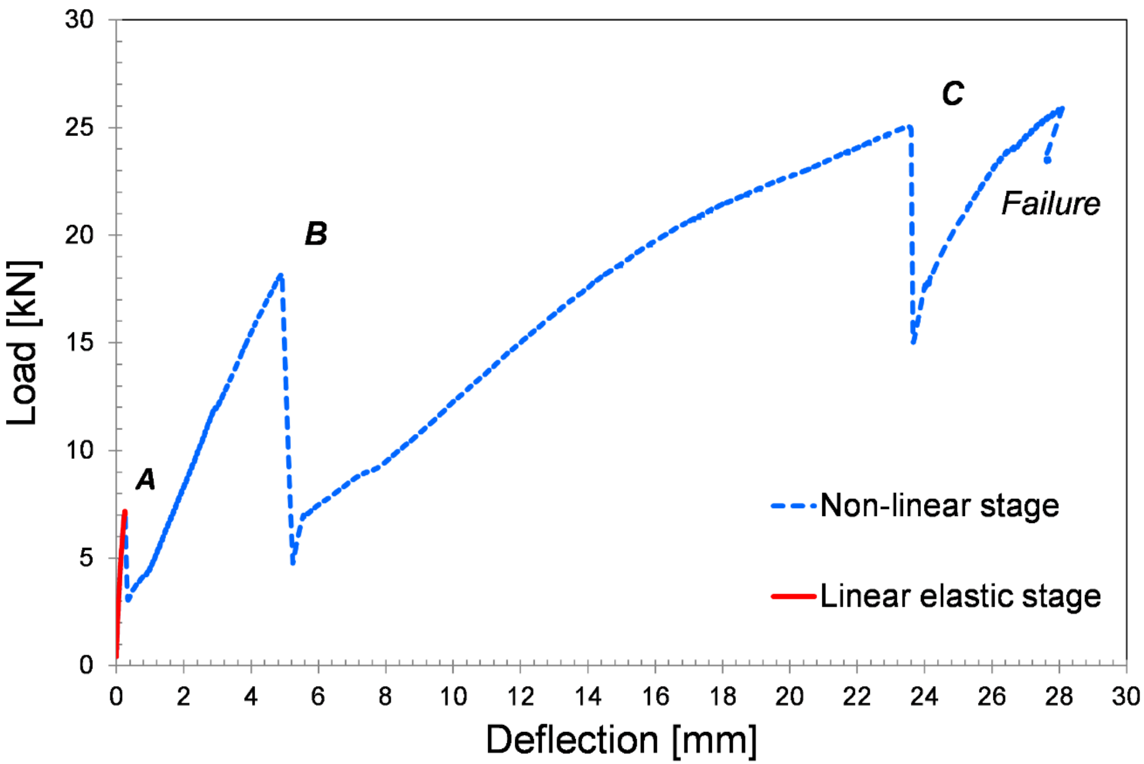
EXPERIMENTAL RESULTS

In terms of overall flexural behaviour it is possible to identify two main stages:

- *Linear elastic stage* (till point A)
- *Non-linear stage* (from point A on)

The *Non-linear stage* can be itself subdivided into:

- *start of cracking stage* (A-B curve)
- *progression of cracking stage* (B-C curve)
- *final failure stage* (curve from point C on)

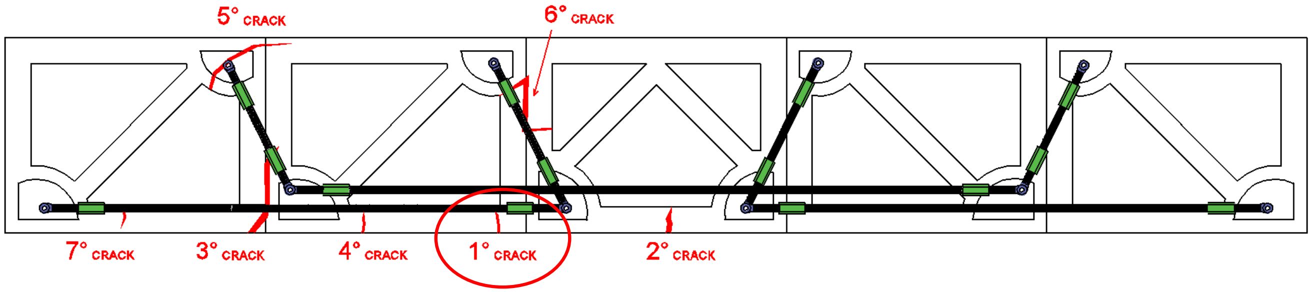


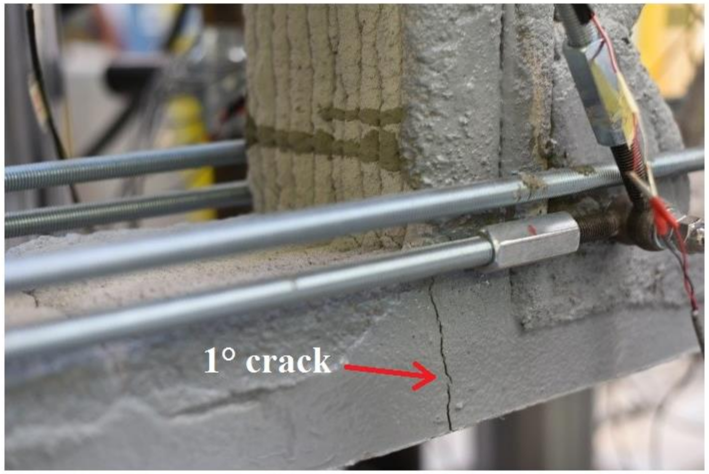
Load-deflection curve from experimental data

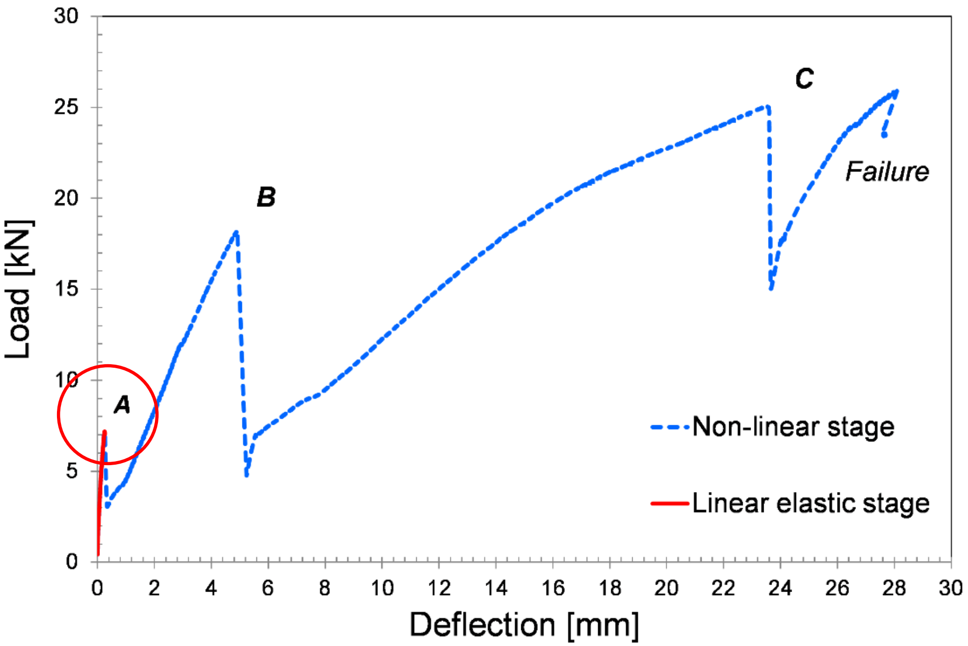
EXPERIMENTAL RESULTS: Linear elastic stage

Formation of the 1° tensile crack (point A) in the concrete (bottom side): point of transition from the *linear elastic stage* (in which the beam is intact) to the *non-linear stage*.

- Moderate loss of carrying load capacity associated to the reduction of the tensile resistant cross-sectional area



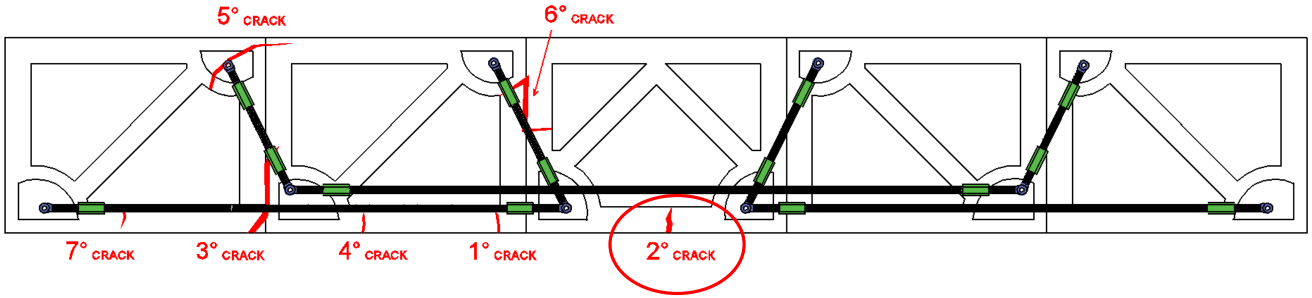


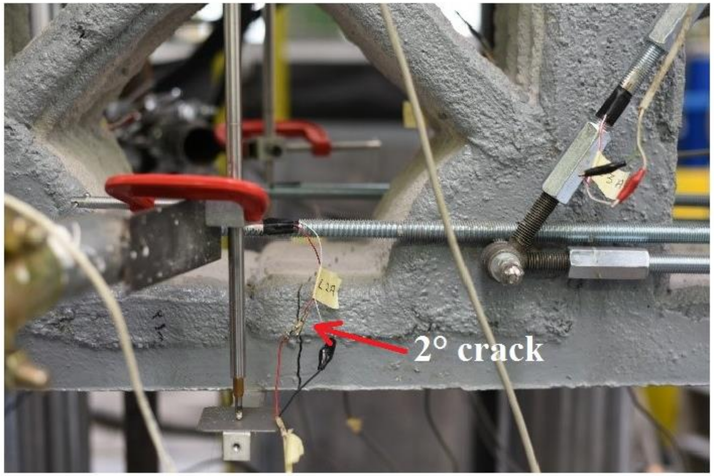


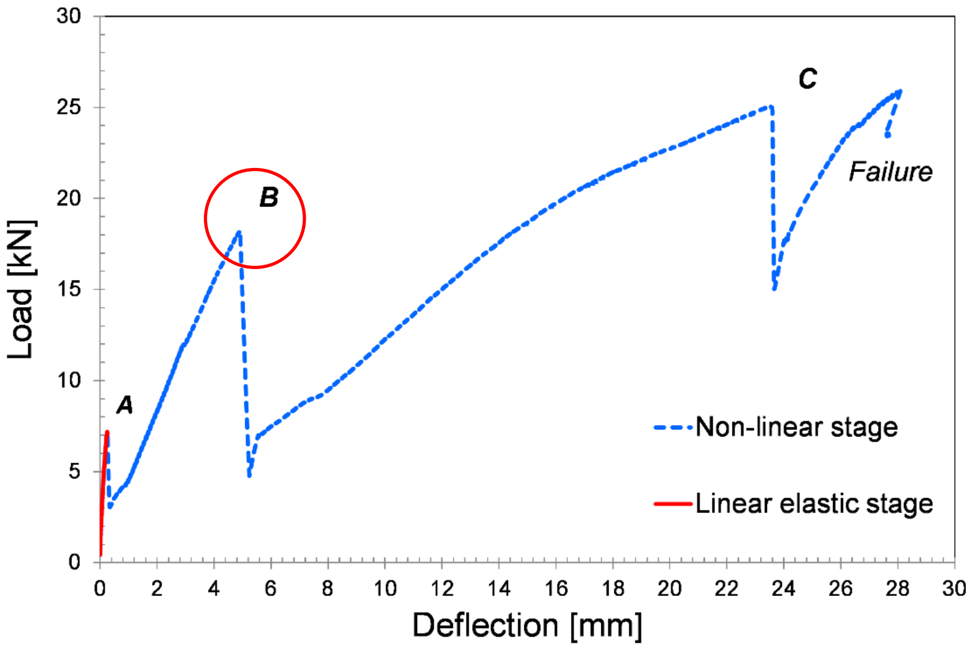
EXPERIMENTAL RESULTS: Start of cracking stage

Load increase with an almost linear trend until the formation of the second major crack (point B)

- The curve still maintains a linear slope; the beam system is still reacting as a monolithic element since bars connector system is preserved



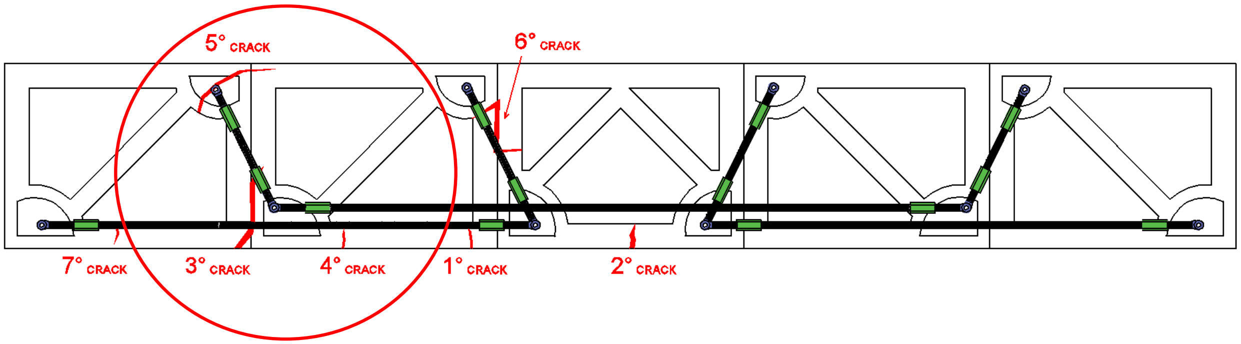


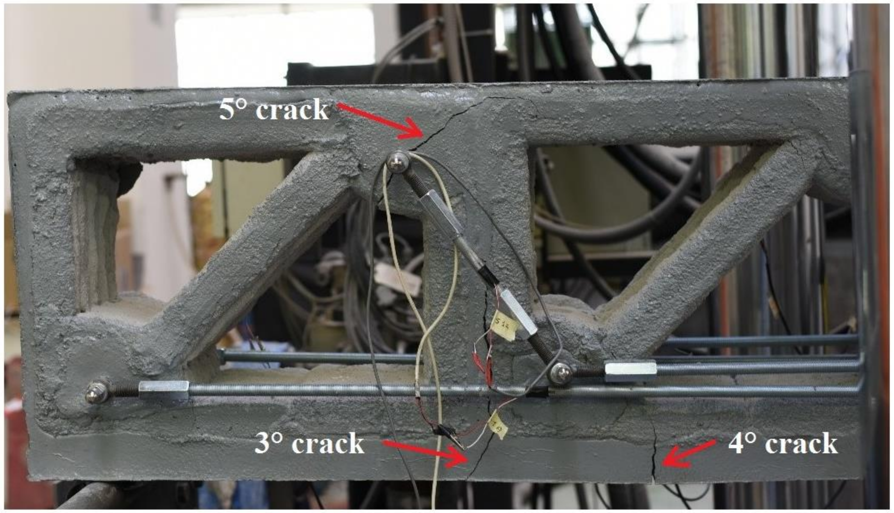


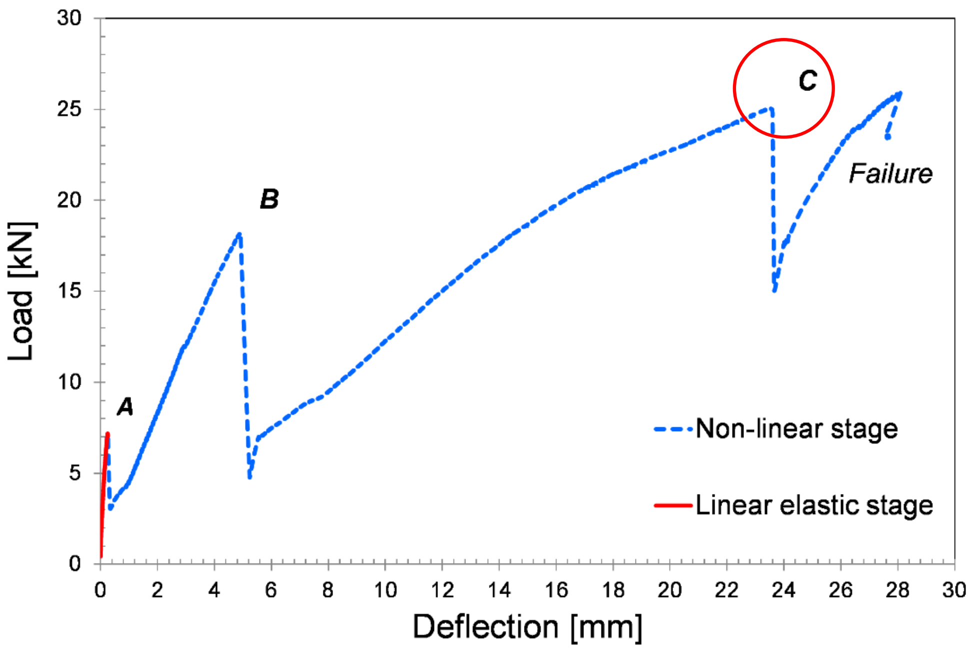
EXPERIMENTAL RESULTS: Progression of cracking stage

Formation of the 3°, 4° and 5° major cracks which do not develop in a distinct manner
⟶ not possible to clearly identify which one determines the third peak load (point C).

- Complex mechanism of fracture involving concrete segments (tensile cracks), the interface connection surface (interface opening and relative sliding), and bars connection system (shear failure of the anchoring substrate made of concrete material).



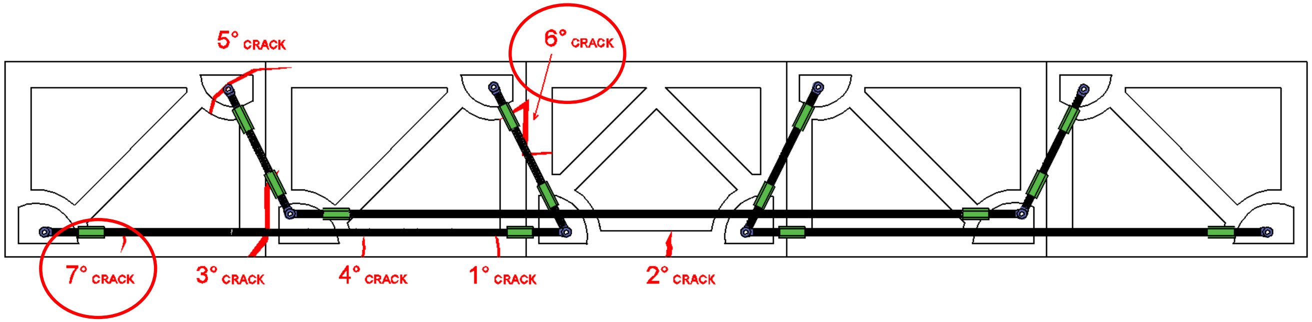


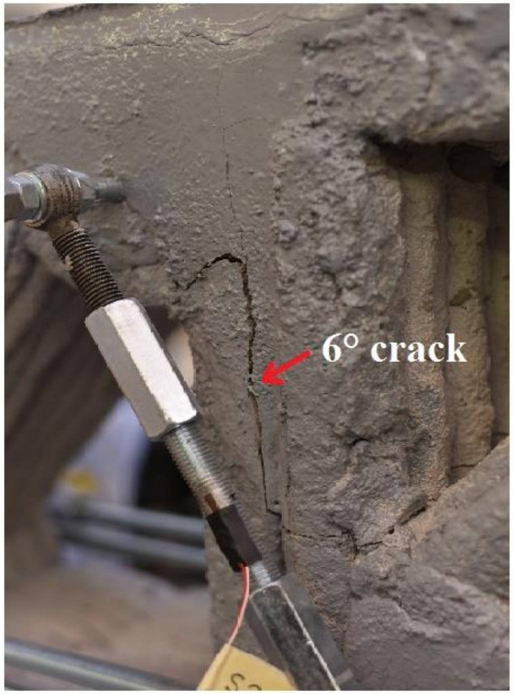


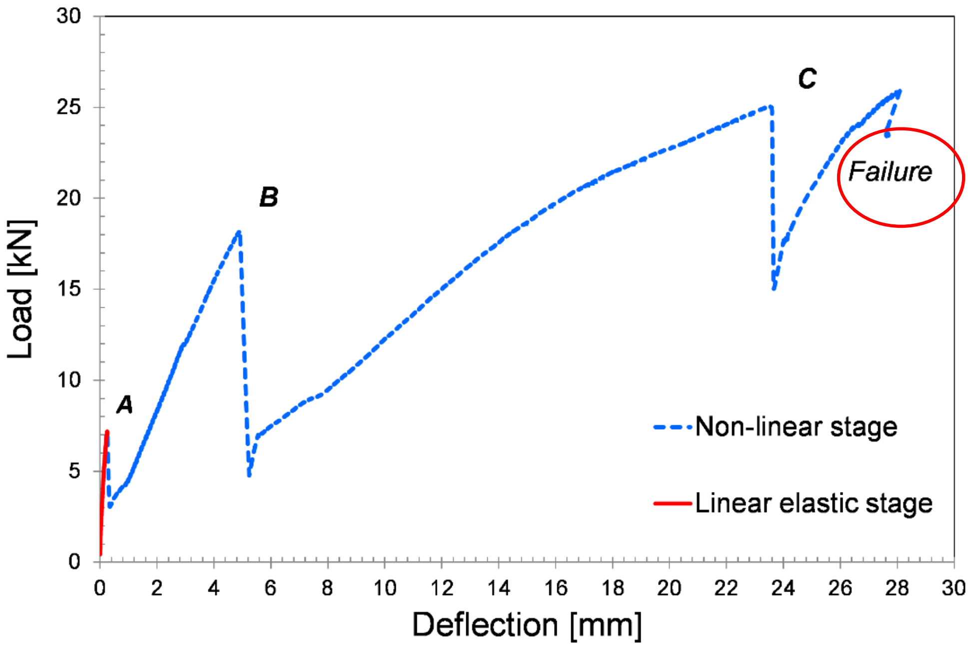
EXPERIMENTAL RESULTS: Final failure stage

After reaching of the ultimate peak load (point C) severe damage occurs in correspondence of the connection system between the steel reinforcement and the central concrete segment.

- Global failure of the beam



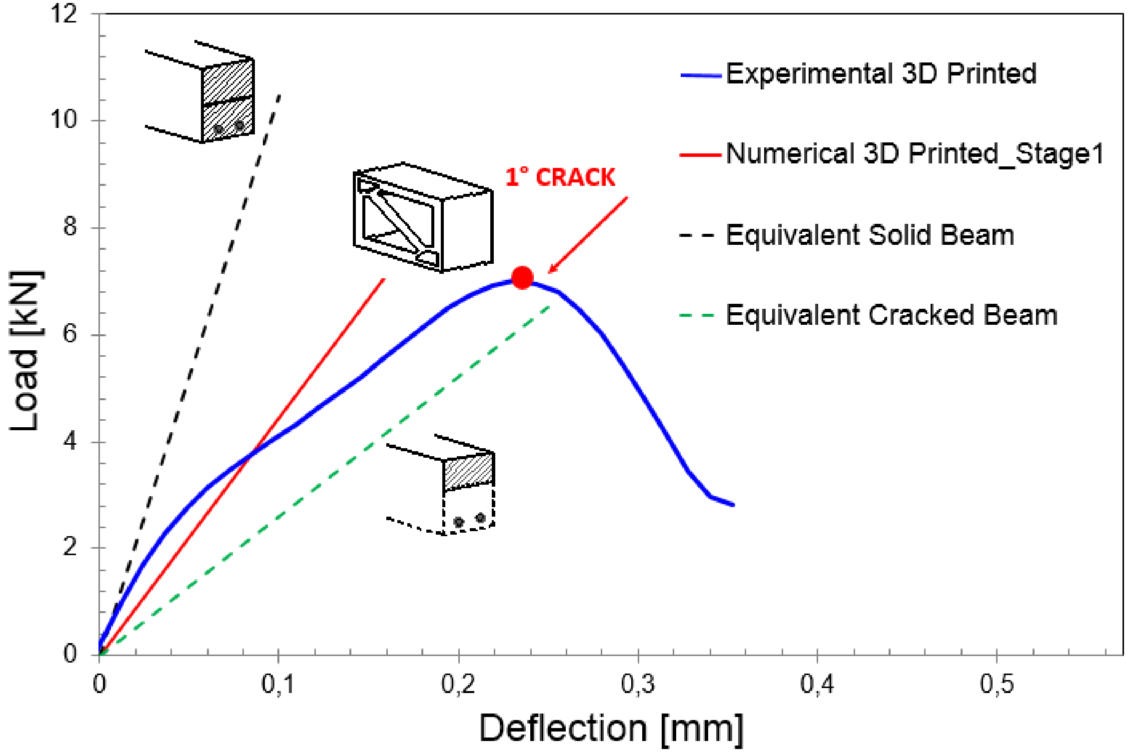




COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

Linear elastic stage

- The behaviour of the 3D printed RC beam is intermediate between that of the Equivalent Solid Beam and the Equivalent Cracked beam



Linear elastic stage: load-deflection curves from experimental data and simulations.

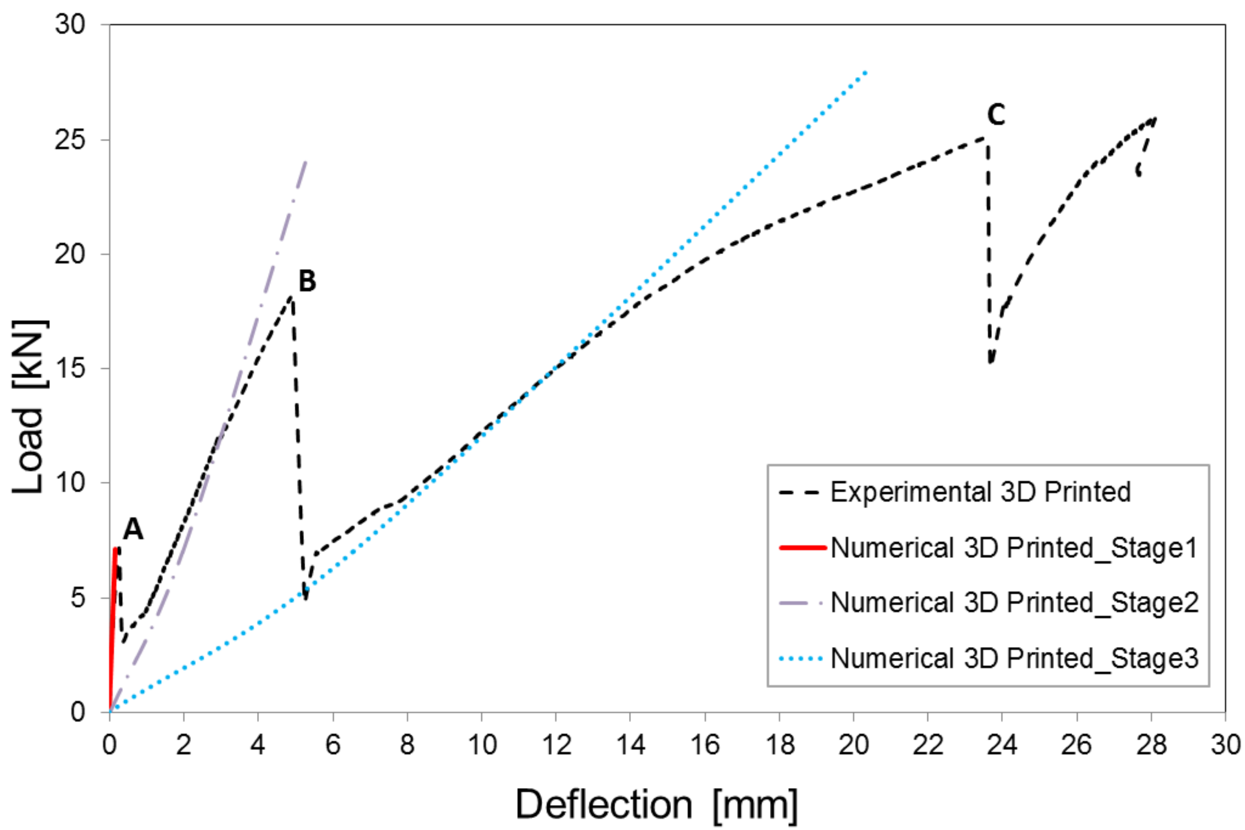
COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

Non-linear stage

- *start of cracking stage* (A-B curve) the numerical curve matches quite well the experimental data

- *progression of cracking stage* (B-C curve) the numerical curve matches quite well the experimental one until a load of around 15 kN.

The worsening in the response prediction is due to the local effects induced by the strong non-linear crack mechanism not captured by the simplified numerical model.



Load-deflection curves from experimental data and simulations

The present thesis wanted both to deepen the performance of numerical methods for the design of complex shapes and to present an innovative 3D printing method for the production of RC elements and possible compatible topology optimization tools.

MODELING OF NON-PRISMATIC ELEMENTS

- The discussion of the results has highlighted a good response of the NP-Model
- The simplicity of derivation makes possible the implementation of the NP-Model in commercial software

- Common codified methods are often unable to account for the varying section shapes of non-prismatic elements, suffering of an ineffective modelling capability.

Software SAP2000 VS NP-Model → CAUTIOUS MODELING OF NON-PRISMATIC ELEMENTS WHIT COMMERCIAL SOFTWARE!

The present thesis wanted both to deepen the performance of numerical methods for the design of complex shapes and to present an innovative 3D printing method for the production of RC elements and possible compatible topology optimization tools.

MANUFACTURING AND OPTIMIZATION METHODS

Topology optimization

- Several issues in the implementation of topology optimization for concrete AM purposes

KRATOS software → ✓ *Stages of the design process*

PSTOpt algorithm → ✓ *Topology optimization problem*

✓ *Printing Material*

LACK OF A CODE WICH COVERS ALL ISSUES + TECHNOLOGY PECULIARITIES

3D Printing approach

- Potential of the technology proved in practice by full-scale 3D printed beams and preliminary outcomes from an experimental activity and numerical analysis.

✓ *Overall ductility*
✓ *Local failure*
✓ *Rebar system*
✓ *…*

→ MORE INVESTIGATIONS ARE NEEDED TO ADDRESS CRITICAL ISSUES AND EXTEND THE METHOD TO INNOVATIVE PRACTICAL CASES

FUTURE STEPS

- To bring together the achievements obtained, combining the design step (using the non-prismatic beam model and a renewed topology optimization tool) with the manufacturing one (presented 3D printing approach) in a real application.
- Printed objects integrated in a real building ⟶ monitoring over the time.

SUCCESSFUL IMPLEMENTATION OF AM IN BUILDING INDUSTRY

CHALLENGES

- ❑ To distinguish between "printing process" and "building system“
  ⟶ Systematic classification of the available AM concrete-based technologies and related obtainable products.

- ❑ To facilitate understanding of concrete 3D printing to engineers and designers.
  Targeted research topics:
  - material
  - analytic/numerical method for the calculation
  - new proven experimental/statistical data to support theoretical advances

- V. Mercuri, G. Balduzzi, D. Asprone and F. Auricchio. 2D Non-prismatic beam model for stiffness matrix evaluation. Conference paper from *World Conference on Timber Engineering (WCTE2016)*, November 2016.

- G. Scalet, E. Boatti, M. Ferraro, V. Mercuri, D. J. Hartl and F. Auricchio, V. Mercuri. Explicit finite element implementation of a shape memory alloy constitutive model and associated analyses. Conference paper from *XIV International Conference on Computational Plasticity*. Fundamentals and Applications, September 2016.

- V. Mercuri, G. Balduzzi, D. Asprone and F. Auricchio. Non-prismatic planar beam: stiffness matrix evaluation and application to reinforced concrete frames. Preprint submitted to *International Journal of Advanced Structural Engineering (IJAS)* - Springer, 27 September 2017.

- C. Menna, D. Asprone, F. Auricchio and V. Mercuri. 3D printing of reinforced concrete elements: technology and design approach. *Construction & Building Materials, 165 (2018): 218-231.*

THANKS FOR YOUR ATTENTION!