

Pavia, 6 November 2012

A nonuniform TFA homogenization
technique based on piecewise interpolation
functions of the inelastic field

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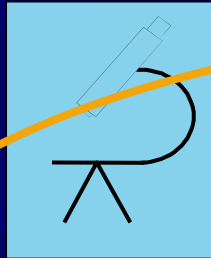
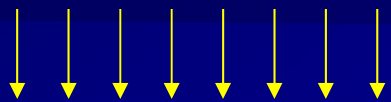
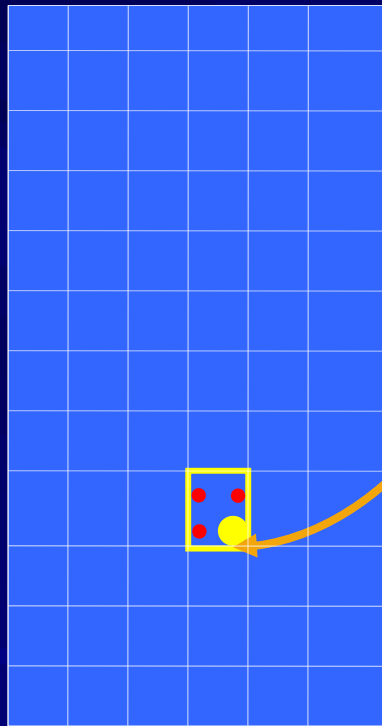
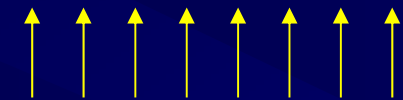
Dipartimento di Ingegneria Civile e
Meccanica



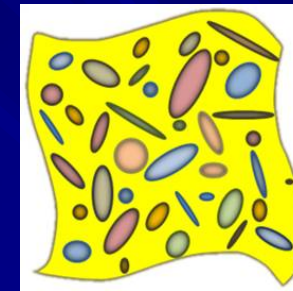
Outline of the presentation

- Motivations
- Nonuniform TFA homogenization
- Applications to
 - Periodic composites
 - SMA and plastic constituents
- Numerical Procedure
- Numerical applications
- Conclusions and future developments

Motivations: micro-macro analysis



Micromechanical problem
through homogenization



RVE

$$\bar{\epsilon} \xrightarrow{\text{blue arrow}} \bar{\sigma} = \bar{C}(\bar{\epsilon} - \bar{p})$$

Main idea:

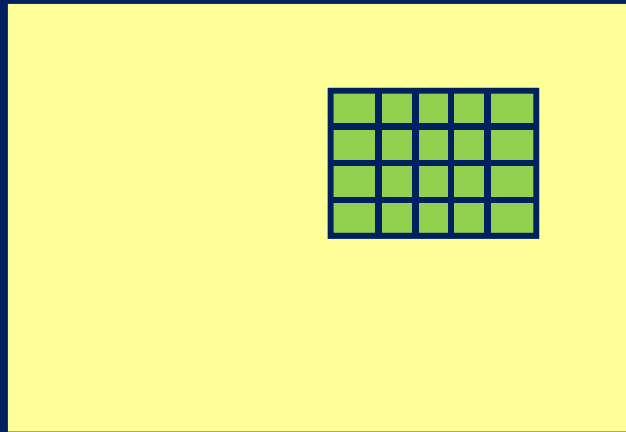
develop a homogenization procedure

- *able to determine the overall constitutive behavior of the composite considering the inelastic effects*
- *in order to avoid the nonlinear FEA at each iteration in each Gauss point of each element*

TFA:

Transformation Field Analysis

Composite material



Nonlinear effect in the inclusion

Constant inelastic strain in the inclusion
(Dvorak, 1992)

Piecewise constant inelastic strain in the
inclusion (Dvorak et al. 1994, Chaboche,
2000)

Original idea: internal coactions by Colonnetti

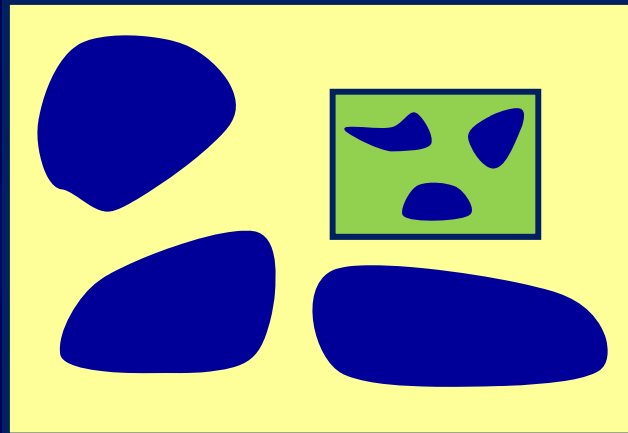
G. COLONNETTI, *Su certi stati di coazione elastica che non dipendono da azioni esterne*,
«Rendiconti della R. Accademia dei Lincei», giugno 1917, pp. 43-47.

A. DANUSSO, *Le autotensioni. Spunti teorici ed applicazioni pratiche*, «Rendiconti del seminario
matematico e fisico di Milano», VIII, 1934, pp. 217-246.

NUTFA:

NonUniform Transformation Field Analysis

Composite material



Inelastic modes $\boldsymbol{\mu}_k(\mathbf{x})$

determined numerically as *actual inelastic strain fields* by simulating the response of the composite along monotone loading paths.

Evolutionary problem

solved through the reformulation of the equations in terms of reduced variables

Nonuniform TFA (Michel-Suquet, 2003)

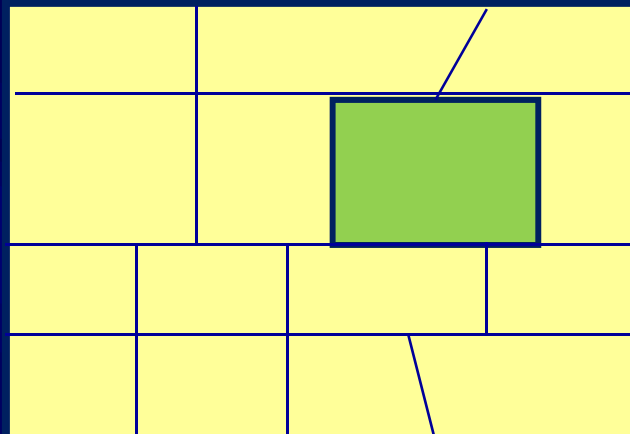
Inelastic strain within each nonlinear phase decomposed on a set of fields, **inelastic modes**:

$$\boldsymbol{\pi}(\mathbf{x}, t) = \sum_{k=1}^M \underbrace{\pi_k(t)}_{\text{Scalar variables}} \underbrace{\boldsymbol{\mu}_k(\mathbf{x})}_{\text{Tensorial fields}}$$

Scalar variables

Tensorial fields

PWNUTFA: PieceWise NonUniform Transformation Field Analysis



p phases

each phase divided in subsets

$$\Omega = \bigcup_{i=1}^N \Omega^i \quad V = \sum_{i=1}^N V^i$$

Constitutive law for the subsets :

$$\boldsymbol{\sigma}^i = \mathbf{C}^i \left[\boldsymbol{\varepsilon}^i - \boldsymbol{\pi}^i \right]$$

inelastic strain accounting
for all the nonlinear effects

Assumption : the inelastic strain $\boldsymbol{\pi}^i$ in the subsets is not constant

Localization

RVE subjected to:

- average elastic strain
- inelastic strains

elastic strain $\bar{\mathbf{e}}$

localization of the strain

$$\boldsymbol{\varepsilon}_{\bar{\mathbf{e}}}(\mathbf{x}) = \mathbf{A}(\mathbf{x}) \bar{\mathbf{e}}$$

average strain $\bar{\mathbf{e}}$

average stress $\bar{\boldsymbol{\sigma}}^{\bar{\mathbf{e}}} = \frac{1}{V^{\Omega}} \sum_{i=1}^N \left(\int_{\Omega^i} \mathbf{C}^i \mathbf{A}^i(\mathbf{x}) \bar{\mathbf{e}} dV \right) = \bar{\mathbf{C}} \bar{\mathbf{e}}$

→ $\bar{\mathbf{C}}$ overall elastic tensor

Localization

RVE subjected to:

- average elastic strain
- inelastic strains

inelastic strain in subset j

$$\boldsymbol{\pi}^j(\mathbf{x}, t) = \sum_{k=1}^{M^j} \boldsymbol{\pi}_k^j(t) \mu_k^j(\mathbf{x})$$

Assumption: null average strain

tensor internal
variables

scalar
modes

localization of the strain in the subset i

M^j number of modes

$$\boldsymbol{\varepsilon}_p^{i,j}(\mathbf{x}) = \sum_{k=1}^{M^j} \mathbf{P}_k^{i,j}(\mathbf{x}) \boldsymbol{\pi}_k^j$$

average stress

$$\bar{\boldsymbol{\sigma}}^{\pi^j} = \sum_{k=1}^{M^j} \mathbf{S}_k^j \boldsymbol{\pi}_k^j$$

Strain field localization

$$\begin{array}{l|l} \bar{\mathbf{e}} & \boldsymbol{\pi}^j(\mathbf{x}, t) = \sum_{k=1}^{M^j} \boldsymbol{\pi}_k^j(t) \mu_k^j(\mathbf{x}) \\ \boldsymbol{\varepsilon}_{\bar{\mathbf{e}}}^i(\mathbf{x}) = \mathbf{A}^i(\mathbf{x}) \bar{\mathbf{e}} & \boldsymbol{\varepsilon}_p^{i,j}(\mathbf{x}) = \sum_{k=1}^{M^j} \mathbf{P}_k^{i,j}(\mathbf{x}) \boldsymbol{\pi}_k^j \end{array}$$

average stress

$$\bar{\boldsymbol{\sigma}}^{\bar{\mathbf{e}}} = \bar{\mathbf{C}} \bar{\mathbf{e}} \quad \bar{\boldsymbol{\sigma}}^{\pi^j} = \sum_{k=1}^{M^j} \mathbf{S}_k^j \boldsymbol{\pi}_k^j$$

average strain

$$\bar{\mathbf{e}} \quad \mathbf{0}$$

total stress

$$\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{\sigma}}^{\bar{\mathbf{e}}} + \sum_{j=1}^N \bar{\boldsymbol{\sigma}}^{\pi^j}$$

total strain

$$\bar{\boldsymbol{\varepsilon}} = \bar{\mathbf{e}}$$

Overall stress-strain relationship

Overall inelastic strain

$$\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{\sigma}}^{\bar{\mathbf{e}}} + \sum_{j=1}^N \bar{\boldsymbol{\sigma}}^{\pi^j} = \bar{\mathbf{C}} \bar{\mathbf{e}} + \sum_{j=1}^N \sum_{k=1}^{M^j} \mathbf{S}_k^j \boldsymbol{\pi}_k^j = \bar{\mathbf{C}} (\bar{\mathbf{e}} - \bar{\mathbf{p}}) \quad \bar{\mathbf{p}} = -\bar{\mathbf{C}}^{-1} \sum_{j=1}^N \sum_{k=1}^{M^j} \mathbf{S}_k^j \boldsymbol{\pi}_k^j$$

Total strain in the subset i

$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \mathbf{A}^i(\mathbf{x}) \bar{\mathbf{e}} + \sum_{j=1}^N \sum_{k=1}^{M^j} \mathbf{P}_k^{i,j}(\mathbf{x}) \boldsymbol{\pi}_k^j$$

Reduced strain in the subset i respect to mode l

$$\hat{\mathbf{e}}_l^i = \left\langle \boldsymbol{\mu}_l^i(\mathbf{x}) \cdot \boldsymbol{\varepsilon}^i(\mathbf{x}) \right\rangle = \mathbf{a}_l^i \bar{\mathbf{e}} + \sum_{j=1}^N \sum_{k=1}^{M^j} \mathbf{D}_{lk}^{i,j} \boldsymbol{\pi}_k^j$$

$$\mathbf{a}_l^i = \left\langle \boldsymbol{\mu}_l^i(\mathbf{x}) \cdot \mathbf{A}^i(\mathbf{x}) \right\rangle = \frac{1}{V} \int_{\Omega} \boldsymbol{\mu}_l^i(\mathbf{x}) \mathbf{A}(\mathbf{x}) dV$$

$$\mathbf{D}_{lk}^{i,j} = \left\langle \boldsymbol{\mu}_l^i(\mathbf{x}) \cdot \mathbf{P}_k^{i,j}(\mathbf{x}) \right\rangle = \frac{1}{V} \int_{\Omega} \boldsymbol{\mu}_l^i(\mathbf{x}) \mathbf{P}_k^{i,j}(\mathbf{x}) dV$$

The localization tensors and their reduced forms are evaluated solving linear elastic problems, each one characterized by the RVE subjected to only one non-zero element of $\bar{\mathbf{e}}$ and $\boldsymbol{\pi}_k^i$ respectively

Evolutionary problem

best approximation of the inelastic field with respect to the adopted representation form

Total strain in the subset i represented in the same form adopted for the inelastic strain

$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{e}_k^i \mu_k^i(\mathbf{x})$$

The coefficient of the linear combination \mathbf{e}_k^i can be evaluated in function of $\hat{\mathbf{e}}_l^i$ inverting equation

$$\hat{\mathbf{e}}_l^i = \frac{1}{V} \sum_{k=1}^{M^i} \mathbf{e}_k^i \int_{\Omega} \mu_l^i(\mathbf{x}) \mu_k^i(\mathbf{x}) dV = \sum_{k=1}^{M^i} \mathbf{M}_{lk}^i \mathbf{e}_k^i$$

Constitutive law of the subset i

$$\boldsymbol{\sigma}^i(\mathbf{x}) = \mathbf{C}^i \sum_{k=1}^{M^i} (\mathbf{e}_k^i - \boldsymbol{\pi}_k^i) \mu_k^i(\mathbf{x})$$

Inelastic problem solved at each point of the subset i
computing the evolution of inelastic strain $\boldsymbol{\pi}^i(\mathbf{x})$ with the
equations of the continuum in terms of the strain and stress

$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{e}_k^i \mu_k^i(\mathbf{x})$$

$$\boldsymbol{\sigma}^i(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{C}^i [\mathbf{e}_k^i - \boldsymbol{\pi}_k^i] \mu_k^i(\mathbf{x})$$



determine $\boldsymbol{\pi}^i(\mathbf{x})$



desired representation $\sum_{k=1}^{M^i} \boldsymbol{\pi}_k^i \mu_k^i(\mathbf{x})$

Compute the variables π_k^i ($k = 1, \dots, M_i$) in each subset i solving the minimum problem:

$$\min_{\pi_k^i} \mathcal{E}(\pi_1^i, \pi_2^i, \dots, \pi_{M^i}^i)$$

$$\mathcal{E}(\pi_1^i, \pi_2^i, \dots, \pi_{M^i}^i) = \int_{\Omega^i} \left\| \pi^i(\mathbf{x}) - \sum_{k=1}^{M^i} \pi_k^i \mu_k^i(\mathbf{x}) \right\| dV_x$$

Determine the best approximation of the inelastic field with respect to the adopted representation form

Constitutive models for inelastic subsets

Plastic model with isotropic hardening

Plastic strain $\boldsymbol{\pi}^i(\mathbf{x})$

Limit function $f_P = \sqrt{\|\boldsymbol{\sigma}^i\|^2 - \frac{1}{3}[\text{tr}(\boldsymbol{\sigma}^i)]^2} - \sqrt{\frac{2}{3}}(\sigma_y^i + k^i \alpha^i)$

Evolutionary equations
 $\dot{\boldsymbol{\pi}}^i = \dot{\zeta}_P^i \frac{\partial f_P}{\partial \boldsymbol{\sigma}^i}$
 $\dot{\alpha}^i = \dot{\zeta}_P^i$

Kuhn –Tucker conditions $\dot{\zeta}_P^i \geq 0 \quad f_P \leq 0 \quad \dot{\zeta}_P^i f_P = 0$

Consistency condition $\dot{\zeta}_P^i \dot{f}_P = 0$

SMA model

Transformation strain $\boldsymbol{\pi}^i(\mathbf{x})$ with $0 \leq \|\boldsymbol{\pi}^i(\mathbf{x})\| \leq \varepsilon_L^i$

maximum transformation strain at the end of a phase transformation in an uniaxial test

Thermodynamic force associated to $\boldsymbol{\pi}^i(\mathbf{x})$ $\mathbf{T}^i(\mathbf{x}) = \boldsymbol{\sigma}^i(\mathbf{x}) - \boldsymbol{\alpha}^i(\mathbf{x})$

Backstress $\boldsymbol{\alpha}^i(\mathbf{x}) = \left[\beta^i \langle T - M_f^i \rangle + h^i \|\boldsymbol{\pi}^i(\mathbf{x})\| + \gamma^i \right] \frac{\partial \|\boldsymbol{\pi}^i(\mathbf{x})\|}{\partial \boldsymbol{\pi}^i(\mathbf{x})}$

Souza et al. , 1998 ; Auricchio, Petrini, 2002, 2004; Evangelista, Marfia, Sacco, 2009, 2010.

Limit function $f_s \left(dev(\mathbf{T}^i) \right) = \sqrt{2J_2^i \left(dev(\mathbf{T}^i) \right) + m^i \frac{J_3^i \left(dev(\mathbf{T}^i) \right)}{J_2^i \left(dev(\mathbf{T}^i) \right)} - R^i}$

$$J_2^i = 1/2 \left(dev(\mathbf{T}^i) \right)^2 : \mathbf{I} \quad J_3^i = 1/3 \left(dev(\mathbf{T}^i) \right)^3 : \mathbf{I}$$

Material parameters $m^i = \sqrt{\frac{27}{2}} \frac{\sigma_c^i - \sigma_t^i}{\sigma_c^i + \sigma_t^i} \quad R^i = 2\sqrt{\frac{2}{3}} \frac{\sigma_c^i \sigma_t^i}{\sigma_c^i + \sigma_t^i}$

Uniaxial critical stresses in compression and in tension

Evolutionary equation $\dot{\boldsymbol{\pi}}^i = \dot{\zeta}_s^i \frac{\partial f_s}{\partial \mathbf{T}^i}$

Khun-Tucker conditions $\dot{\zeta}_s^i \geq 0 \quad f_s \leq 0 \quad \dot{\zeta}_s^i f_s = 0$

Consistency condition $\dot{\zeta}_s^i \dot{f}_s = 0$

Numerical Procedure (Voigt notation)

Before solving the nonlinear evolutive problem
perform the linear elastic pre-analyses to evaluate $\bar{\mathbf{C}} \mathbf{a}_l^i \mathbf{D}_{lk}^{i,j} \mathbf{S}_k^j$
 $i, j = 1, \dots, N \quad l = 1, \dots, M^i \quad k = 1, \dots, M^j$

$$\hat{\mathbf{e}}_l^i = \mathbf{a}_l^i \bar{\boldsymbol{\varepsilon}} + \sum_{j=1}^N \sum_{k=1}^{M^j} \mathbf{D}_{lk}^{i,j} \boldsymbol{\pi}_k^j$$

$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{e}_k^i \mu_k^i(\mathbf{x})$$

$$\mathbf{e}^i = \left(\mathbf{M}^i \right)^{-1} \hat{\mathbf{e}}^i$$

$$\mathbf{M}_{lk}^i = \frac{1}{V} \mathbf{I} \int_{\Omega} \mu_l^i(\mathbf{x}) \mu_k^i(\mathbf{x}) dV$$

$$\mathbf{M}^i = \begin{bmatrix} \mathbf{M}_{11}^i & \mathbf{M}_{12}^i & \cdot & \cdot & \mathbf{M}_{1M^i}^i \\ \mathbf{M}_{21}^i & \mathbf{M}_{22}^i & \cdot & \cdot & \mathbf{M}_{2M^i}^i \\ \cdot & & & & \\ \cdot & & & & \\ \mathbf{M}_{M^i1}^i & \mathbf{M}_{M^i2}^i & & & \mathbf{M}_{M^iM^i}^i \end{bmatrix}$$

$$\hat{\mathbf{e}}^i = \begin{Bmatrix} \hat{\mathbf{e}}_1^i \\ \hat{\mathbf{e}}_2^i \\ \cdot \\ \cdot \\ \hat{\mathbf{e}}_{M^i}^i \end{Bmatrix}$$

$$\mathbf{e}^i = \begin{Bmatrix} \mathbf{e}_1^i \\ \mathbf{e}_2^i \\ \cdot \\ \cdot \\ \mathbf{e}_{M^i}^i \end{Bmatrix}$$

Numerical Procedure

Evolutionary nonlinear problem approached adopting a backward Euler integration scheme

Time step solved with a return map technique

Strain driven procedure

Time step $[t_n, t]$

Assign $\bar{\epsilon}$

While $res = \sum_{i=1}^N \sum_{k=1}^{M^i} \left\| \pi_k^i \Big|_{r+1} - \pi_k^i \Big|_r \right\| > tol$ do iteration $r+1$

For each Ω^i with $i = 1, \dots, N$

Set $\pi_k^i = \pi_{k|n}^i$ with $k = 1, \dots, M^i$

Evaluate $\hat{\mathbf{e}}_l^{i(\text{trial})}$ using \mathbf{a}_l^i $\mathbf{D}_{lk}^{i,j}$ and $\mathbf{e}_l^{i(\text{trial})}$ using \mathbf{M}^i ($l = 1, \dots, M^i$)

For $\mathbf{x} \in \Omega^i$

Evaluate $\boldsymbol{\varepsilon}^{i(\text{trial})}(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{e}_k^{i(\text{trial})} \mu_k^i(\mathbf{x})$ $\boldsymbol{\sigma}^{i(\text{trial})}(\mathbf{x}) = \sum_{k=1}^{M^i} \mathbf{C}^i \left[\mathbf{e}_k^{i(\text{trial})} - \pi_k^{i(\text{trial})} \right] \mu_k^i(\mathbf{x})$

Determine $f_*^{(\text{trial})}(\mathbf{x})$

if $f_*^{(\text{trial})}(\mathbf{x}) < 0 \rightarrow \Delta \pi^i(\mathbf{x}) = \mathbf{0}$

if $f_*^{(\text{trial})}(\mathbf{x}) \geq 0 \rightarrow \Delta \pi^i(\mathbf{x}) \neq \mathbf{0}$

Evaluate $\pi_1^i, \pi_2^i, \dots, \pi_{M^i}^i \Big|_{r+1}$ computing and minimizing \mathcal{E}

Compute res

Evaluate $\bar{\sigma} = \bar{\mathbf{C}} \bar{\boldsymbol{\varepsilon}} + \sum_{i=1}^N \sum_{k=1}^{M^i} \mathbf{S}_k^i \pi_k^i$

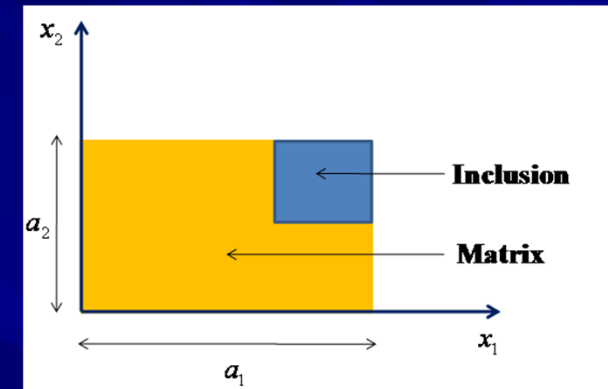
Numerical results

2D plane strain analyses:

- Nonlinear finite element micromechanical analyses

- Homogenization analyses:

Modes: $\mu_1^i = 1$ $\mu_2^i = x_1$ $\mu_3^i = x_2$

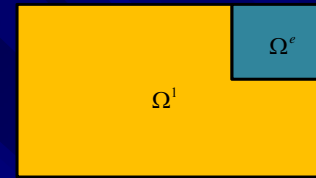


- UTFA uniform TFA: uniform inelastic strain in whole the inelastic part of UC ($N=1$ and $M^i=1$)
- PWUTFA uniform piecewise TFA: uniform inelastic strain in each one of the N subsets ($N>1$ and $M^i=1$)
- NUTFA nonuniform TFA: nonuniform inelastic strain in each one of the N subsets ($N\geq 1$ and $M^i=3$)

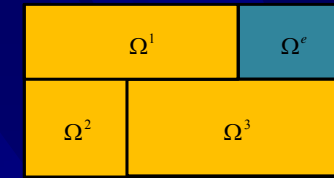
Plastic matrix - Elastic inclusion

Elastic subset Ω^e	$E^e = 25000 \text{ MPa}$ $\nu^e = 0.15$
Plastic subset Ω^i	$E^i = 2500 \text{ MPa}$ $\nu^i = 0.15$ $\sigma_y^i = 3 \text{ MPa}$ $K^i = 1 \text{ MPa}$

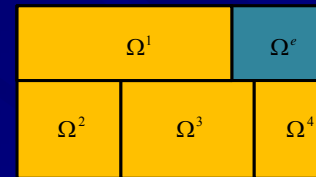
Volume fraction of the inclusion 15%



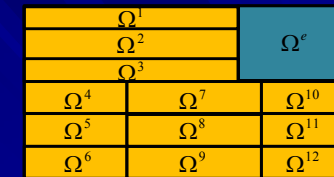
UTFA $N = 1$ $M^i = 1$
NUTFA (i) $N = 1$ $M^i = 3$



PWUTFA (i) $N = 3$ $M^i = 1$

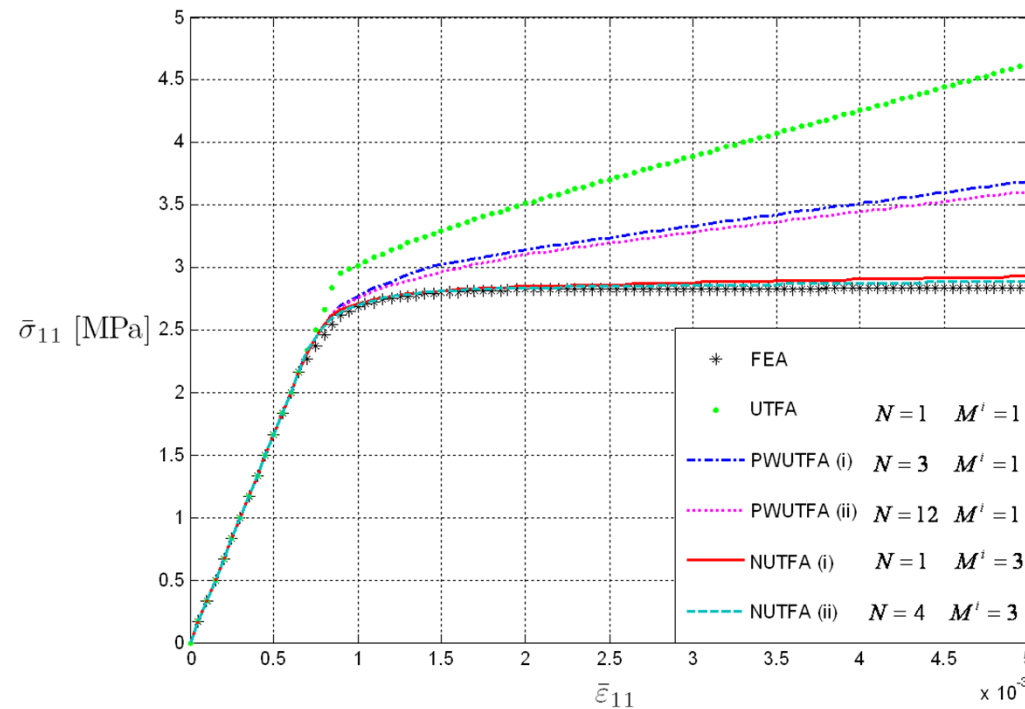


NUTFA (ii) $N = 4$ $M^i = 3$

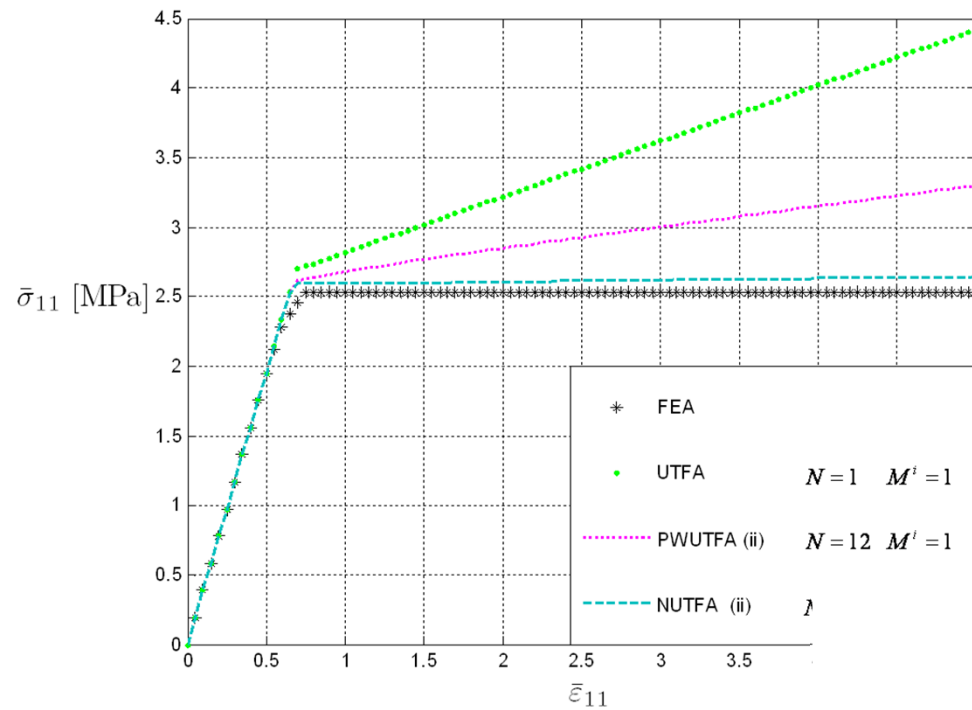


PWUTFA (ii) $N = 12$ $M^i = 1$

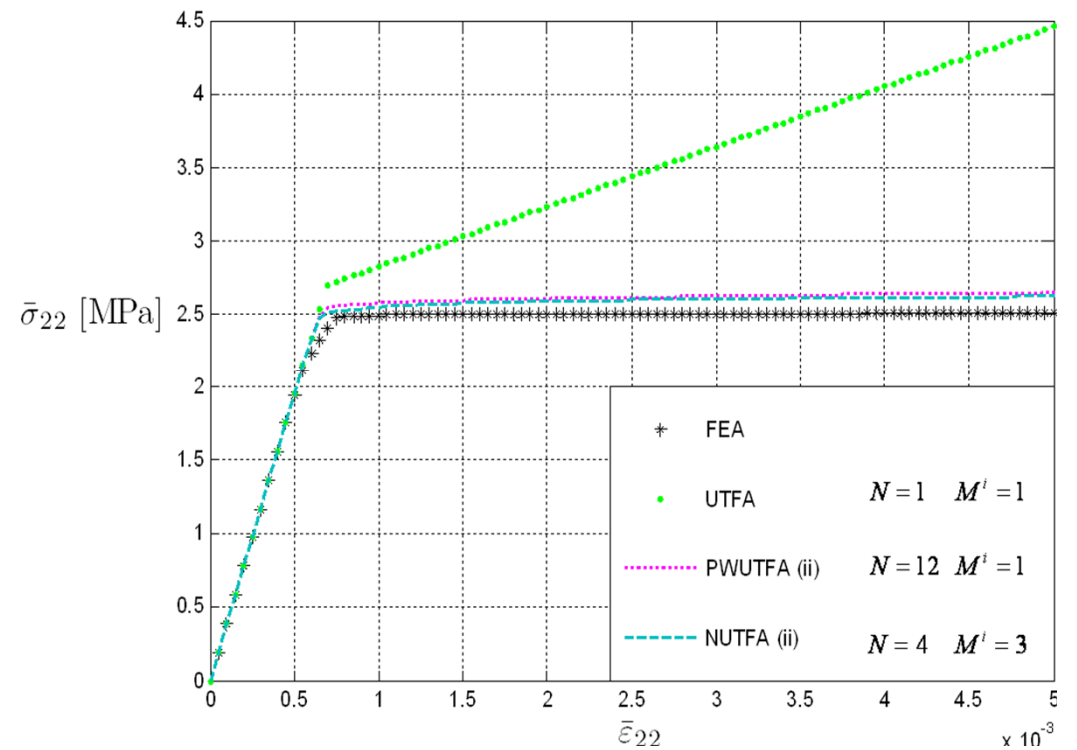
Loading in x_1
direction



Biaxial tensile loading $\bar{\sigma}_{11} = \bar{\sigma}_{22}$



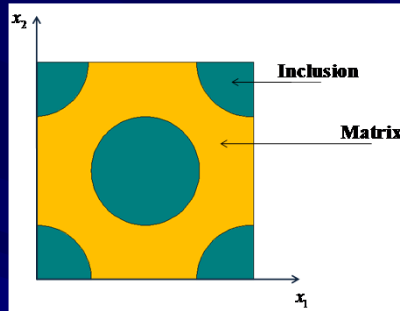
$\bar{\sigma}_{11}$ [MPa]



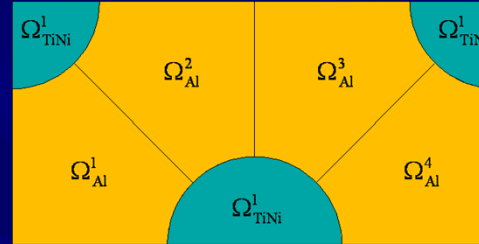
Analysis		Error (%)		Error (%)	CPU times
FEA	2.53	-	2.51	-	225s
UTFA	4.42	74.70	4.46	76.28	5s
PWUTFA(ii)	3.30	30.43	2.65	4.74	27s
NUTFA(ii)	2.64	4.35	2.62	4.38	26s

Plastic matrix – SMA inclusion (Aluminum 6061) (TiNi)

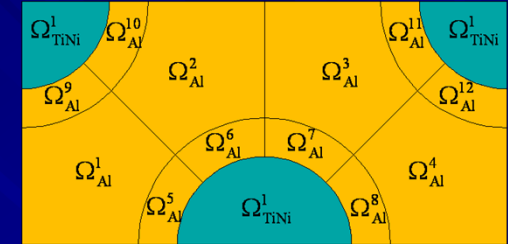
UC



Subset discretization



NUTFA



NUTFA+

Volume fraction of the inclusion 20%

6061 Al subset Ω_{Al}^i	$E_{Al}^i = 70000 \text{ MPa}$ $\sigma_y^i = 245 \text{ MPa}$	$\nu_{Al}^i = 0.33$ $K_{Al}^i = 85 \text{ MPa}$
TiNi subset Ω_{TiNi}^i	$E_{TiNi}^i = 46650 \text{ MPa}$ $h_{TiNi}^i = 1000 \text{ MPa}$ $\varepsilon_{LTiNi}^i = 0.04$ $\sigma_{cTiNi}^i = 72 \text{ MPa}$	$\nu_{TiNi}^i = 0.43$ $\beta_{TiNi}^i = 2.1 \text{ MPaK}^{-1}$ $M_{fTiNi}^i = 280 \text{ K}$ $\sigma_{tTiNi}^i = 56 \text{ MPa}$

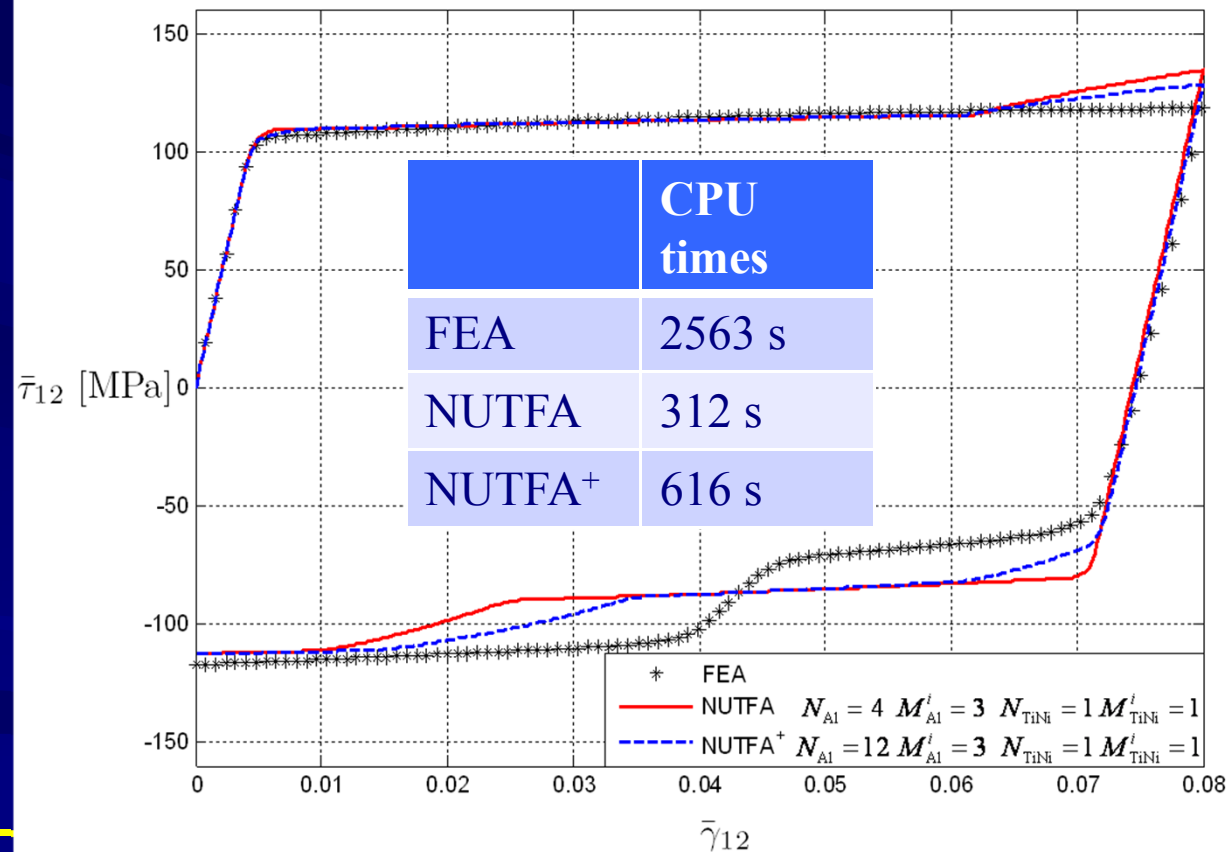
$$\Omega_{TiNi}^i \quad i=1 \quad M^1=1 \quad \Omega_{TiNi}^i \quad i=1 \quad M^1=1$$

$$\Omega_{Al}^i \quad i=1,..,4 \quad M^i=3 \quad \Omega_{Al}^i \quad i=1,..,12 \quad M^i=3$$

$$\mu_1^i = 1, \quad \mu_2^i = x_1, \quad \mu_3^i = x_2$$

Shear response with pseudoelastic effect in the SMA

$t[s]$	0	1	2
$\bar{\gamma}_{12}$	0.00	0.08	0.00
$T[K]$	313	313	313

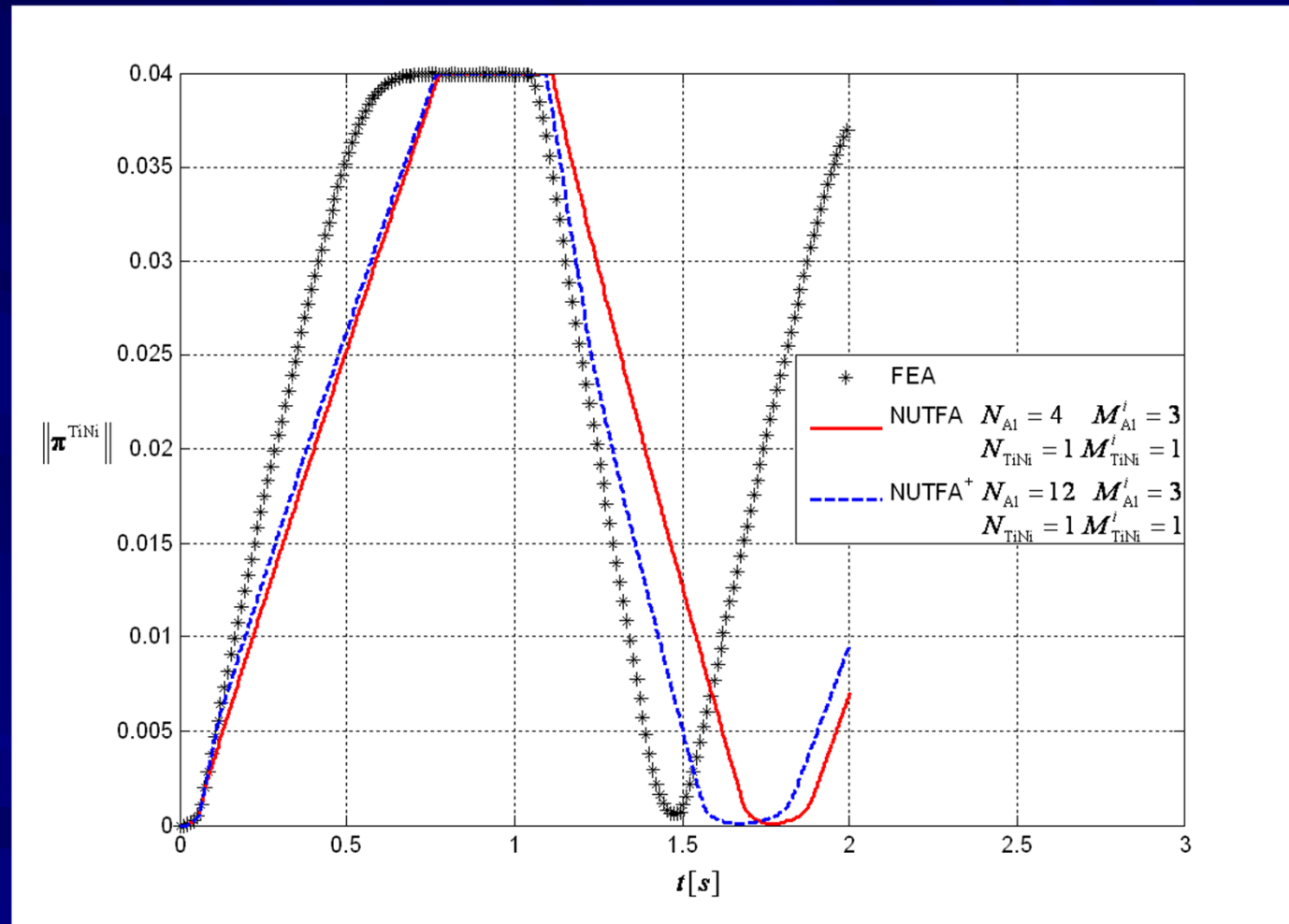


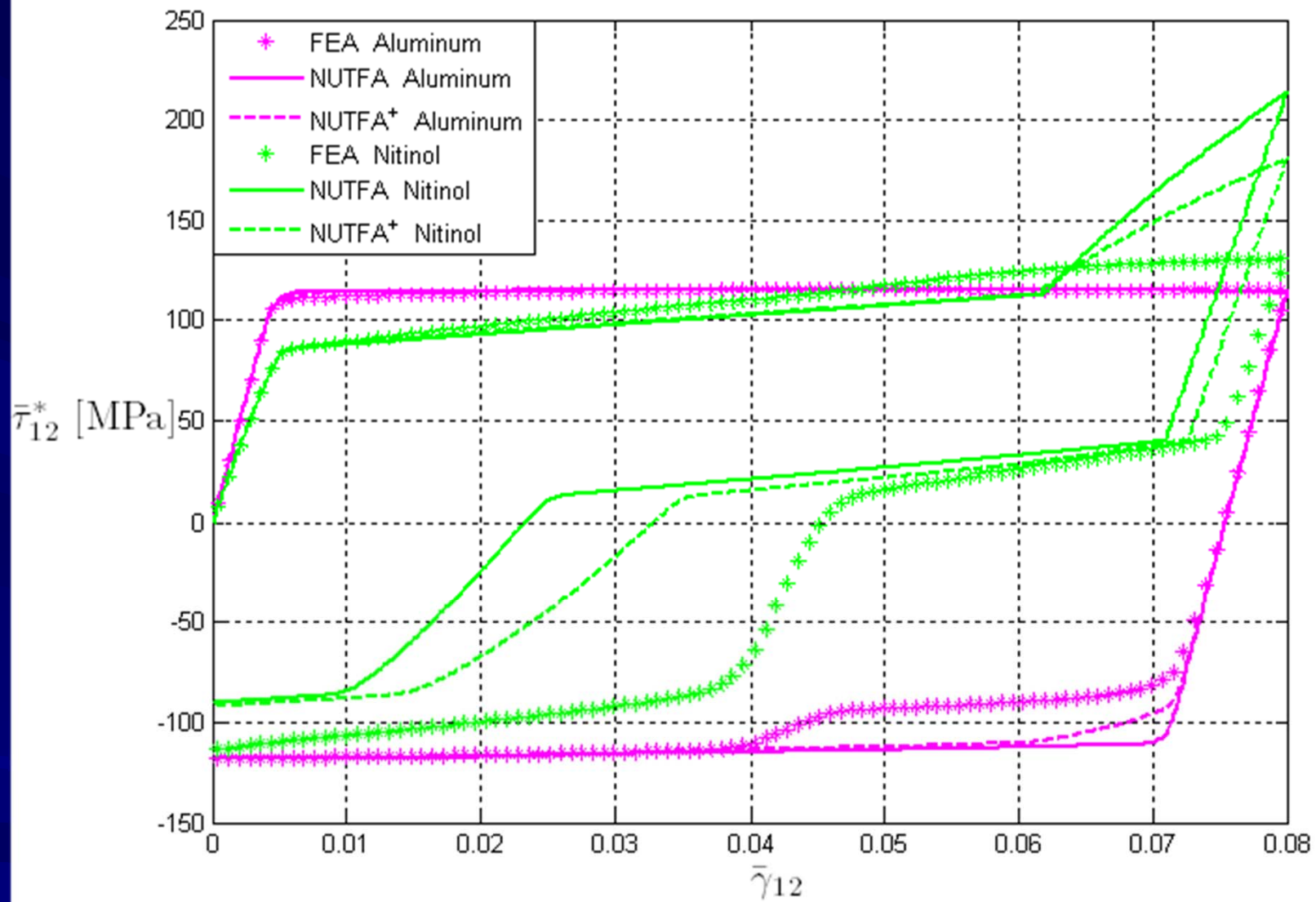
FEA Micromechanical analysis

NUTFA analysis

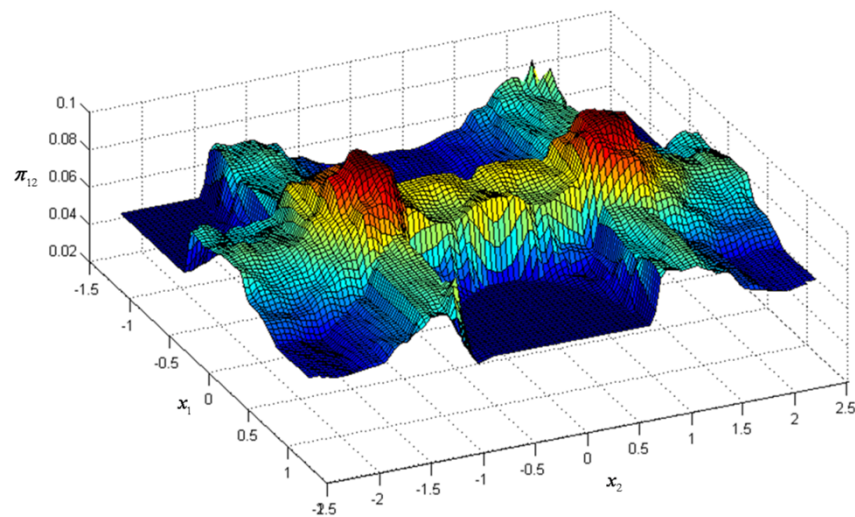
$$\|\boldsymbol{\pi}^{\text{TiNi}}\| = \left\| \frac{1}{V^i} \int_{\Omega^i} \boldsymbol{\pi}(\mathbf{x}) dV_x \right\|$$

$$\|\boldsymbol{\pi}^{\text{TiNi}}\| = \|\boldsymbol{\pi}_1^i\|$$

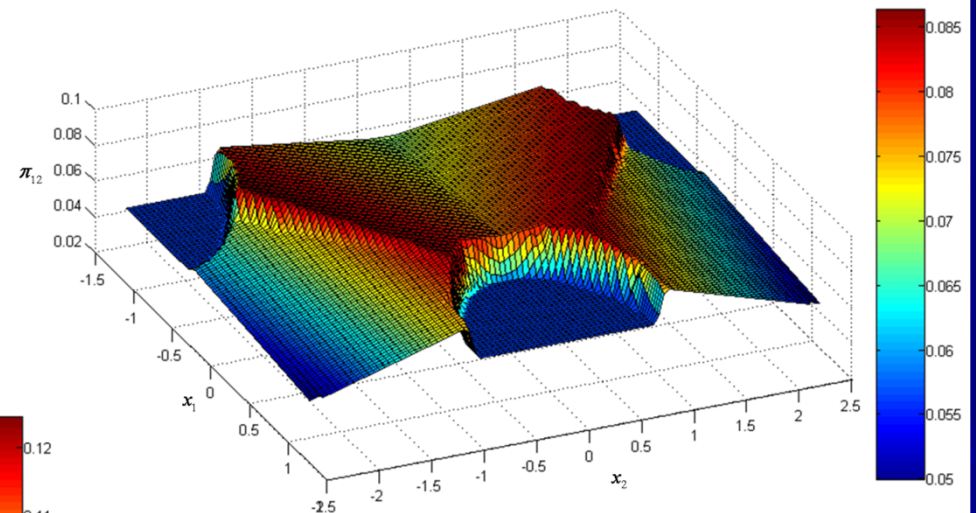




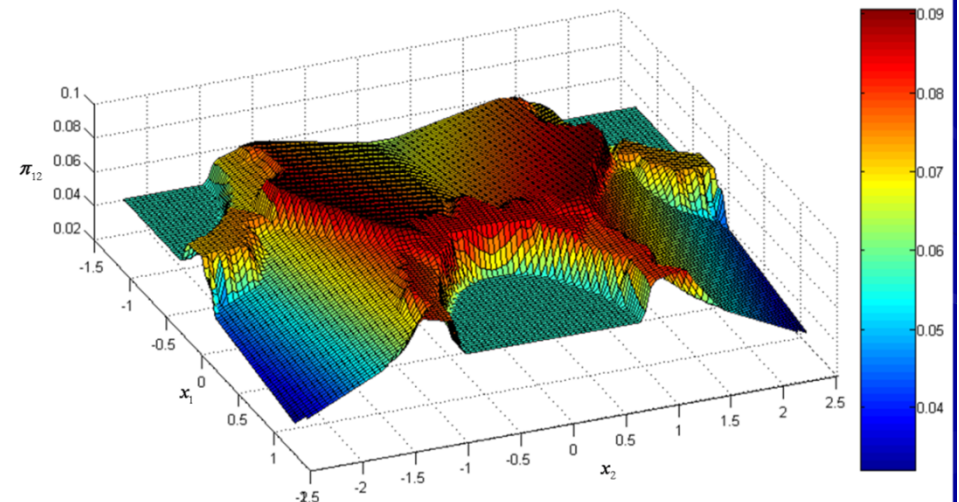
Distribution of the inelastic shear strain at the end of the loading phase



FEA



NUTFA $N_{Al} = 4$ $M_{Al}^i = 3$ $N_{TNNi} = 1$ $M_{TNNi}^i = 1$

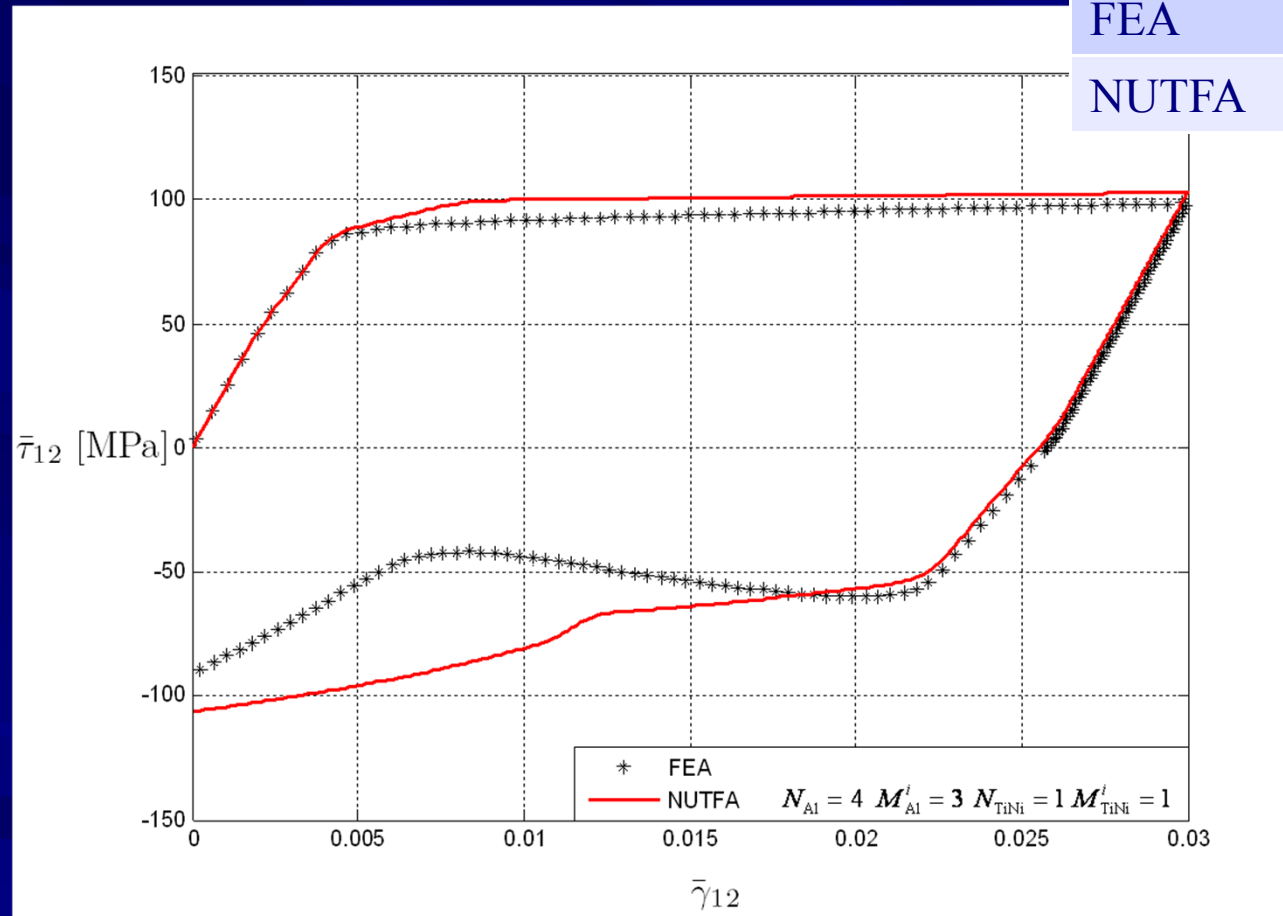


NUTFA+ $N_{Al} = 12$ $M_{Al}^i = 3$ $N_{TNNi} = 1$ $M_{TNNi}^i = 1$

Shear response with shape memory effect in the SMA

$t[s]$	0	1	2	3
$\bar{\gamma}_{12}$	0.0000	0.0300	0.0257	0.0000
$T[K]$	280	280	280	400

	CPU times
FEA	2613 s
NUTFA	212 s

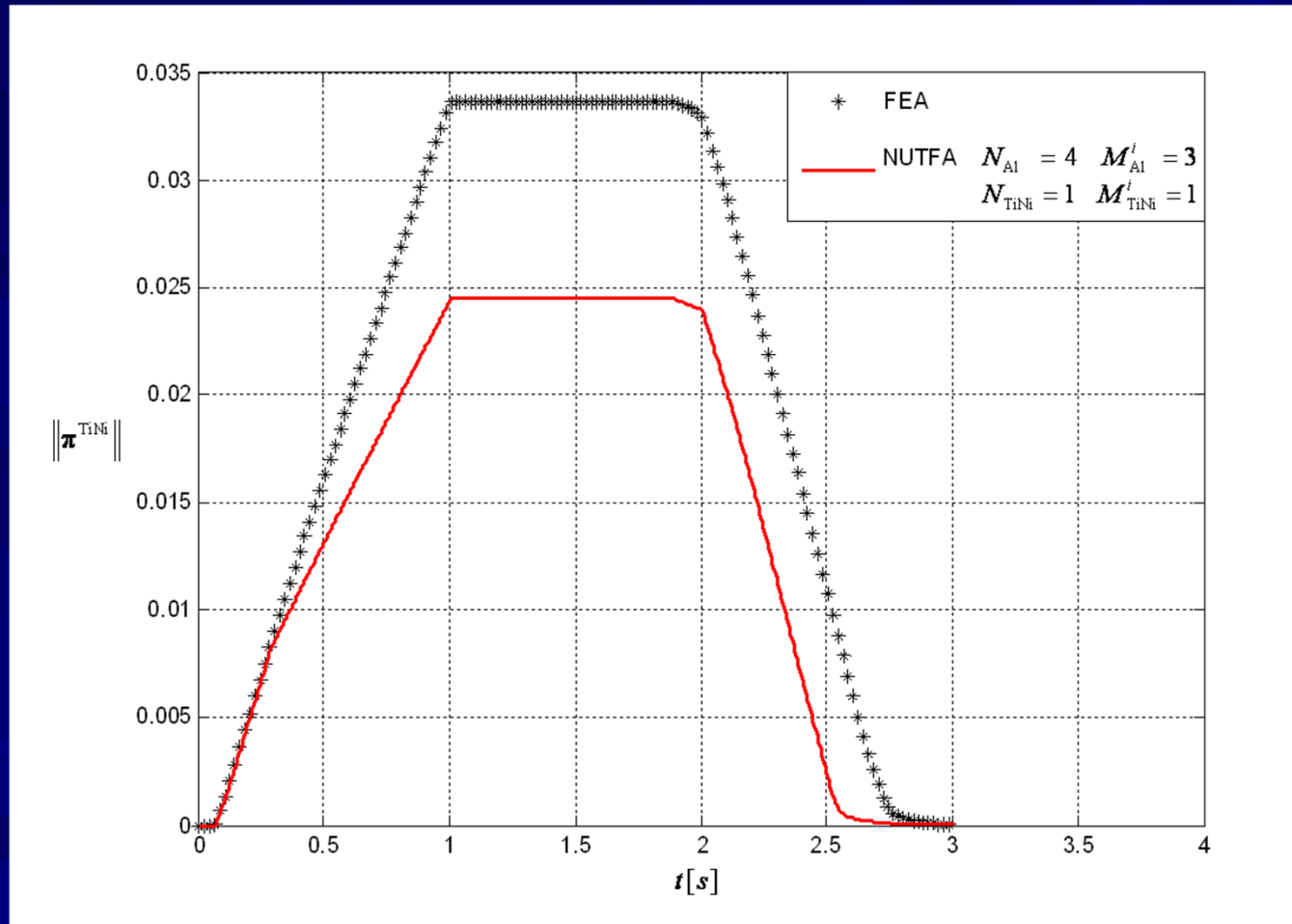


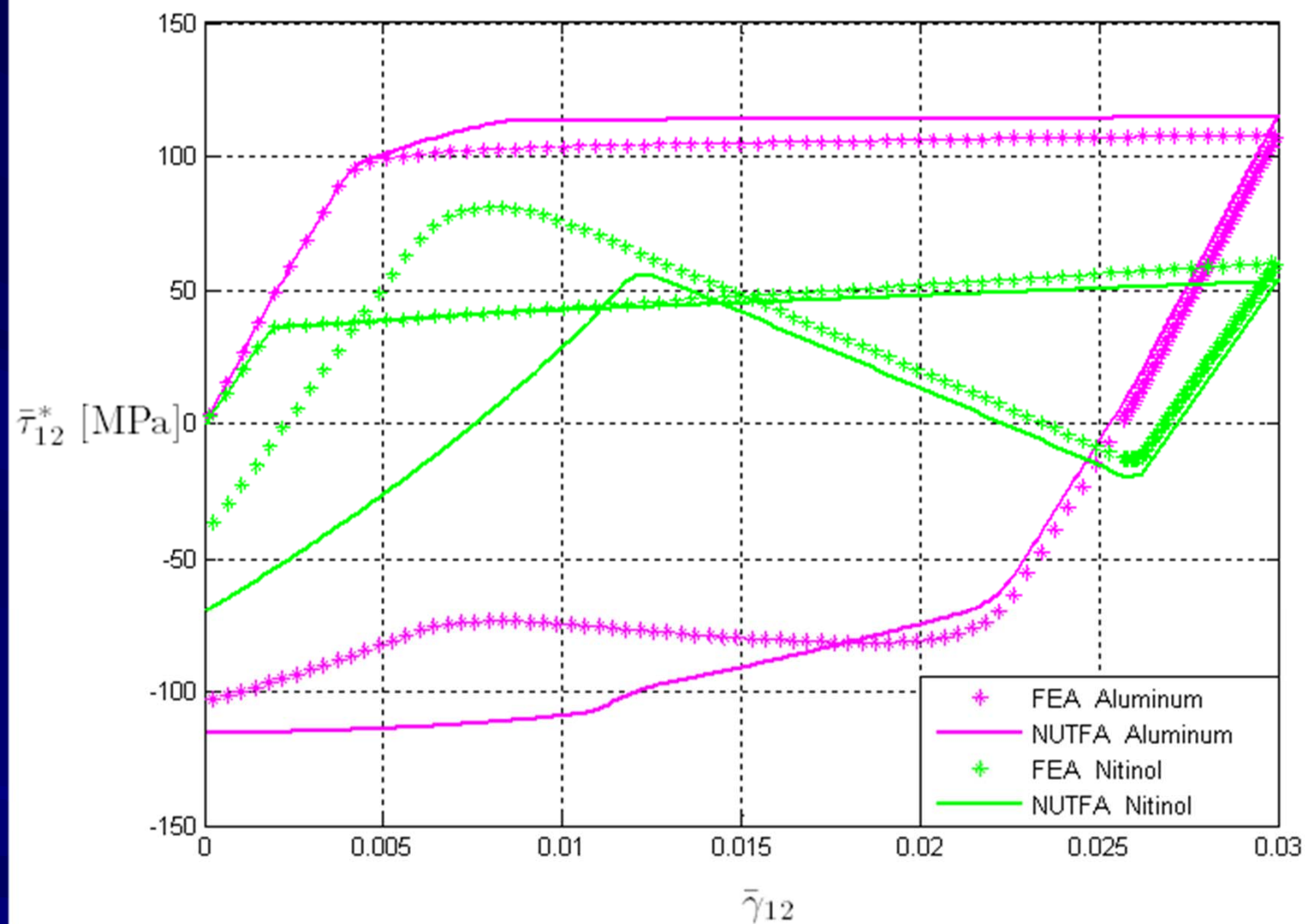
FEA Micromechanical analysis

$$\|\boldsymbol{\pi}^{\text{TiNi}}\| = \left\| \frac{1}{V^i} \int_{\Omega^i} \boldsymbol{\pi}(\mathbf{x}) dV_x \right\|$$

NUTFA analysis

$$\|\boldsymbol{\pi}^{\text{TiNi}}\| = \|\boldsymbol{\pi}_1^i\|$$



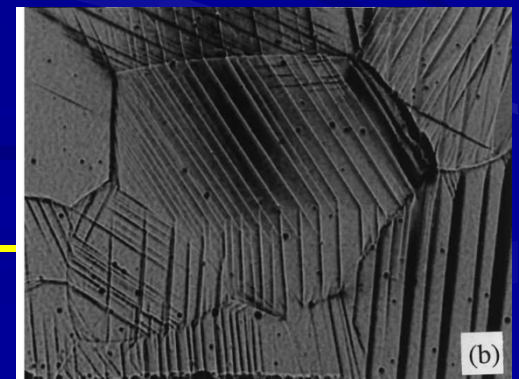


Conclusions

- Proposed homogenization technique based on TFA allows to consider also a nonuniform distribution of inelastic strain in each subset of each phase
 - NUTFA homogenization approximates the nonlinear micromechanical analysis better than uniform and piecewise uniform TFA in the developed numerical applications
 - Time of computation of NUTFA is significantly lower than FEA: important in the framework of multiscale analyses
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Future Development

- Comparisons with the NUTFA by Michel and Suquet/ Fritzen and Böhlke
 - 3D implementation
 - Micromechanical study of the SMA
 - Structural multiscale analysis
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