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# Aeroelastic instabilities in suspension and arch bridges via geometrically exact formulations

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SAPIENZA
UNIVERSITÀ DI ROMA

# Tacoma Narrows Bridge

# Volgograd Bridge

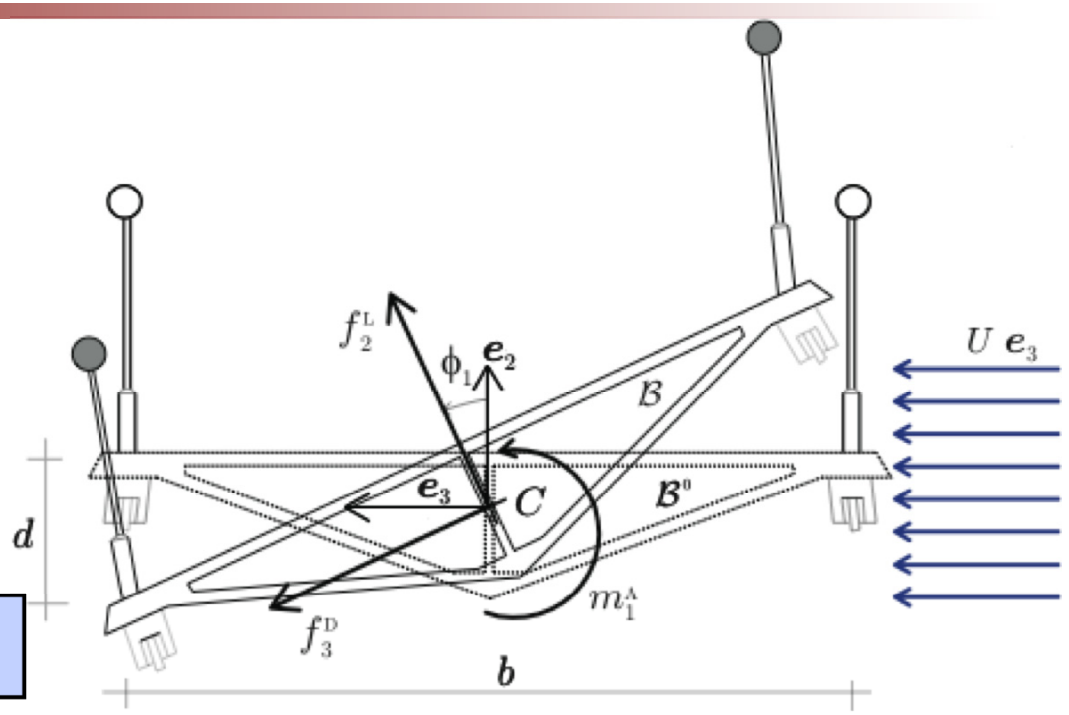
# Aeroelastic phenomena

$$U(x,t) = U(x) + u(x,t)$$

**Static** aeroelastic phenomena

Torsional divergence

Lateral buckling




## Static aeroelastic wind loads

Drag force $$f_3^{\mathrm{D}} = \frac{1}{2}\rho U^2 b C^{\mathrm{D}}(\phi_1)$$

Lift force $$f_2^{\mathrm{L}} = \frac{1}{2}\rho U^2 b C^{\mathrm{L}}(\phi_1)$$

Aerostatic mom $$f_2^{\mathrm{L}} = \frac{1}{2}\rho U^2 b C^{\mathrm{L}}(\phi_1)$$

$$C^{\mathrm{L}}(\phi_1),\ C^{\mathrm{D}}(\phi_1),\ C^{\mathrm{M}}(\phi_1)$$

Linearization about zero wind angle of attack

$$C(\phi_1) = C_0 + \left.\frac{dC}{d\phi_1}\right|_{\phi_1=0} \phi_1 + O(\phi_1^2)$$

# Aerodynamic wind loads

**Lift**

$$f_2^{\mathrm{L}} = \frac{1}{2}\rho b U^2 \left[ K H_1^*(K)\frac{\dot{u}_2}{U} + K H_2^*(K) b \frac{\dot{\phi}_1}{U} + K^2 H_3^*(K)\phi_1 + K^2 H_4^*(K)\frac{u_2}{b} \right]$$

**Drag**

negligible (Simiu-Scanlan)

**Moment**

$$m_1^{\mathrm{A}} = \frac{1}{2}\rho b^2 U^2 \left[ K A_1^*(K)\frac{\dot{u}_2}{U} + K A_2^*(K) b \frac{\dot{\phi}_1}{U} + K^2 A_3^*(K)\phi_1 + K^2 A_4^*(K)\frac{u_2}{b} \right]$$

air density $\rho$

deck's depth $b$

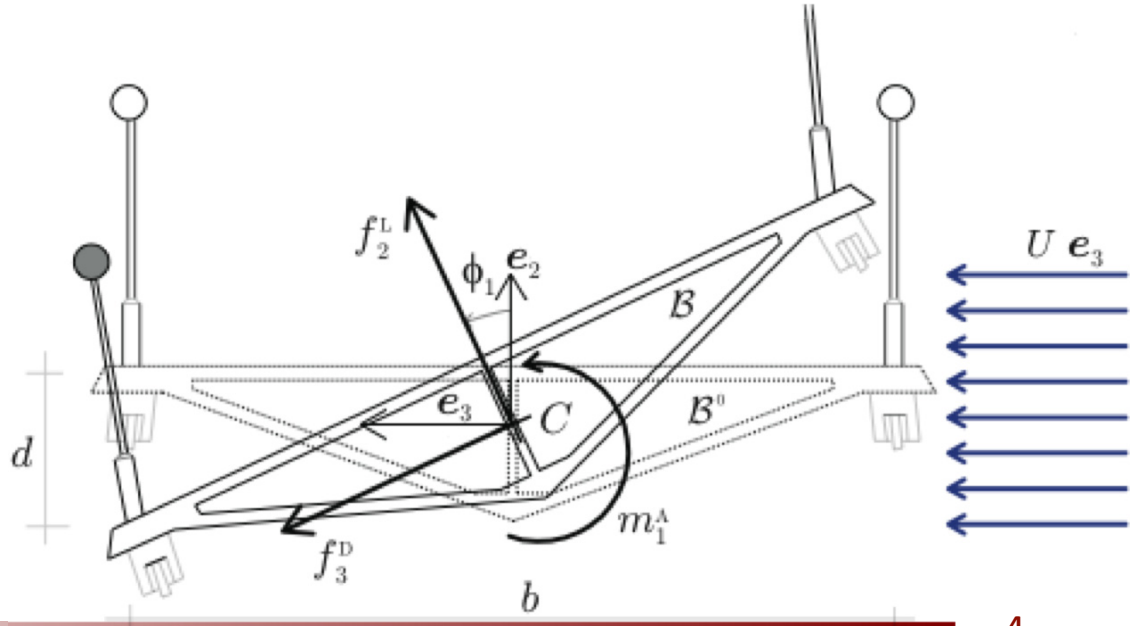
wind speed $U$

**reduced frequency** $K = b\,\omega/U$

$\omega$ is the circular frequency

$H^*_i$ and $A^*_i$ ***aeroelastic derivatives***

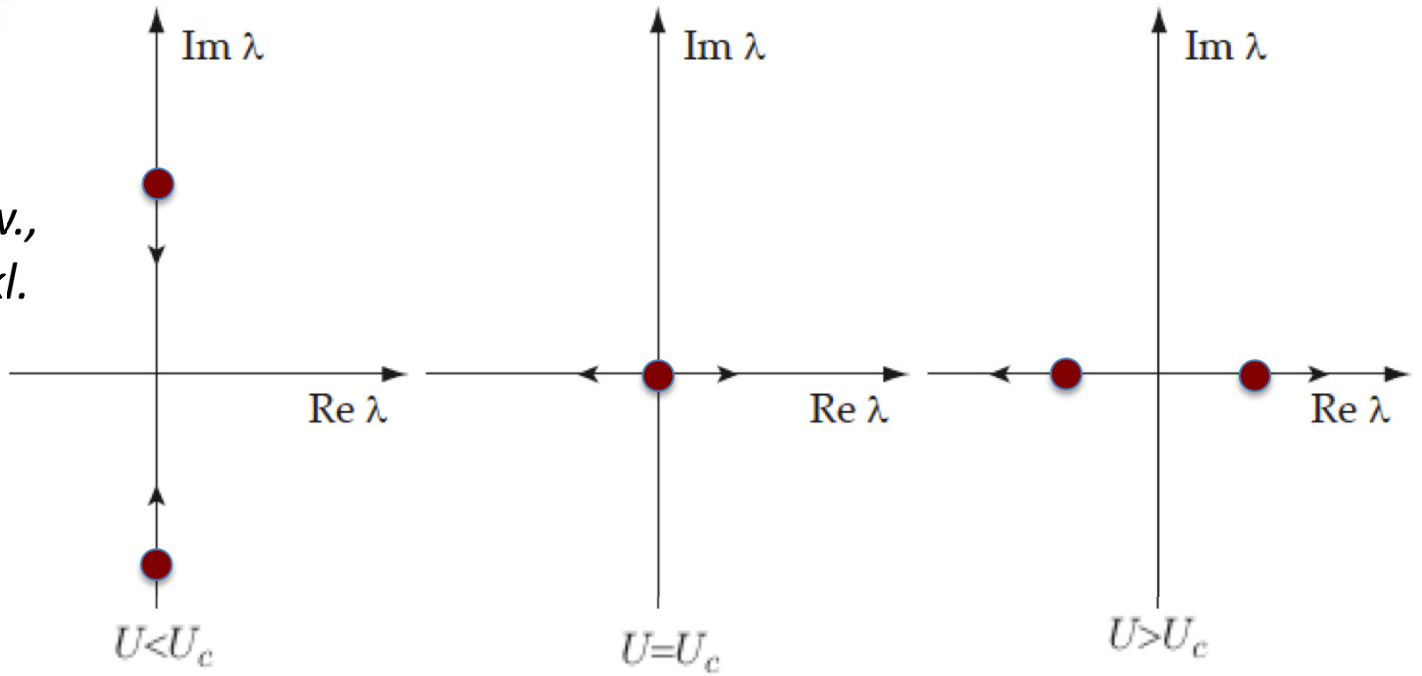
(wind tunnel tests)



# Loss of stability: *divergence* or *Hopf*

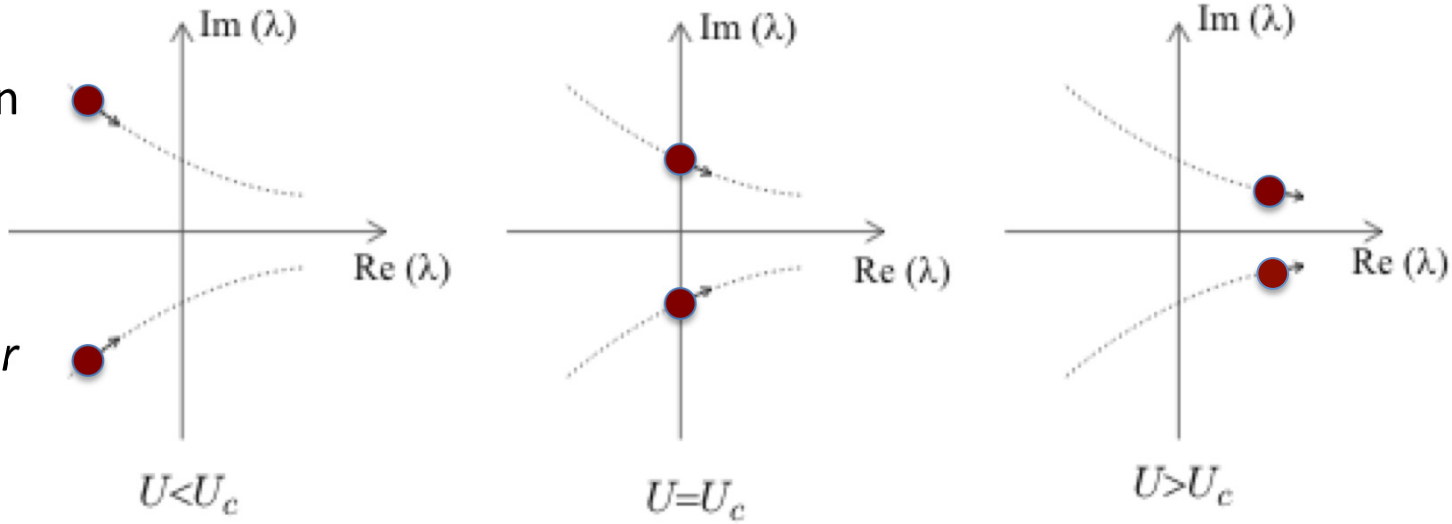
Divergence

*torsional div.,*
*lateral buckl.*
…




Hopf bifurcation

*flutter*
*galloping*
*stall flutter*
*buffeting*
…



# Outline

## Geometrically exact continuum models

Suspension bridges

Arch bridges

- *Kinematics: finite displacements and finite rotations*
- *Total Lagrangian formulation of the equations of motion*
- *Parametric 1D models (one spatial coordinate)*

Main reference

W. Lacarbonara (2010) ***Nonlinear Structural Mechanics***. *Theory, Computational Approaches and Phenomena*, Springer, New York, ISBN 978-1-4419-1275-6. *http://springer.com/978-1-4419-1275-6.*

## Static and dynamic analyses

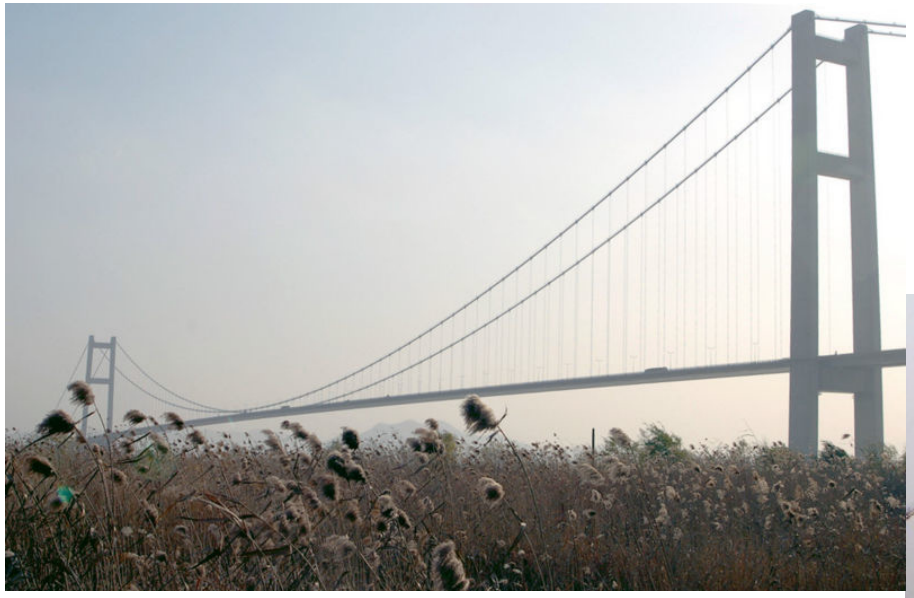
- *Nonlinear asymmetric behaviors:* ***hardening*** *and* ***softening*** *response*

## Aeroelastic analyses

- *The aeroelastic forces and the* ***torsional divergence*** *phenomenon*
- *The aerodynamic forces and the* ***flutter*** *phenomenon*

# Suspension bridges

Runyang Suspension Bridge,
Yangtze river (China) $l = 1490$ m



Hu Men Suspension Bridge,
Zhu Jiang river (China) $l = 888$ m

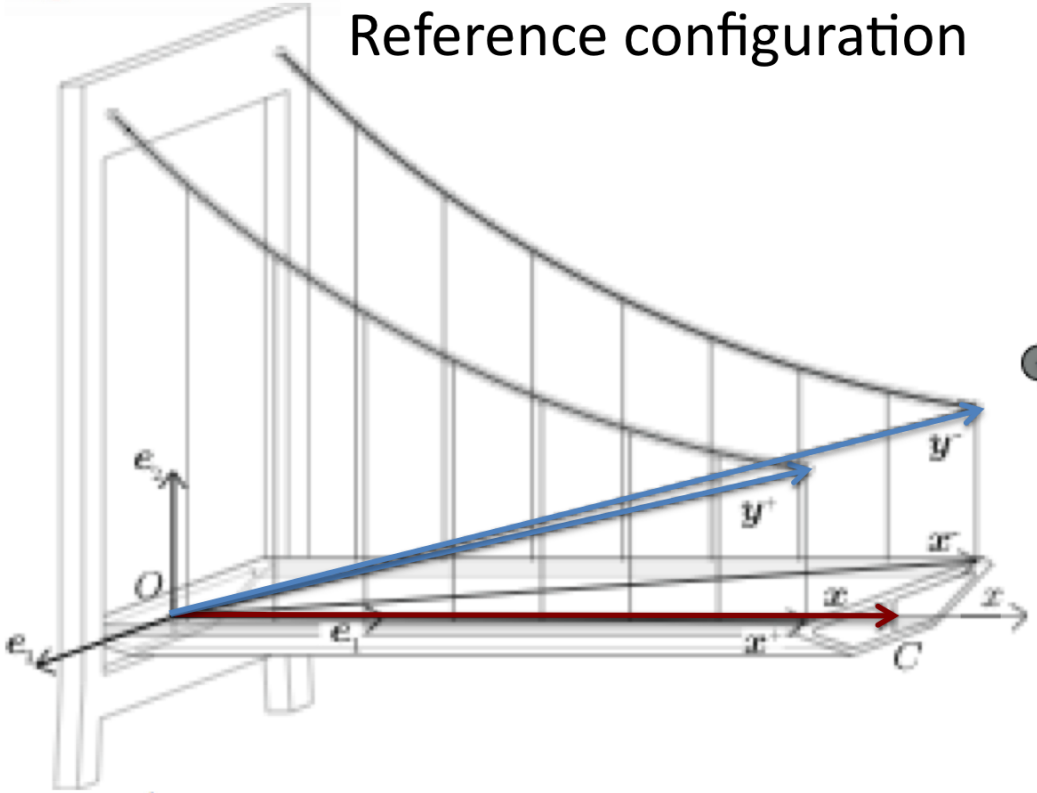
## References


**W.Lacarbonara and A. Arena,**
"Aerostatic torsional divergence of suspension bridges via a fully nonlinear continuum formulation"
in Atti di 10 Convegno Nazionale di Ingegneria del Vento, IN-VENTO 2008, Scopello (TP), 8-11 giugno 2008.

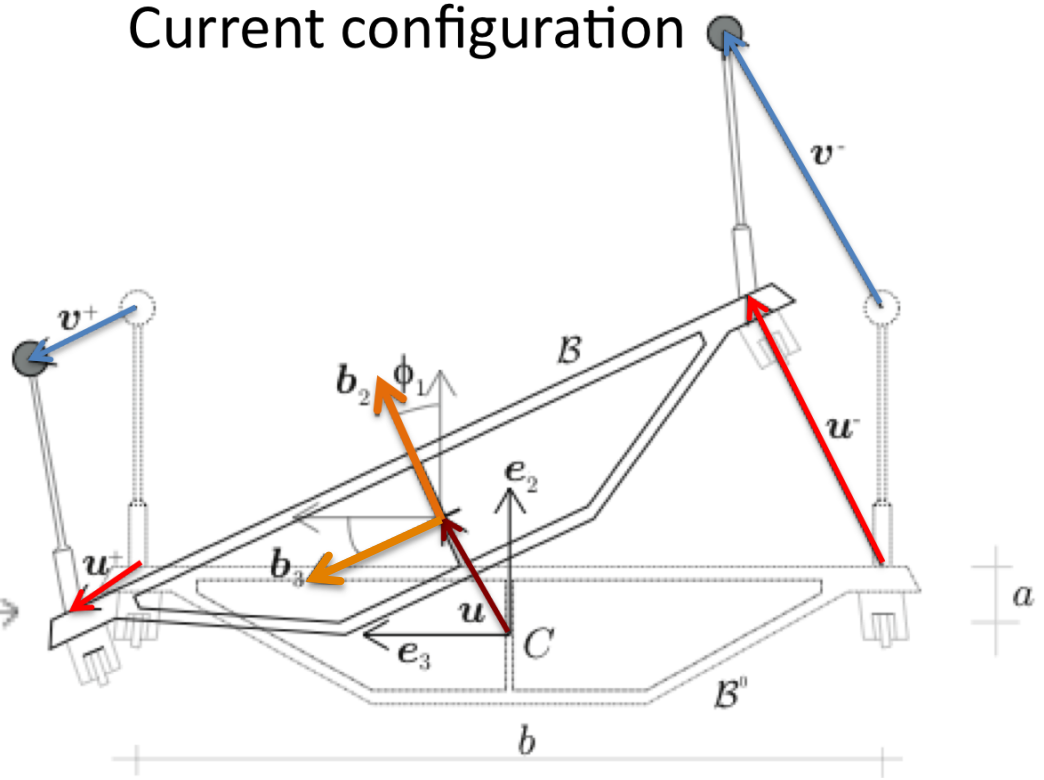
**W.Lacarbonara and A. Arena,**
"Three-dimensional model of suspension bridges via a fully nonlinear continuum formulation"
GIMC 2008, XVII Convegno Italiano di Meccanica Computazionale, Alghero, Sept 10-12, 2008.

# Nonlinear model of suspension bridges

Reference configuration




Current configuration




$\boldsymbol{y}^{\pm}(x)$:= *catenary* cable equilibrium

$$\boldsymbol{r}^{\pm}(x,t) := \boldsymbol{y}^{\pm}(x) + \boldsymbol{v}^{\pm}(x,t)$$

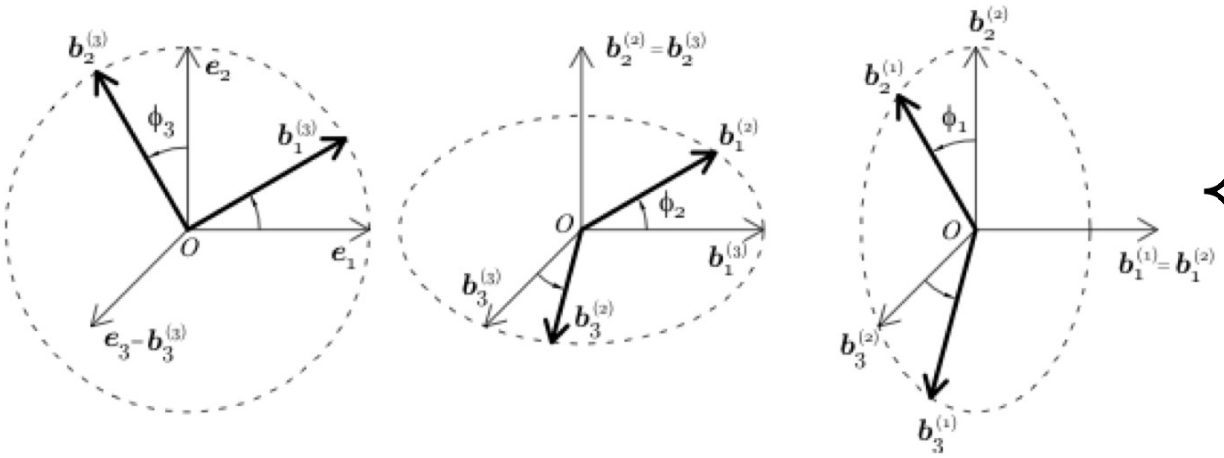
$$\boldsymbol{u}^{\pm}(x,t) = (\boldsymbol{R}(x,t) - \boldsymbol{I}) \cdot \bar{\boldsymbol{x}}^{\pm} + \boldsymbol{u}(x,t)$$

kinematic descriptors (12)

$$\boldsymbol{u}(x,t),\ \boldsymbol{v}^{+}(x,t),\ \boldsymbol{v}^{-}(x,t),\ \boldsymbol{R}[\phi_1(x,t), \phi_2(x,t), \phi_3(x,t)]$$

# Nonlinear model of suspension bridges

Finite rotations of the deck section-fixed directors




1. $\phi_3$ *flexural rotation about* $\boldsymbol{b}_3$
2. $\phi_2$ *flexural rotation about* $\boldsymbol{b}_2$
3. $\phi_1$ *torsional rotation about* $\boldsymbol{b}_1$

Orthogonal tensor (Rotation matrix)

$$\mathbf{R} = \begin{bmatrix} \cos\phi_2\cos\phi_3 & sin\phi_1 sin\phi_2\cos\phi_3 - \cos\phi_1 sin\phi_3 & \cos\phi_1 sin\phi_2\cos\phi_3 + sin\phi_1 sin\phi_3 \\ \cos\phi_2 sin\phi_3 & \cos\phi_1\cos\phi_3 + sin\phi_1 sin\phi_2 sin\phi_3 & \cos\phi_1 sin\phi_2 sin\phi_3 - sin\phi_1\cos\phi_3 \\ -sin\phi_2 & \cos\phi_2 sin\phi_1 & \cos\phi_1\cos\phi_2 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{b}_1(x,t) \\ \boldsymbol{b}_2(x,t) \\ \boldsymbol{b}_3(x,t) \end{bmatrix} = \mathbf{R}^{\mathrm{T}}(x,t) \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix}$$

# Nonlinear model of suspension bridges

CABLES

$$\boldsymbol{\nu}^{\pm} = \partial_s \boldsymbol{r}^{\pm} = \nu^{\pm} \boldsymbol{b}_1^{\pm}$$

stretches

$$\nu^{\pm} := \sqrt{\left(1 + \partial_s v_1^{\pm}\right)^2 + \left(\tan\theta^0 + \partial_s v_2^{\pm}\right)^2 + \left(\partial_s v_3^{\pm}\right)^2}$$

DECK

$$\boldsymbol{\nu} = \partial_x \boldsymbol{r} = \nu \boldsymbol{b}_1 + \eta_2 \boldsymbol{b}_2 + \eta_3 \boldsymbol{b}_3$$

stretch

shear strains

$$\nu = (1 + \partial_x u_1)\cos\phi_2 \cos\phi_3 + \partial_x u_2 \cos\phi_2 \sin\phi_3 - \partial_x u_3 \sin\phi_2$$

$$\begin{aligned}\eta_2 = {} & (1 + \partial_x u_1)(\sin\phi_1 \sin\phi_2 \cos\phi_3 - \cos\phi_1 \sin\phi_3) \\ & + \partial_x u_2 (\cos\phi_1 \cos\phi_3 + \sin\phi_1 \sin\phi_2 \sin\phi_3) \\ & + \partial_x u_3 \sin\phi_1 \cos\phi_2 \\ \eta_3 = {} & (1 + \partial_x u_1)(\cos\phi_1 \sin\phi_2 \cos\phi_3 + \sin\phi_1 \sin\phi_3) \\ & + \partial_x u_2 (\cos\phi_1 \sin\phi_2 \sin\phi_3 - \sin\phi_1 \cos\phi_3) \\ & + \partial_x u_3 \cos\phi_1 \cos\phi_2\end{aligned}$$

$$\partial_x \boldsymbol{b}_k = \boldsymbol{\mu} \times \boldsymbol{b}_k$$

curvature vector

$$\begin{aligned}\mu_1 &= \partial_x \phi_1 - \partial_x \phi_3 \sin\phi_2 \\ \mu_2 &= \partial_x \phi_2 \cos\phi_1 + \partial_x \phi_3 \sin\phi_1 \cos\phi_2 \\ \mu_3 &= \partial_x \phi_3 \cos\phi_1 \cos\phi_2 - \partial_x \phi_2 \sin\phi_1\end{aligned}$$

HANGERS

$$\nu_H^{\pm}(x,t) := \sqrt{\left(v_1^{\pm} - u_1^{\pm}\right)^2 + \left(h^0 + v_2^{\pm} - u_2^{\pm}\right)^2 + \left(v_3^{\pm} - u_3^{\pm}\right)^2}/h^0$$

# Nonlinear model of suspension bridges

Generalized stress resultants (dual to strains)

Cables tensions

$$\boldsymbol{n}^{\pm} := \int_A \boldsymbol{T}^{\pm} \cdot \boldsymbol{b}_1^{\pm\circ} dA = N^{\pm}\boldsymbol{b}_1^{\pm} \quad \longleftrightarrow \quad \boldsymbol{\nu}^{\pm}$$

Deck contact forces & couples

$$\boldsymbol{n} := \int_A \boldsymbol{T} \cdot \boldsymbol{b}_1^{\circ} dA = N\boldsymbol{b}_1 + Q_2\boldsymbol{b}_2 + Q_3\boldsymbol{b}_3 \quad \longleftrightarrow \quad \boldsymbol{\nu}$$

$$\boldsymbol{m} := \int_A \breve{\boldsymbol{x}} \times (\boldsymbol{T} \cdot \boldsymbol{b}_1^{\circ}) dA = T\boldsymbol{b}_1 + M_2\boldsymbol{b}_2 + M_3\boldsymbol{b}_3 \quad \longleftrightarrow \quad \boldsymbol{\mu}$$

Hangers forces

$$\boldsymbol{n}_{\mathrm{H}}^{\pm}(x,t) = \sum_{p=1}^{M} N_p^{\pm}(x,t)\boldsymbol{e}_p^{\pm}(x,t)\delta(x - x_p)$$

$$\breve{\boldsymbol{x}} = \boldsymbol{R} \cdot \bar{\boldsymbol{x}}$$

Mechanical data (forces & couples)

$$\begin{cases} \boldsymbol{f}^{\pm}(x,t) = f_1^{\pm}(x,t)\boldsymbol{e}_1 + \left[-mg(x_s^0)^{-1} + f_2^{\pm}(x,t)\right]\boldsymbol{e}_2 + f_3^{\pm}(x,t)\boldsymbol{e}_3 \\ \boldsymbol{f}(x,t) = f_1(x,t)\boldsymbol{e}_1 + \left[-\rho Ag + f_2(x,t)\right]\boldsymbol{e}_2 + f_3(x,t)\boldsymbol{e}_3, \\ \boldsymbol{c}(x,t) = c_1(x,t)\boldsymbol{e}_1 + c_2(x,t)\boldsymbol{e}_2 + c_3(x,t)\boldsymbol{e}_3 \end{cases}$$

# Nonlinear model of suspension bridges

Equations of motion (local statement of balance of mom.)

$$\partial_x \boldsymbol{n}^+ - \boldsymbol{n}_{\mathrm{H}}^+ + \boldsymbol{f}^+ = \rho A_{\mathrm{C}} \ddot{\boldsymbol{r}}^+$$
$$\partial_x \boldsymbol{n}^- - \boldsymbol{n}_{\mathrm{H}}^- + \boldsymbol{f}^- = \rho A_{\mathrm{C}} \ddot{\boldsymbol{r}}^-$$

suspension cables

$$\partial_x \boldsymbol{n} + \boldsymbol{n}_{\mathrm{H}}^+ + \boldsymbol{n}_{\mathrm{H}}^- + \boldsymbol{f} = \rho A \ddot{\boldsymbol{r}}$$
$$\partial_x \boldsymbol{m} + \boldsymbol{\nu} \times \boldsymbol{n} + \tfrac{b}{2} \boldsymbol{b}_3 \times (\boldsymbol{n}_{\mathrm{H}}^+ - \boldsymbol{n}_{\mathrm{H}}^-) + \boldsymbol{c} = \rho \boldsymbol{J} \cdot \dot{\boldsymbol{\omega}}$$

deck

Constitutive laws

$$N(x,t) = EA(x)(\nu(x,t) - 1),$$
$$T(x,t) = GJ(x)\mu_1(x,t),$$
$$M_k(x,t) = EJ_k(x)\mu_k(x,t), \quad k = 1,2$$

$$N^{\pm}(x,t) = N^{0}(x) + EA_{\mathrm{C}}(x)[\nu^{\pm}(x,t)/\nu^{0}(x) - 1]$$

$$N_{\boldsymbol{p}}^{\pm}(x_{\boldsymbol{p}},t) = EA_{\boldsymbol{p}}\left(\nu_{\boldsymbol{p}}^{\pm}(x_{\boldsymbol{p}},t) - 1\right)$$

# Nonlinear model of suspension bridges

CABLE Equations of motion in the *fixed* basis

$$\partial_x \left[ N^{\pm} \cos\theta^0 / \nu^{\pm} \left( 1 + \partial_x v_1^{\pm} \right) \right] - h_1^{\pm} + f_1^{\pm} = \rho A_{\mathrm{C}} \ddot{v}_1^{\pm}$$

$$\partial_x \left[ N^{\pm} \cos\theta^0 / \nu^{\pm} \left( \tan\theta^0 + \partial_x v_2^{\pm} \right) \right] - h_2^{\pm} + f_2^{\pm} = \rho A_{\mathrm{C}} \ddot{v}_2^{\pm}$$

$$\partial_x \left( N^{\pm} \cos\theta^0 / \nu^{\pm} \partial_x v_3^{\pm} \right) - h_3^{\pm} + f_3^{\pm} = \rho A_{\mathrm{C}} \ddot{v}_3^{\pm}$$

DECK Equations of motion in the *local* basis

$$\partial_x N - \mu_3 Q_2 + \mu_2 Q_3 + f_1 + h_1^+ + h_1^- = \rho A \ddot{\boldsymbol{r}} \cdot \boldsymbol{b}_1,$$

$$\partial_x Q_2 + \mu_3 N - \mu_1 Q_3 + f_2 + h_2^+ + h_2^- = \rho A \ddot{\boldsymbol{r}} \cdot \boldsymbol{b}_2,$$

$$\partial_x Q_3 - \mu_2 N + \mu_1 Q_2 + f_3 + h_3^+ + h_3^- = \rho A \ddot{\boldsymbol{r}} \cdot \boldsymbol{b}_3,$$

$$\partial_x T - \mu_3 M_2 + \mu_2 M_3 - \frac{b}{2} \left( h_2^+ - h_2^- \right) + c_1 = (\rho \boldsymbol{J} \cdot \dot{\boldsymbol{\omega}}) \cdot \boldsymbol{b}_1,$$

$$\partial_x M_2 + \mu_3 T - \mu_1 M_3 - \nu Q_3 + \frac{b}{2} \left( h_1^+ - h_1^- \right) + c_2 = (\rho \boldsymbol{J} \cdot \dot{\boldsymbol{\omega}}) \cdot \boldsymbol{b}_2,$$

$$\partial_x M_3 - \mu_2 T + \mu_1 M_2 + \nu Q_2 + c_3 = (\rho \boldsymbol{J} \cdot \dot{\boldsymbol{\omega}}) \cdot \boldsymbol{b}_3.$$

# Discretization (PDE mode)

Balance equations implemented in a FE solver (COMSOL Multiphysics v. 3.5 )

$$c_a \frac{\partial^2 \boldsymbol{u}}{\partial t^2} + d_a \frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot \boldsymbol{\Gamma} = \mathbf{F}, \quad \Omega$$

$$-\boldsymbol{n} \cdot \boldsymbol{\Gamma} = G + \left( \frac{\partial R}{\partial \boldsymbol{u}} \right)^{\top} \boldsymbol{\mu}, \quad \partial\Omega \quad \text{Neumann}$$

$$0 = R, \quad \partial\Omega \quad \text{Dirichlet}$$

***General form*** in COMSOL

- $\Omega$ is the computational domain, (1-D bridge span [0,1])
- $\partial\Omega$ is the boundary where the Neumann and Dirichlet bc's are prescribed

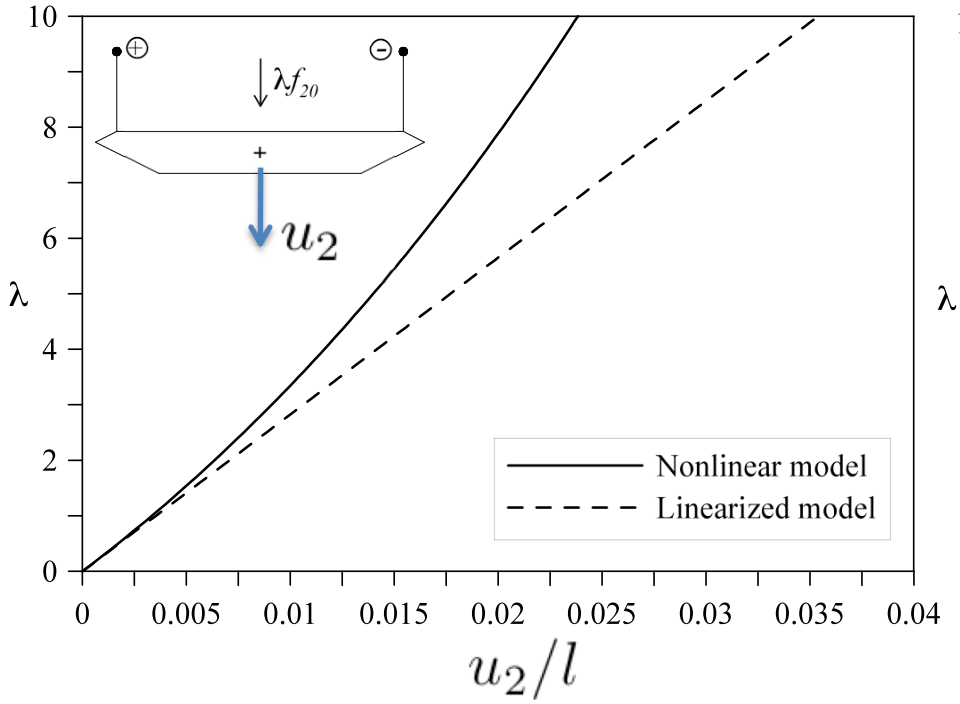
MAIN STEPS

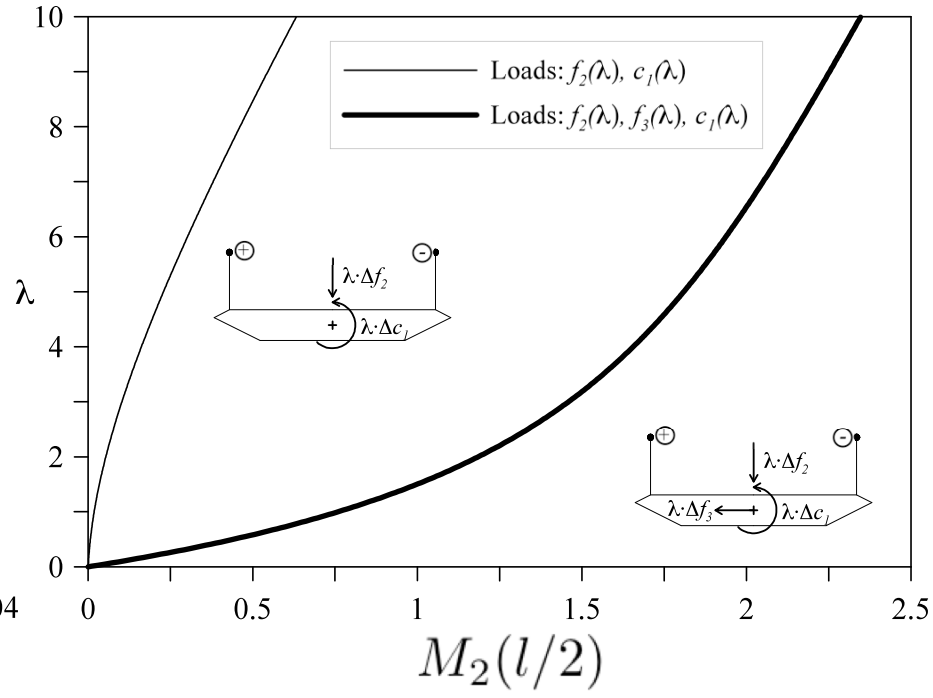
1. Mesh of the domain $\Omega$ and the FE type
2. Type of analysis (time dependent, stationary or parametric)
3. PDEs in the form $\nabla \cdot \Gamma = F$ (i.e., n×1 vectors in 1-D space):

$\Gamma$ general function of the independent variables and their derivatives

# Suspension bridges: equilibrium paths

λ multiplier of the dead loads



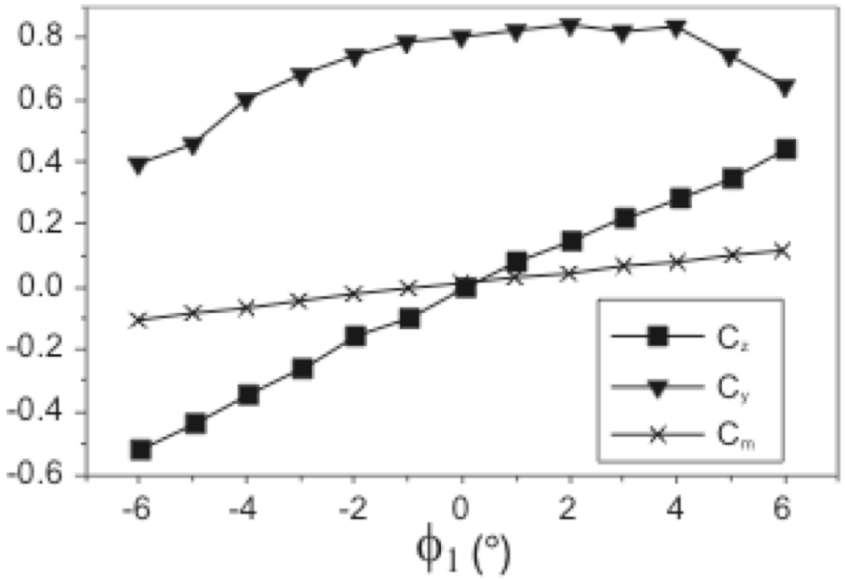

The ***hardening*** effect (cable geometric nonlinearity) is appreciable, even at low values of the weight multiplier λ

Without horizontal forces $f_3$, the flexural coupling induces a moment about the $\boldsymbol{e}_2$-axis

# Suspension bridges: torsional divergence

Hu Men Suspension Bridge, Zhu Jiang river

Drag, Lift and Aerostatic Moment Coefficients




Flexural-torsional motions linearly uncoupled

$$k^{\mathrm{T}}\phi_1 = \tfrac{1}{2}\rho U^2 b^2 \left(C_0^{\mathrm{M}} + C_1^{\mathrm{M}}\phi_1\right)$$

$$\left(k^{\mathrm{T}} - \lambda k^{\mathrm{A}}\right)\phi_1 = \tfrac{1}{2}\rho U^2 b^2 C_0^{\mathrm{M}}$$

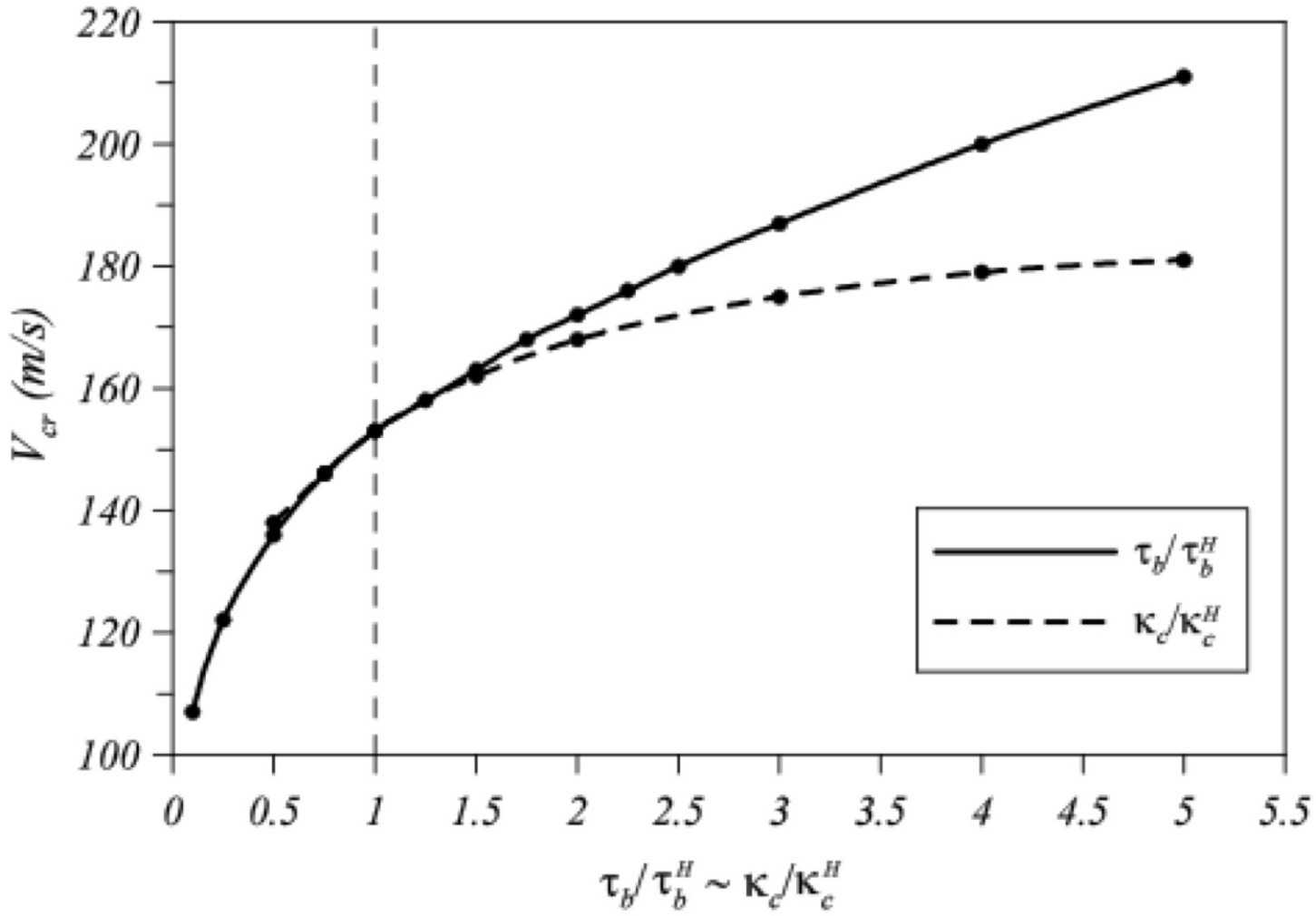
$$U_{\mathrm{cr}} = \sqrt{\frac{2k^{\mathrm{T}}}{\rho b^2 C_1^{\mathrm{M}}}}$$

Torsional divergence and lateral buckling within nonlinear formulation: *flexural-torsional coupling affects divergence condition*

Drag force considered only for suspension cables $C_{D,c} = 0.7$ (Xiang, 1996)

# Suspension bridges: torsional divergence

Sensitivity to main stiffness parameters



# Ponte della Musica

"Ponte della Musica", render by courtesy of ***Powell-Williams Architects***

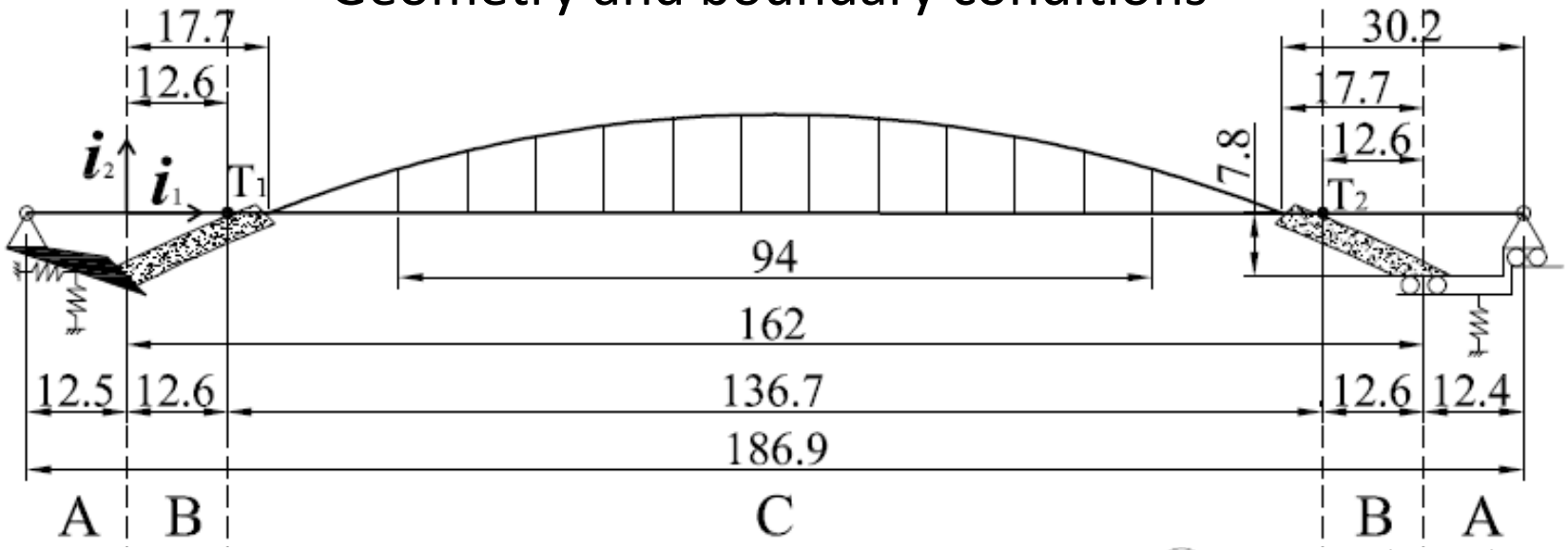
**W. Lacarbonara and A. Arena** (2009)
"Static and aeroelastic limit states of the Bridge of Sound via a fully nonlinear continuum model",
Pap. No. 236, Proc. of XIX Congresso AIMETA, Ancona, 14–17 settembre 2009.

**W. Lacarbonara, A. Arena** (2010)
"Flutter of an arch bridge via a fully nonlinear continuum Formulation", *Journal of Aerospace Engineering*, in press.

"*Ponte della Musica*" (pedestrian-vehicular)
*Lungotevere Flaminio* Rome

# Ponte della Musica

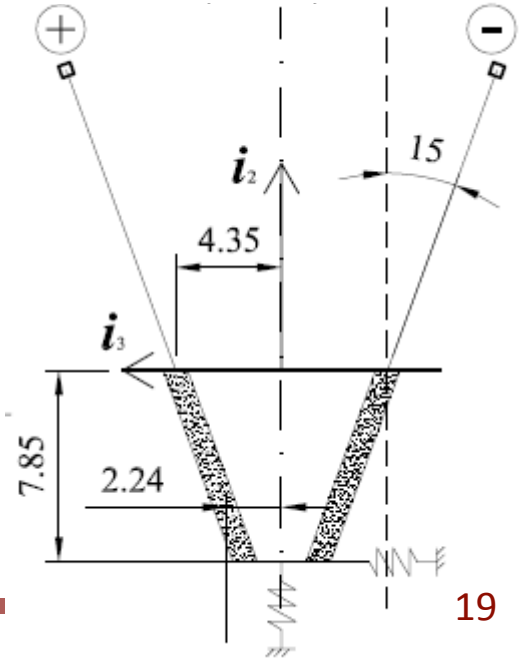
## Geometry and boundary conditions




Two parabolic arches inclined by 15°,
*high-resistance r.c. & steel,* span l=162 m

Through-type arch bridge: *self-absorbed thrust*

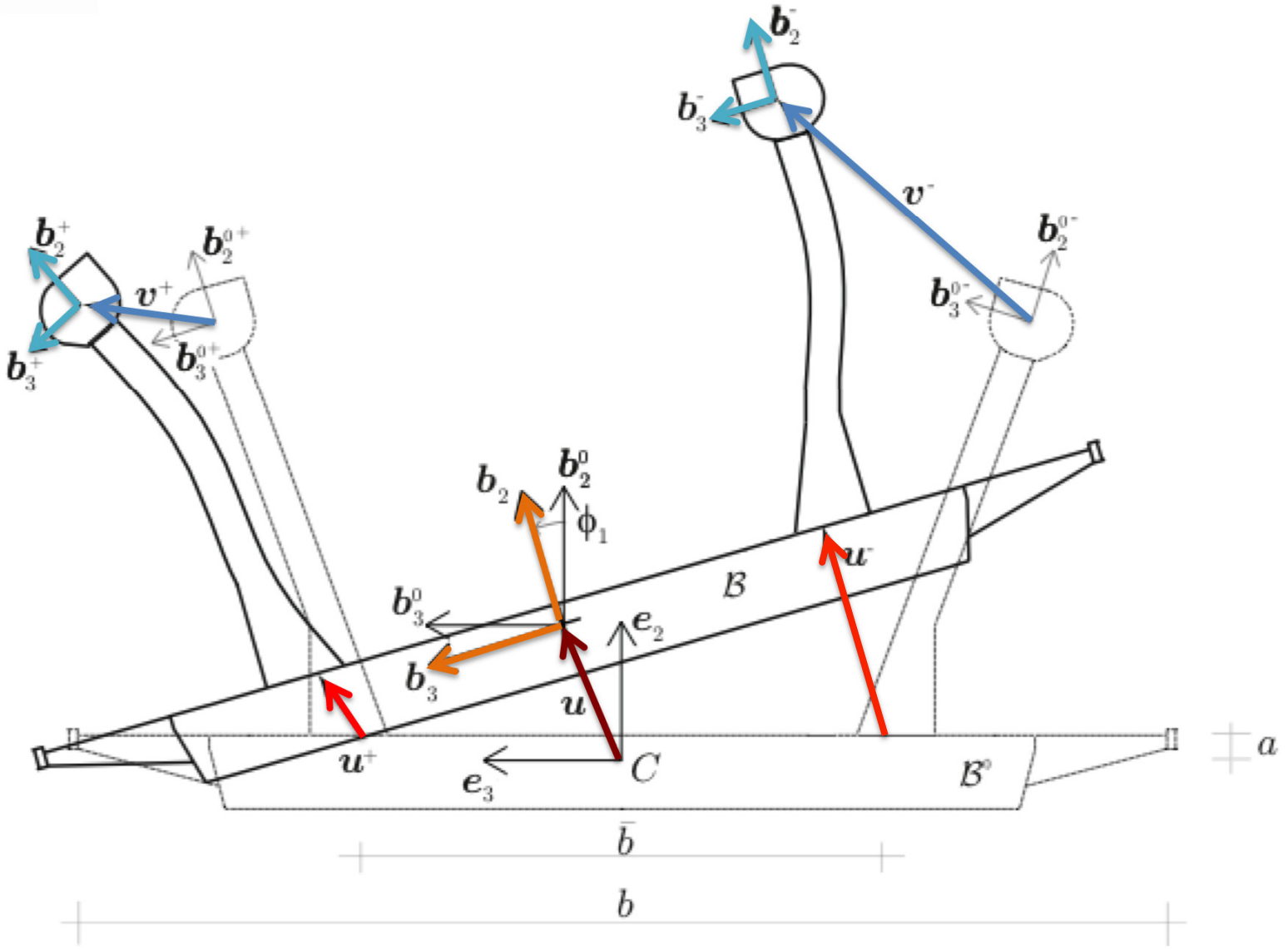
Deck main sizes: l=186.90 m, $h_{max}$=1.40 m, $b_{car}$=19.50 m



# Ponte della Musica

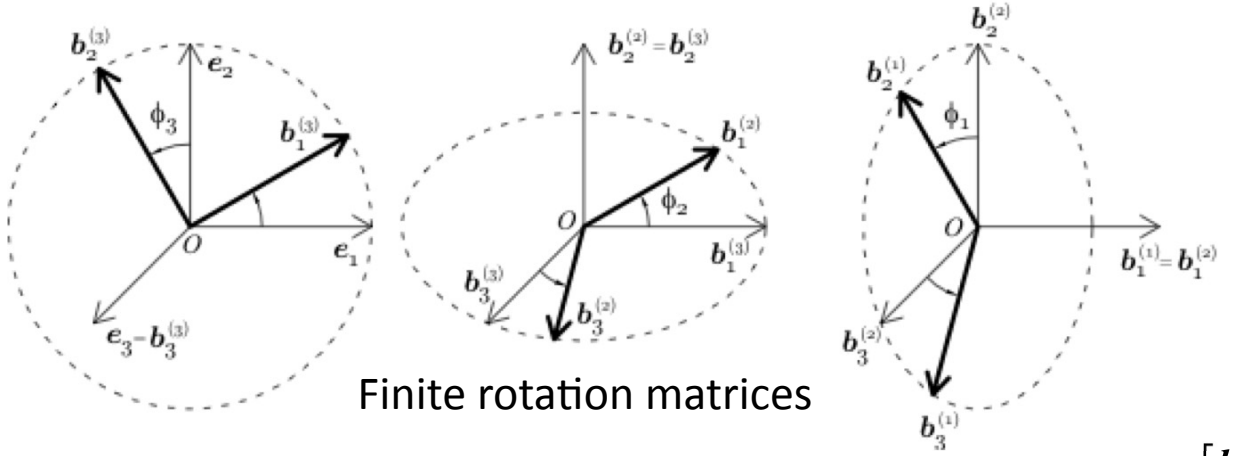
*While being built and assembled in the factory*

# Arch bridges: kinematics



# Arch bridges: kinematics

Finite rotations of the deck and arches section-directors



1. *Flexural rotation* $\phi_3$ $(\phi_3^{\pm})$ *about* $\boldsymbol{b}_3$ $(\boldsymbol{b}_3^{\pm})$
2. *Flexural rotation* $\phi_2$ $(\phi_2^{\pm})$ *about* $\boldsymbol{b}_2$ $(\boldsymbol{b}_2^{\pm})$
3. *Torsional rotation* $\phi_1$ $(\phi_1^{\pm})$ *about* $\boldsymbol{b}_1$ $(\boldsymbol{b}_1^{\pm})$

Finite rotation matrices

$$\mathbf{Q}^{\pm} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha^{\pm} & -\sin\alpha^{\pm} \\ 0 & \sin\alpha^{\pm} & \cos\alpha^{\pm} \end{bmatrix} \qquad \mathbf{Q}^{0}(x) = \begin{bmatrix} \cos\theta(x) & -\sin\theta(x) & 0 \\ \sin\theta(x) & \cos\theta(x) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{b}_1^{\pm}(x,t) \\ \boldsymbol{b}_2^{\pm}(x,t) \\ \boldsymbol{b}_3^{\pm}(x,t) \end{bmatrix} = \left(\mathbf{Q}^{\pm}\cdot\mathbf{Q}^{0}(x)\cdot\mathbf{R}^{\pm}(x,t)\right)^{\mathrm{T}} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \cos\phi_2\cos\phi_3 & \sin\phi_1\sin\phi_2\cos\phi_3 - \cos\phi_1\sin\phi_3 & \cos\phi_1\sin\phi_2\cos\phi_3 + \sin\phi_1\sin\phi_3 \\ \cos\phi_2\sin\phi_3 & \cos\phi_1\cos\phi_3 + \sin\phi_1\sin\phi_2\sin\phi_3 & \cos\phi_1\sin\phi_2\sin\phi_3 - \sin\phi_1\cos\phi_3 \\ -\sin\phi_2 & \cos\phi_2\sin\phi_1 & \cos\phi_1\cos\phi_2 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{b}_1(x,t) \\ \boldsymbol{b}_2(x,t) \\ \boldsymbol{b}_3(x,t) \end{bmatrix} = \mathbf{R}^{\mathrm{T}}(x,t) \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix}$$

***Arches*** **(*6x2*)**

$$\boldsymbol{\nu}^{\pm} = \partial_s \boldsymbol{r}^{\pm}$$
$$\partial_s \boldsymbol{b}_k^{\pm} = \boldsymbol{\mu}^{\pm} \times \boldsymbol{b}_k^{\pm} + \boldsymbol{R}^{\pm} \cdot \boldsymbol{\mu}_k^{\pm\circ} \times \boldsymbol{b}_k^{\pm\circ}$$

**Strains** (18)

***Deck*** **(*6*)**

$$\boldsymbol{\nu} = \partial_x \boldsymbol{r}$$
$$\partial_x \boldsymbol{b}_k = \boldsymbol{\mu} \times \boldsymbol{b}_k$$

# Arch bridges: Equations of motion

Total Lagrangian formulation

$$\partial_x \boldsymbol{n}^{\pm}(x,t) + \boldsymbol{f}^{\pm}(x,t) + \boldsymbol{h}_j^{\pm}(x,t) = \rho A^{\mathrm{a}} \sec\theta \; \ddot{\boldsymbol{v}}^{\pm}(x,t)$$

$$\partial_x \boldsymbol{m}^{\pm}(x,t) + \partial_x \boldsymbol{r}^{\pm}(x,t) \times \boldsymbol{n}^{\pm}(x,t) + \boldsymbol{c}^{\pm}(x,t) + \boldsymbol{c}_j^{\pm}(x,t) = \rho \boldsymbol{J}^{\mathrm{a}} \sec\theta \cdot \dot{\boldsymbol{\omega}}^{\pm}(x,t)$$

$$\partial_x \boldsymbol{n}(x,t) + \boldsymbol{f}(x,t) + \boldsymbol{h}_i^{+}(x,t) + \boldsymbol{h}_i^{-}(x,t) = \rho A^{\mathrm{b}} \ddot{\boldsymbol{u}}(x,t)$$

$$\partial_x \boldsymbol{m}(x,t) + \partial_x \boldsymbol{r}(x,t) \times \boldsymbol{n}(x,t) + \frac{\overline{b}}{2} \boldsymbol{b}_3 \times \left(\boldsymbol{h}_i^{+}(x,t) - \boldsymbol{h}_i^{-}(x,t)\right) + \left(\boldsymbol{c}_i^{+}(x,t) + \boldsymbol{c}_i^{-}(x,t)\right)$$

$$+ \boldsymbol{c}(x,t) = \rho \boldsymbol{J}^{\mathrm{b}} \cdot \dot{\boldsymbol{\omega}}(x,t)$$

Connecting beams between arches (node *j*) and deck (node *i*)

$$\boldsymbol{h}_i^{\pm}(x,t) = \sum_{p=1}^{N_p} \delta(x - x_p)\, \boldsymbol{h}_{p,i}^{\pm}(t) \qquad \boldsymbol{h}_j^{\pm}(x,t) = \sum_{p=1}^{N_p} \delta(x - x_p)\, \boldsymbol{h}_{p,j}^{\pm}(t)$$

$$\boldsymbol{c}_i^{\pm}(x,t) = \sum_{p=1}^{N_p} \delta(x - x_p)\, \boldsymbol{c}_{p,i}^{\pm}(t) \qquad \boldsymbol{c}_j^{\pm}(x,t) = \sum_{p=1}^{N_p} \delta(x - x_p)\, \boldsymbol{c}_{p,j}^{\pm}(t)$$

Linear elastic constitutive equations

Balance equations in the fixed frame

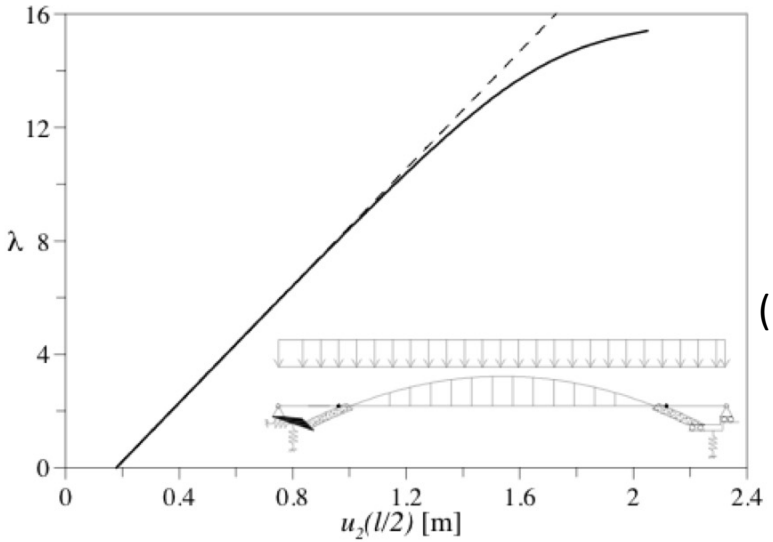
***18** scalar equations of motions in **18** kinematic independent parameters*

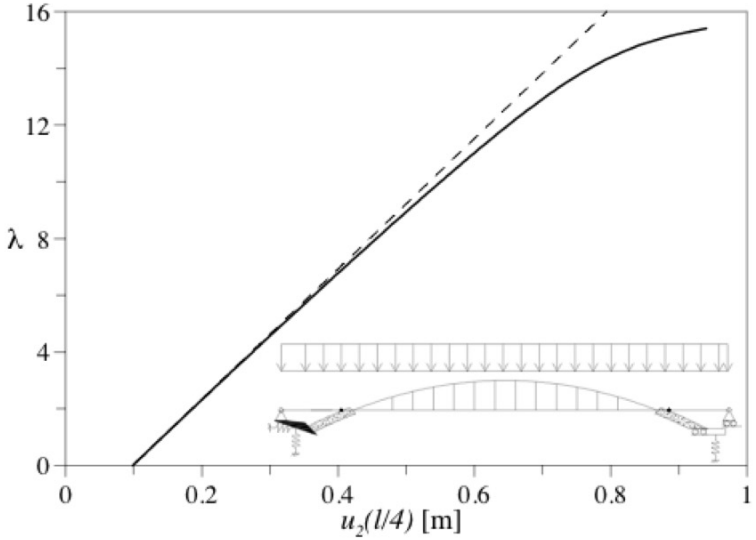
$$v_1^{\pm}(x,t), v_2^{\pm}(x,t), v_2^{\pm}(x,t), \phi_1^{\pm}(x,t), \phi_2^{\pm}(x,t), \phi_3^{\pm}(x,t)$$

$$u_1(x,t), u_2(x,t), u_3(x,t), \phi_1(x,t), \phi_2(x,t), \phi_3(x,t)$$

# Ponte della Musica: softening behavior

Deck vertical displacement

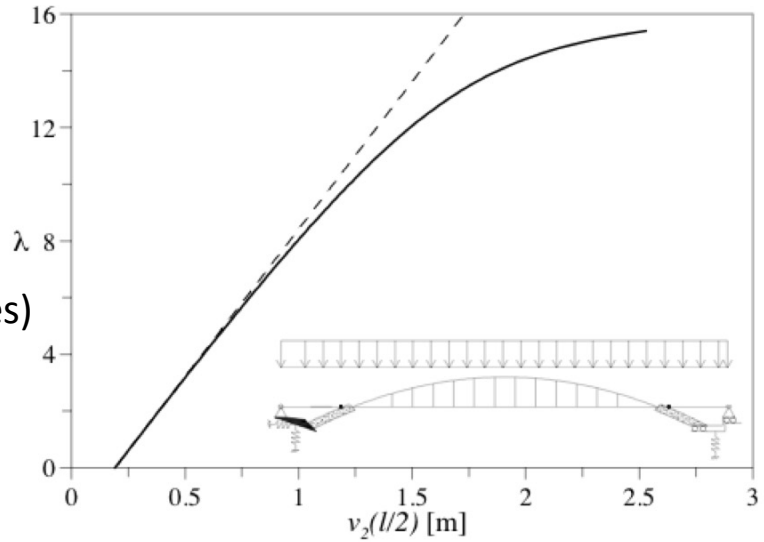


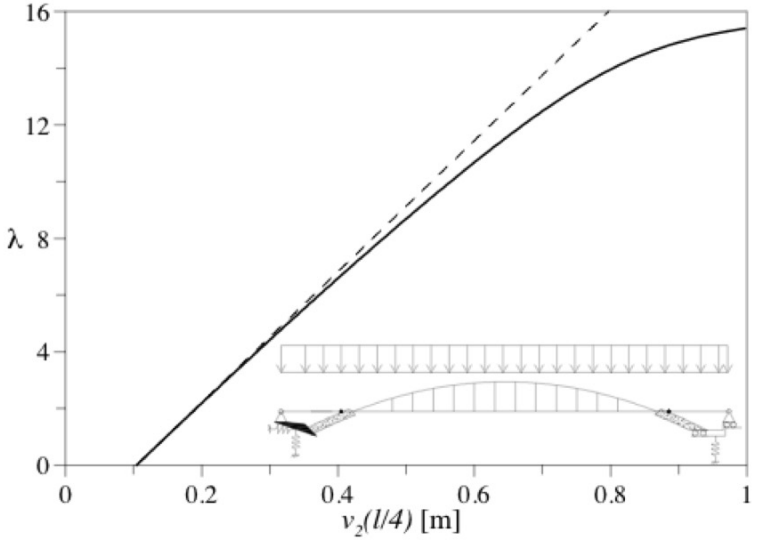




λ= multiplier of accidental loads (pedestrian + vehicles)

Limit point: λ≈14÷15

Arches vertical displacement

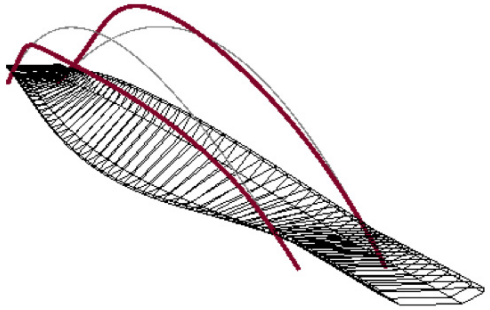


# Ponte della Musica: modal properties

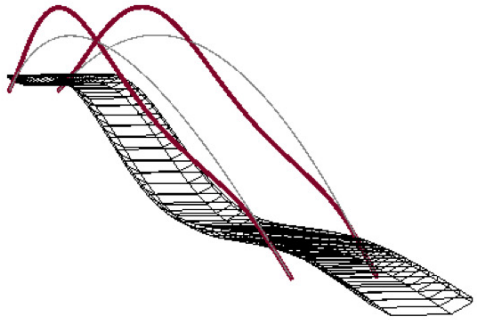
**Mode 1** Flex.-Tor. *symm*

$f = 0.726Hz$



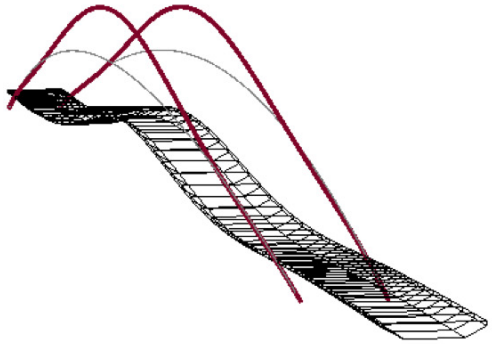
**Mode 2** Flex. *skew*

$f = 0.770Hz$



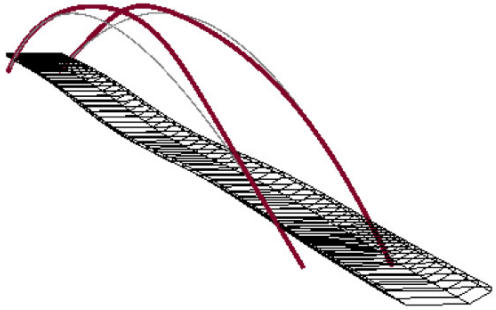
**Mode 3** Flex. *symm*

$f = 0.905Hz$



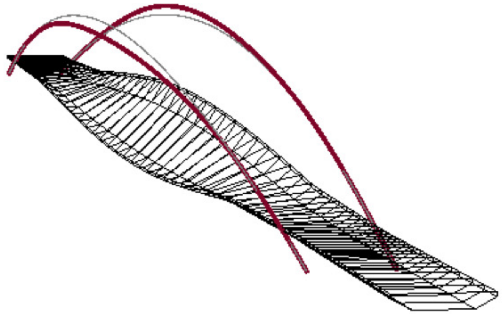
**Mode 4** Flex. *symm*

$f = 1.261Hz$



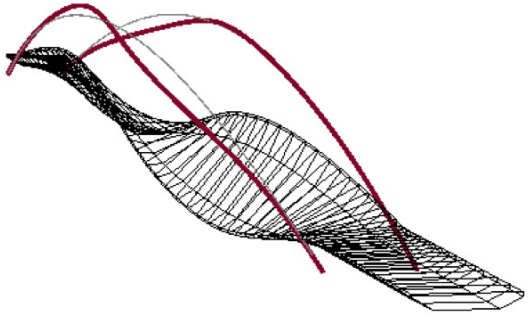
**Mode 5** Flex.-Tor. *symm*

$f = 1.354Hz$



**Mode 6** Flex.-Tor. *skew*

$f = 1.692Hz$



# Ponte della Musica: frequencies

Continuum nonlinear model

| | Modo 1 Flex-Tor *symm* $u_3, v_3^{\pm}, \phi_1, v_2^{\pm}$ | Modo 2 Flex *skew* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 3 Flex *symm* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 4 Flex *symm* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 6 Flex-Tor *skew* $u_3, v_3^{\pm}, \phi_1, v_2^{\pm}$ |
|---|---|---|---|---|---|
| $T_i[s]$ | 1.37682 | 1.29931 | 1.10461 | 0.79299 | 0.5894 |
| $f_i[Hz]$ | 0.726 | 0.770 | 0.905 | 1.261 | 1.6965 |

FE linear model (working design)

| | Modo 1 Flex-Tor *symm* $u_3, v_3^{\pm}, \phi_1, v_2^{\pm}$ | Modo 2 Flex *symm* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 3 Flex *skew* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 4 Flex *symm* $u_2, v_2^{\pm}, (v_3^{\pm})$ | Modo 5 Flex-Tor *skew* $u_3, v_3^{\pm}, \phi_1, v_2^{\pm}$ |
|---|---|---|---|---|---|
| $T_i[s]$ | 1.40919 | 1.40013 | 1.30784 | 0.72421 | 0.64822 |
| $f_i[Hz]$ | 0.710 | 0.714 | 0.765 | 1.381 | 1.543 |

Experimental test results from sectional model (wind tunnel)

| Modi | Descrizione | $f_z[Hz]$ | $f_\alpha[Hz]$ |
|---|---|---|---|
| 1 | Simmetrico ad un ventre | - | 0.7013 |
| 2-5 | Antisimmetrici a due ventri | 0.7617 | 1.5380 |
| 3-6 | Simmetrici ad un ventre | 0.8256 | 1.8748 |
| 4-6 | Simmetrici rispettivamente a tre (flessione verticale) ed un ventre (torsione) | 1.4058 | 1.8748 |

# Ponte della Musica: flutter

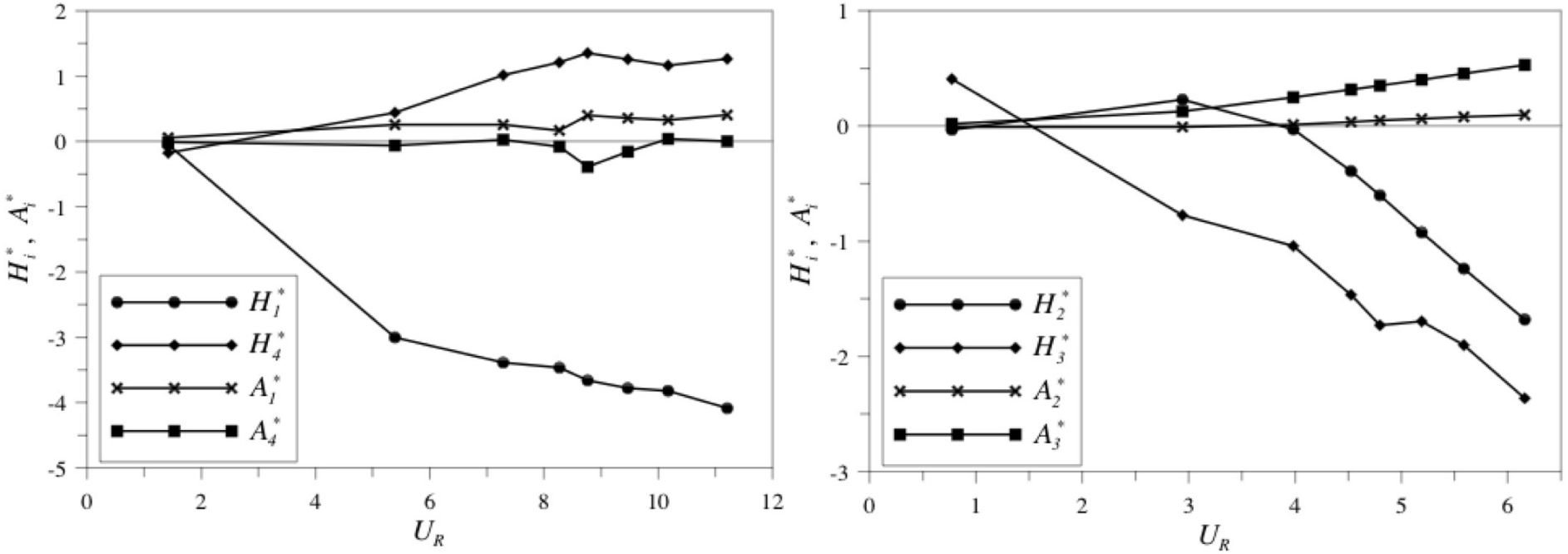
Flutter eigenvalue problem

$$\mathbf{B}(v^{\pm},\phi^{\pm},u,\phi)+\mathbf{H}(v^{\pm},\phi^{\pm},u,\phi)=\mathbf{M}(x)\,\ddot{\mathbf{U}}+\left(\mathbf{C}-\mathbf{C}^{A}\right)\dot{\mathbf{U}}-\mathbf{K}^{A}\mathbf{U}$$

$$\left(u(x,t),\phi(x,t),v^{\pm}(x,t),\phi^{\pm}(x,t)\right)=\left(\tilde{u}(x),\tilde{\phi}(x),\tilde{v}^{\pm}(x),\tilde{\phi}^{\pm}(x)\right)e^{i\sigma t}$$

$\sigma = \sigma_R + i\,\sigma_I$ *Flutter* condition at $\sigma_I = 0$

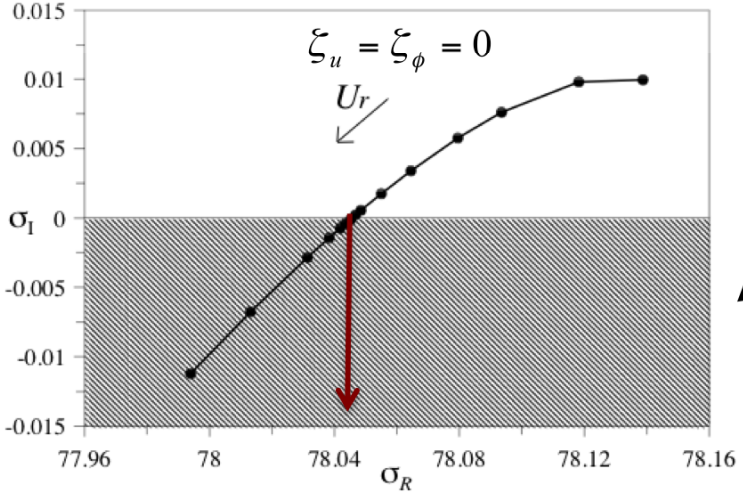
Aeroelastic derivatives at wind angle of attack +3° (*CRIACIV*, Prato, 2009)



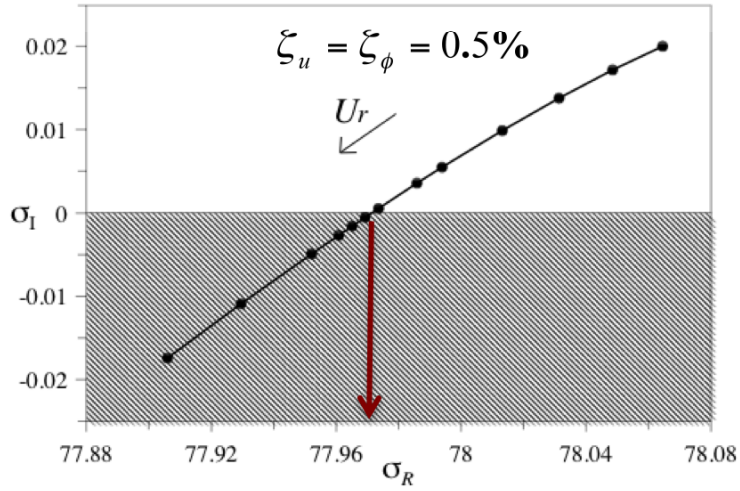

Courtesy of CRIACIV, Prato, Italy

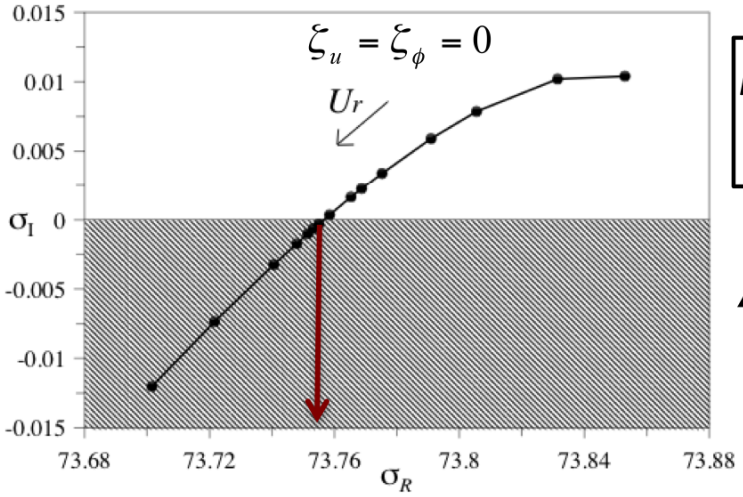
# Ponte della Musica: flutter

Eigenvalues real and complex parts : stable regions (white) and unstable (grey)

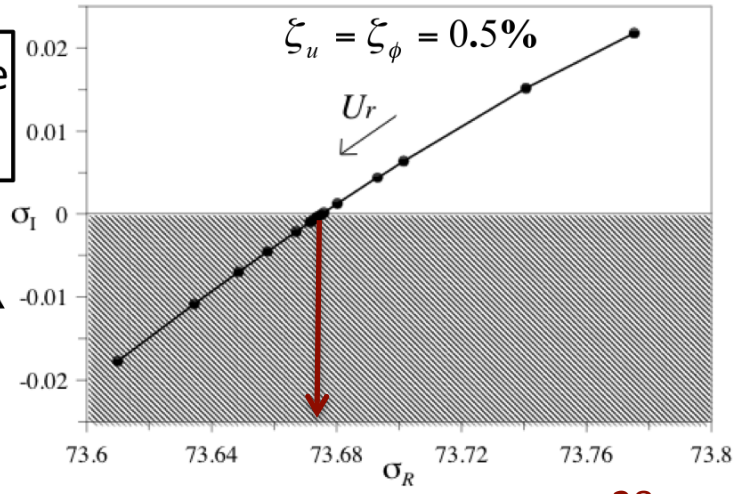



*Stress-free*




*Pre-stressed* structure (permanent loads)

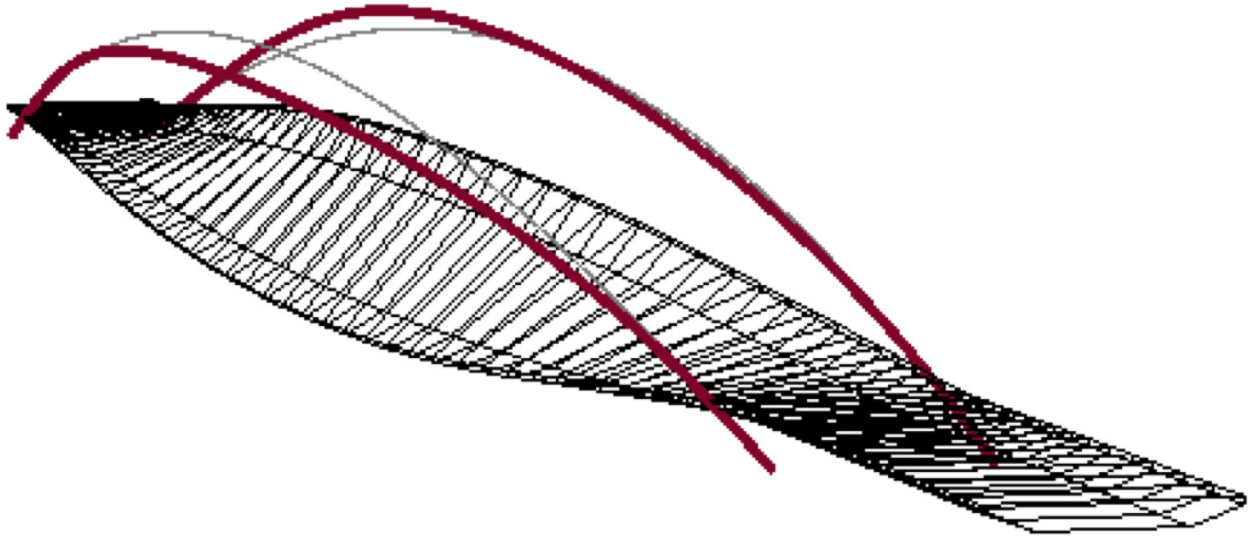


# Ponte della Musica: flutter

| | Undamped | | | | $\zeta_u = \zeta_\phi = 0.5\%$ | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $U_{r,c}$ | $f_c[Hz]$ | $U_c[m/s]$ | $U_c[km/h]$ | $U_{r,c}$ | $f_c[Hz]$ | $U_c[m/s]$ | $U_c[km/h]$ |
| Wind tunnel experimental results | | | | | | | | |
| 0° | $> 16.3$ | | | | $> 16.3$ | | | |
| +3° | 3.61 | 0.66 | 46.7 | 168.12 | 4.21 | 0.65 | 53.4 | 192.24 |
| Continuum nonlinear model results | | | | | | | | |
| 0° | $> 16.3$ | | | | $> 16.3$ | | | |
| +3° | 3.29 | 0.729 | 47.18 | 169.85 | 4.275 | 0.728 | 60.68 | 218.45 |
| Continuum nonlinear model results, pre-stress contribution | | | | | | | | |
| 0° | $> 16.3$ | | | | $> 16.3$ | | | |
| +3° | 3.275 | 0.689 | 43.97 | 158.31 | 4.305 | 0.688 | 57.74 | 207.86 |

Neglecting prestress: overestimation of flutter speed by 7 %!

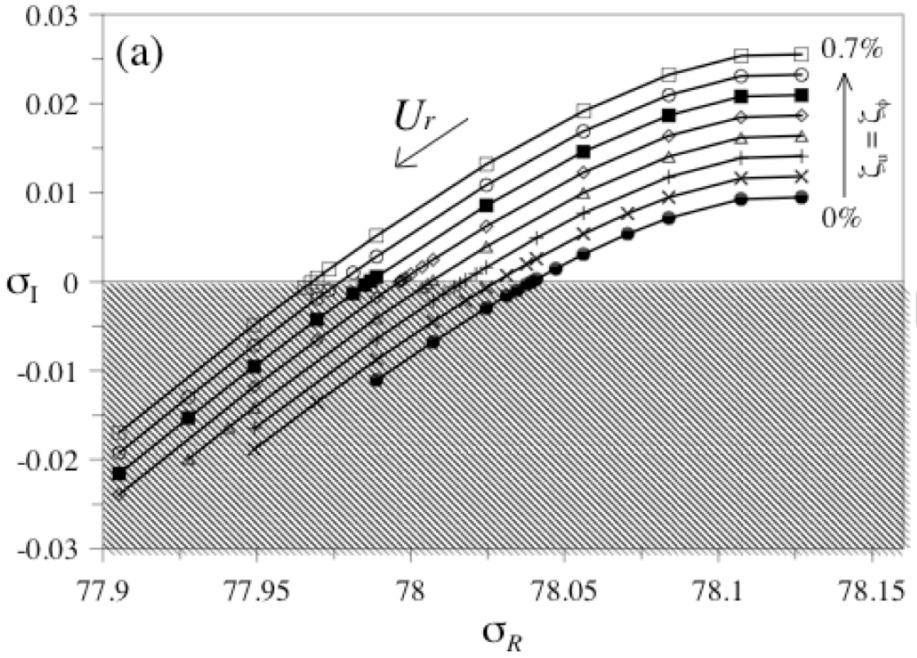
# Ponte della Musica: flutter mode

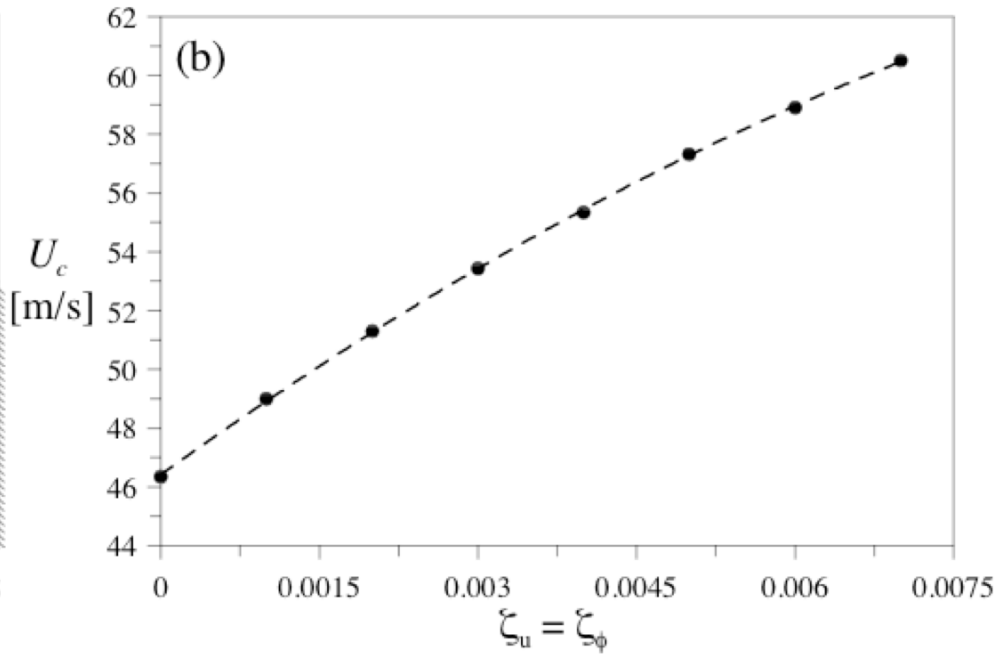


Lowest *flutter* mode ($f = 0.73$ Hz, $U_c = 47.18$ m/s)

# Ponte della Musica: sensitivity analyses

Effect of damping ratio $\zeta_u = \zeta_\phi$



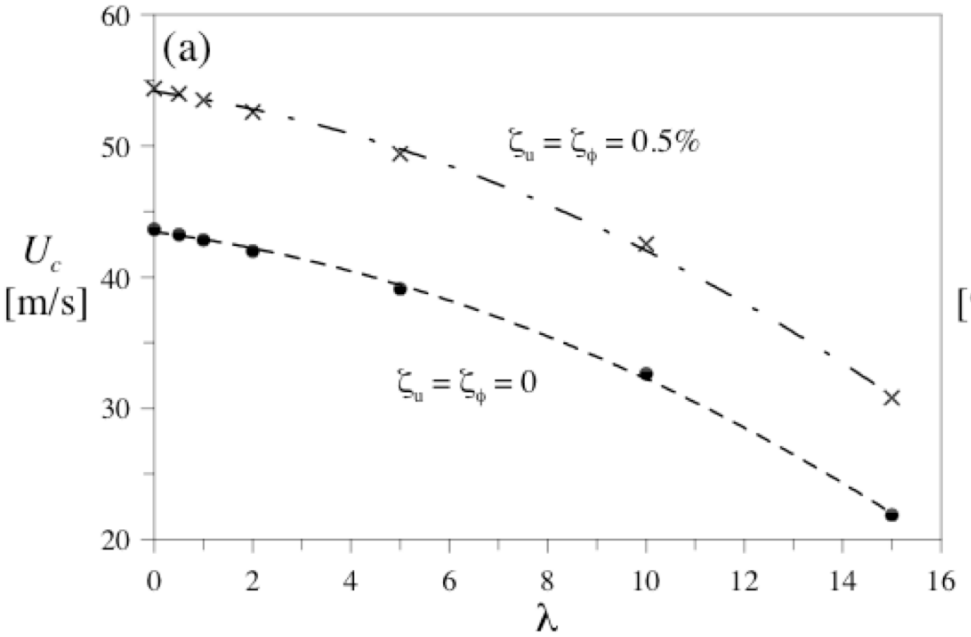



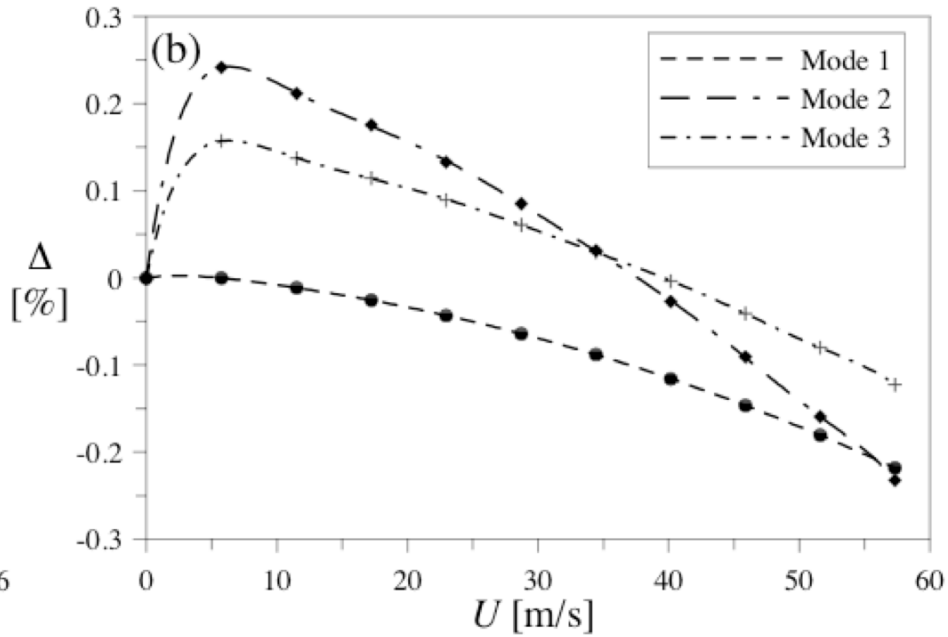

a) Complex space eigenvalue $\sigma$

b) Critical wind velocity vs damping ratio

# Ponte della Musica: sensitivity analyses

Effect of the structure pre-stress




a) Critical wind velocity for different pre-stress levels λ

b) Lowest three natural frequencies in the pre-critical wind speeds

# On-going work: indicial functions theory

On-going project with Prof. **Pier Marzocca**, Clarkson University, Potsdam, USA

Time response of unsteady aerodynamic coefficients due to a step change in the state variables

- Suitable for use in routine aeroelastic computations for flutter and nonlinear aeroelastic response, aeroservoelasticity
- Reduction of the computational costs as compared to full FSI based on CFD simulations
- Efficient formulation in preliminary design and optimization
- Accurate approach for the aerodynamic characteristics of bridge sections including nonlinear effects
- Unsteady airloads on bridge sections undergoing arbitrary motions.

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# Indicial functions theory

- **Linear theory**
  - Aerodynamic coefficients linearized with respect to the state variables if their variations are smooth functions of those states.
  - The linear step-change response can be convoluted with the input to predict the output of a linear system, the nonlinear indicial theory is a generalization of this concept.

- **Nonlinear theory**
  - Linear formalism in the form of a generalized superposition integral, states evolution depends on the entire history of the motion.
  - The aeroelastic response to an arbitrary input can be constructed by integrating a nonlinear functional, that involves the knowledge of the time-dependent input and the indicial response.

# Indicial functions theory

**Linear indicial function**

Cumulative response:
Duhamel convolution integral

$$C_f(t) = C_f^0(\alpha_m) + \frac{dC_f}{d\alpha}\left[\alpha(0)\Phi_f(t) + \int_0^t \frac{d\alpha}{dt}(\tau)\Phi_f(t-\tau)\,d\tau\right]$$

**Nonlinear indicial function**

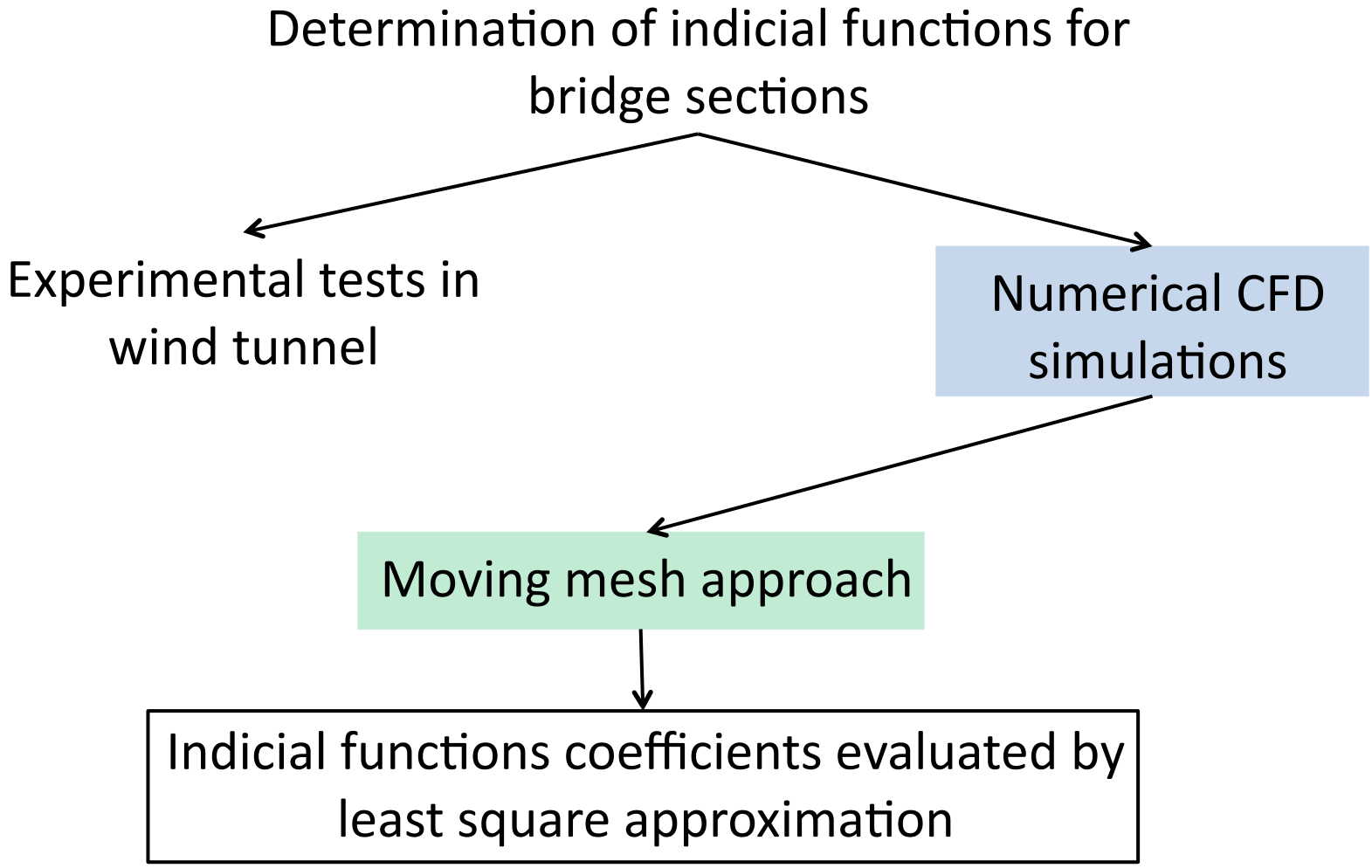
$$C_f(t) = C_f\left(t, \alpha(0)\right) + \int_0^t \left.\frac{dC_f}{d\alpha}\right|_\tau \frac{d\alpha}{dt}(\tau)\Phi_f(\alpha, t-\tau)\,d\tau$$

A simplification has been made by assuming $\Phi_f$ dependent only on the input and on the elapsed time ($t$-$\tau$)

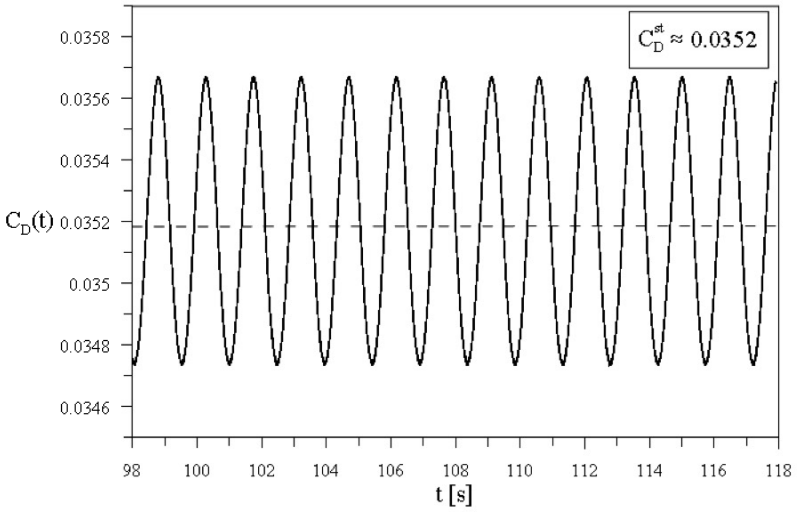
**Nonlinear effects due to**

- Amplitude in the step input (i.e. $\Delta\alpha$)
- Mean wind angle of attack $\alpha_m$
- Viscous effects in the range of wind speed *(Re)*

# Linear/nonlinear indicial functions



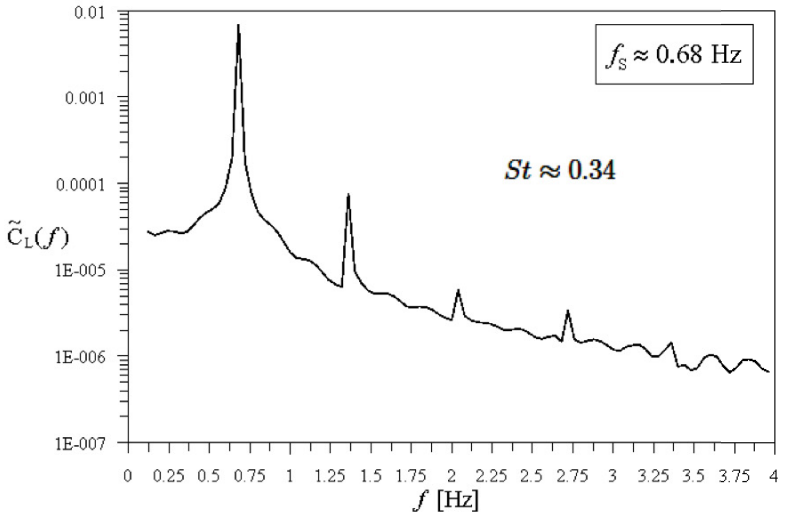
# CFD calculations



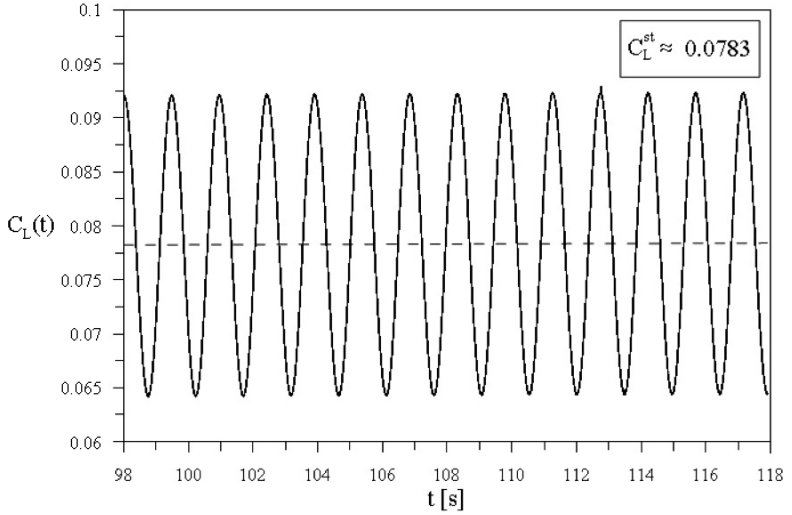

$$C_D(t) := \frac{F_D(t)}{\frac{1}{2}\rho U^2 B}$$

$$C_L(t) := \frac{F_L(t)}{\frac{1}{2}\rho U^2 B}$$

$$St := \frac{f\ D}{U}$$



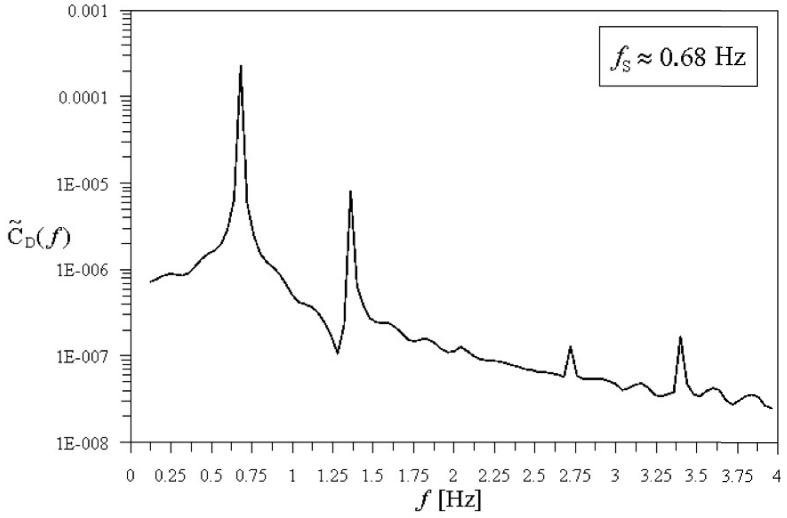

**Wide range of CFD results in literature**




$$-0.21 < C_L < 0.09$$

$$0.05 < C_D < 0.10$$

$$0.08 < St < 0.28$$



# Unsteady aerodynamics lag states

Use of indicial functions to generate unsteady aerodynamic forces

$$\Phi_\xi^j(s) = 1 - \sum_{i=1}^{N} A_i^{(j,\xi)}\, e^{-B_i^{(j,\xi)} s} \longrightarrow F^j(t) = G^j \sum_\xi C_\xi^j \left[ \Phi_\xi^j(0)\xi(t) + \int_0^t \dot{\Phi}_\xi^j(t-\tau)\xi(\tau)\, d\tau \right]$$

Additional lag states

$$W_\xi^j(t) := \int_0^t \dot{\Phi}_\xi^j(t-\tau)\xi(\tau)\, d\tau$$

$$W_{\xi,i}^j(t) := \int_0^t \varphi_i^{(j,\xi)}(t-\tau)\,\xi(\tau)\, d\tau$$

Evolution equation associated with the unsteady aerodynamic loads

$$\dot{W}_{\xi,i}^j(t) = \xi(t) - \hat{B}_i^{(j,\xi)} W_{\xi,i}^j(t)$$

- *N* : number of exponential terms *i*
- *M* : number of state variables $\xi$ (step change)
- *3* : number of aerodynamic loads (*Lift, Drag, Moment*)

Aerodynamic governing equations including
*3×(N·M)* state equations

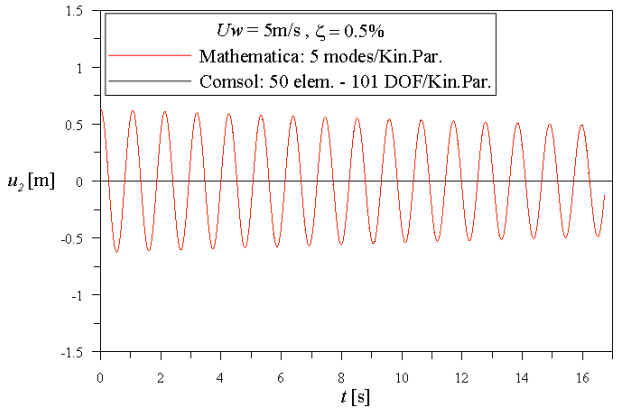
Reference: A. Arena, P. Marzocca, W. Lacarbonara,
Nonlinear dynamic stall in long flexible wings via geometrically exact approach,
*AIAA Journal*, to be submitted, 2010.

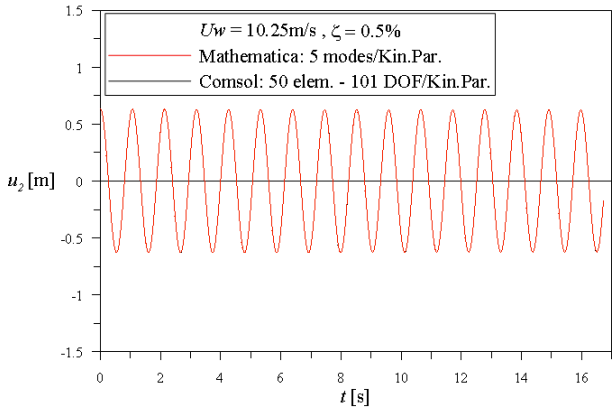
# Numerical approaches

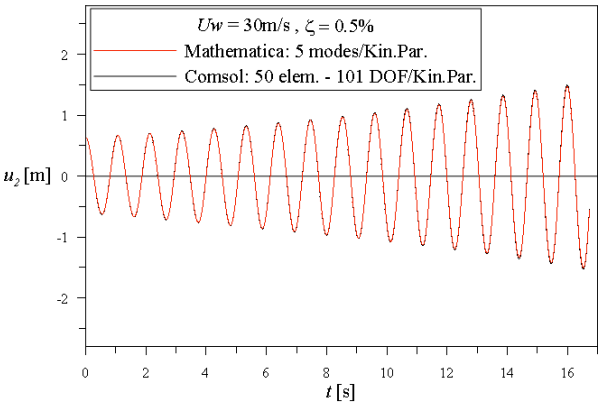
*Time-domain analyses*

Simply supported (linear) beam, rectangular section B/D = 14

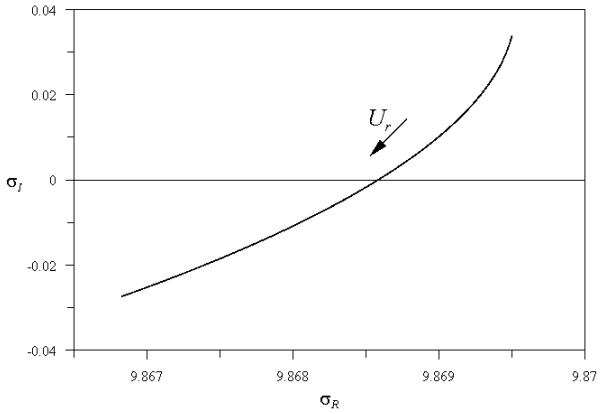
- Modal space projection and numerical integration of aerodynamic equations
- FE integration of the aerodynamic equations

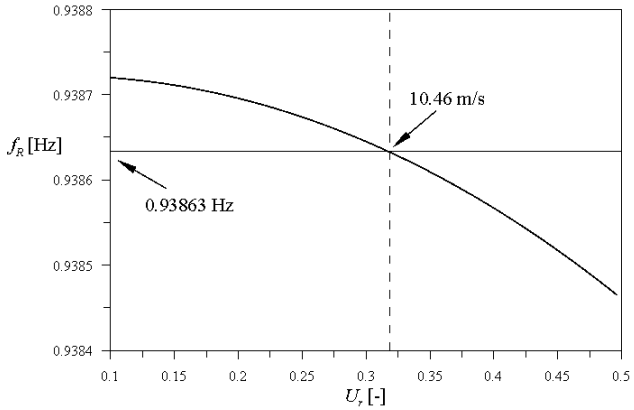



*Frequency-domain analyses*, eigenvalue problem (Scanlan)




Frequency- to time-domain relations

$$(A_i^*, H_i^*, P_i^*) \rightarrow f\left(C_f^0, \frac{dC_f}{d\alpha}, A_i, B_i\right)$$

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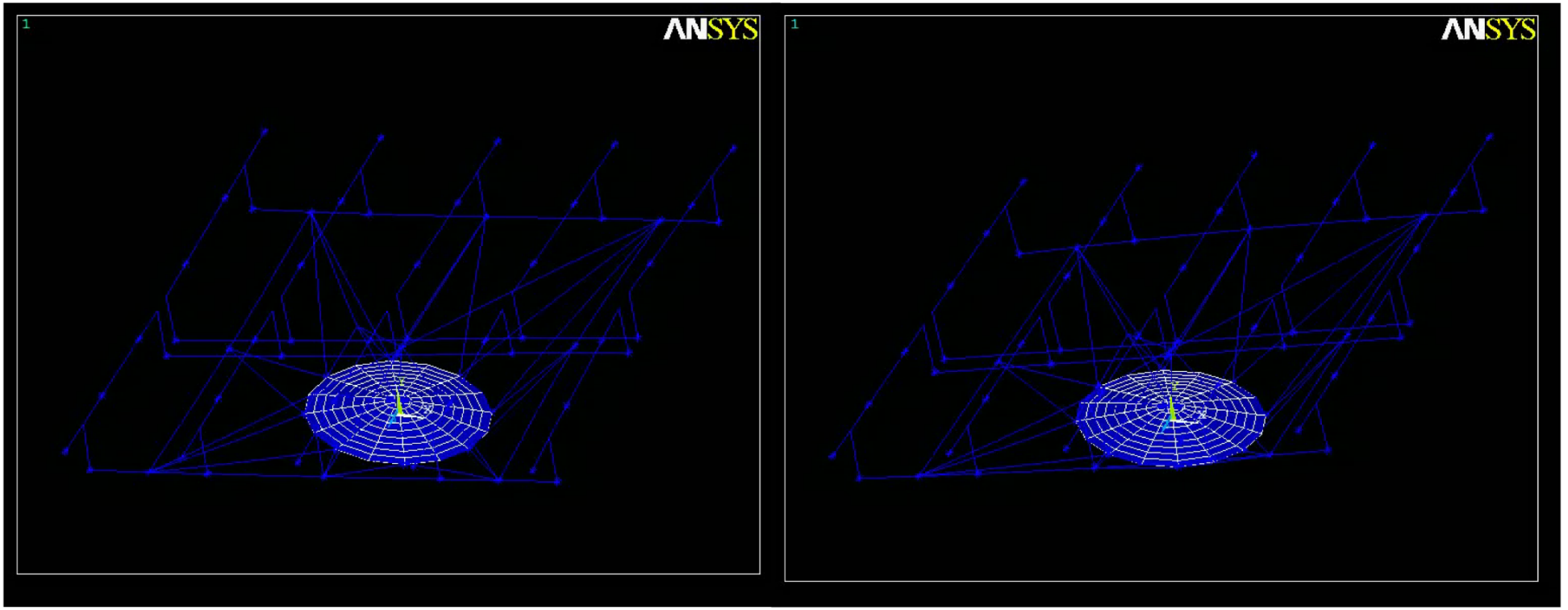
# Summary

- Obtaining nonlinear indicial functions for typical suspension and arch bridge sections (single-box sharp edges)
- Implementing and solving the nonlinear aerodynamic equations in time-domain by use of the aerodynamic lag states
- **VIV** and related nonlinear phenomena
- Sensitivity analyses carried out via parametric nonlinear models of suspension and arch bridges
- Approaching the problem to twin-box sections for super-long-span suspension bridge
- Aerodynamics enhancement by TMDs

# Solar tracker: flutter and passive control

Rome, December, 2009




U=23.65 m/s  Flutter speed increased by 50 % with 2 TMDs

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