

EUCENTRE - European Centre for Training and Research in Earthquake Engineering



in collaboration with Università degli Studi di Pavia Dipartimento di Meccanica Strutturale

Complementary energy of elastic simple bodies undergoing large strains

In linear elasticity, the complementary energy is obtained by Legendre transform of the elastic energy on the basis of energy convexity. So techniques of convex duality can be then applied. Their use is not so immediate when large strains occur because we have to abandon convexity if we want that the energy be objective. So the standard path cannot be followed easily and we need to think over the question.

In my talk I discuss a definition of complementary energy for non-linear elastic simple bodies in large strain regime, which takes into account the polyconvex nature of the elastic energy. It addresses toward the variational analysis of problems with stress constraints. The proposal is based on an in-depth analysis of the kinematics of simple bodies, and related extended definition of inner power from which polyconvexity of the elastic energy emerges naturally. An ancillary result is the decomposition of the first Piola-Kirchhoff stress in two factors: one has energetic origin and is the value of a form, the other is a geometric ingredient determined directly by the deformation.

Anatomy of models of condensed matter

Human attempts of describing the mechanisms of physical world are a continuous sources of mathematical problems. Models are, in fact, representations of classes of phenomena inspired and supported by observations intended here as interpretative cataloguing of events. A model has then to be considered not as a manner of justifying the development of a more or less difficult exercise in mathematics, rather it is an occasion of exploring by a language both qualitative and quantitative, as mathematics is, the intricate aspects of the physical world.

A far-reaching playground where models can be constructed and analyzed in non-trivial way is the territory of condensed matter physics, above all when we try to describe the mechanics of deforming materials in which microscopic events influence the macroscopic behavior through actions which are hardly portrayed by means of standard stresses (let us call them complex, or non-simple if you want). The effort does not necessarily reduce just to the choice of appropriate constitutive structures, as a naïve beginner could instinctively imagine. All the necessary steps to the construction of a continuum model of deformable bodies have to be revisited, in principle.

In my talk I discuss reasons for claiming that *even* the notion of observer and the definition of its changes are essentially structural ingredients of a continuum mechanical model of deformable (simple or complex) materials.

The path I follow leads to the proof of a varied version of Marsden-Hughes theorem involving the notion of relative power (clarified with respect to another previous proposal of mine), leading to (covariant) configurational balances obtained without the need of presuming a priori the existence of configurational actions but the sole dissipative driving force, and without the use of inverse-mapping techniques.

Prof. Paolo Maria Mariano DICeA, University of Florence, Firenze, Italy & Scuola Normale Superiore, Pisa, Italy Friday 27 January, 10.30 – 12.30 MS1 Conference Room, Department of Structural Mechanics, Via Ferrata,1 – Pavia