POLITECNICO DI MILANO

Centre for Computational Structural and Materials Mechanics

On the analysis of solid and fluid dissipation in MEMS resonators

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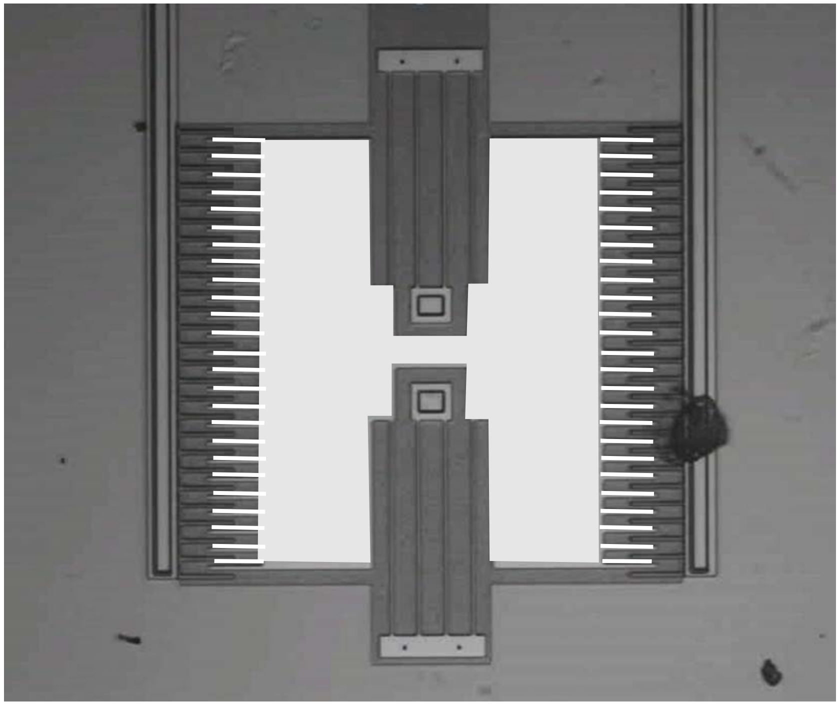
http://www.stru.polimi.it/mems/

LEISURE APPLICATIONS FOR BIAXIAL ACCELEROMETER

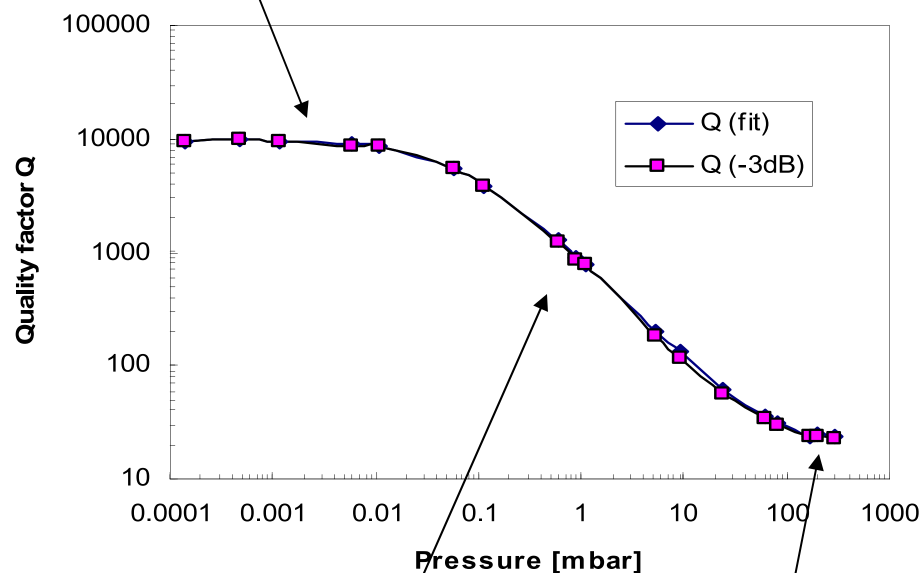
accelerometers, gyroscopes, magnetometers

DIFFERENT SOURCES OF DISSIPATION

Example: Tang resonator



solid/surface damping: internal friction, thermoelastic damping (resonators)



fluid damping: rarefied regime (gyroscopes)

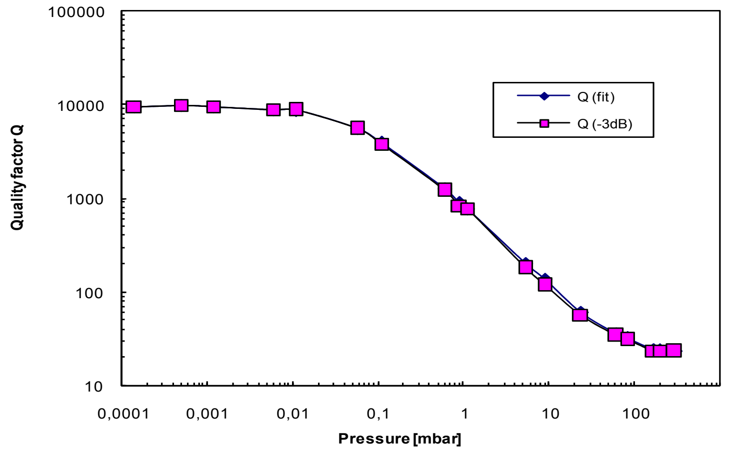
fluid damping: continuum regime (accelerometers)

KNUDSEN NUMBER AND FLOW MODELS

Knudsen number $Kn = \lambda/L$

λ: mean free path of molecules
L: characteristic length scale

λ=0.069 µm at SATP, $\lambda \sim 1/p$



- **Navier-Stokes equations (no-slip):** $Kn \leq 10^{-3}$
- **Navier-Stokes equations (slip):** $10^{-3} \leq Kn \leq 10^{-1}$
- **Transition regime:** $10^{-1} \leq Kn \leq 10$
- **Free molecular flow:** $Kn > 10$

Overview

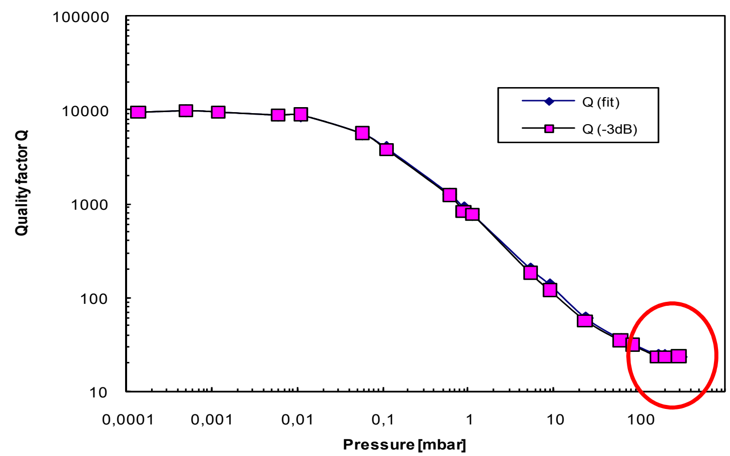
1. *Fluid* damping
   - Continuum regime
   - Rarefied (free molecule) regime

2. *Solid* damping
   - Thermoelastic dissipation
   - Anchor losses
   - Surface losses

Knudsen number $Kn = \lambda/L$

λ: mean free path of molecules
L: characteristic length scale

$\lambda = 0.069\ \mu m$ at SATP, $\lambda \sim 1/p$



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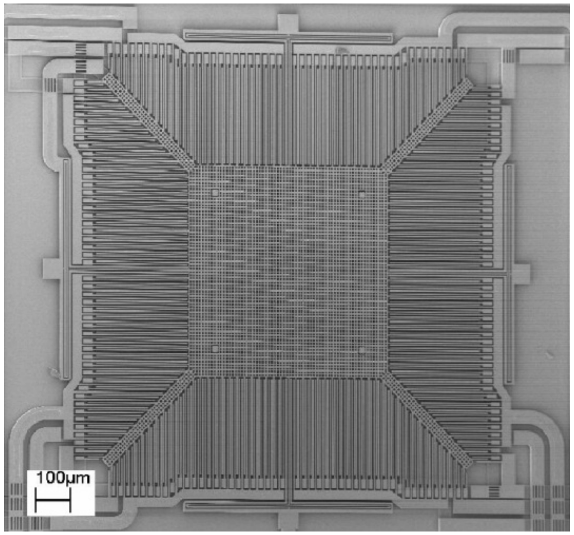
Full Navier-Stokes model and simplifications

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \eta\Delta\mathbf{u} \qquad \frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{u} = 0$$

- Reynolds number Re = UL/ν (U typical speed, L typical dimension, ν kinematic viscosity)
  if Re << 1 neglect non-linear convective terms in Navier Stokes
- Mach number M = U/c (U typical speed, c speed of sound)
  if M << 1 set ∇·u = 0 (incompressibility) in Navier-Stokes
- Stokes number St = fL^2/ν (f vibration frequency)
  if St << 1 neglect inertia terms in Navier-Stokes

Example of biaxial accelerometer (SI units):
L ~ 2.6 $\times 10^{-6}$ m; f = 4400Hz; ν =1.5 10^{-5}; U ~ 2πfD ;
D < 1/10 L (D amplitide of oscillation)
U ~ 7×10^{-3} Re ~ 10^{-3} M ~ 7×10^{-4} St ~ 7×10^{-3}



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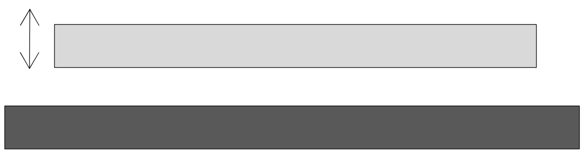
Incompressible (quasi-static) Stokes formulation

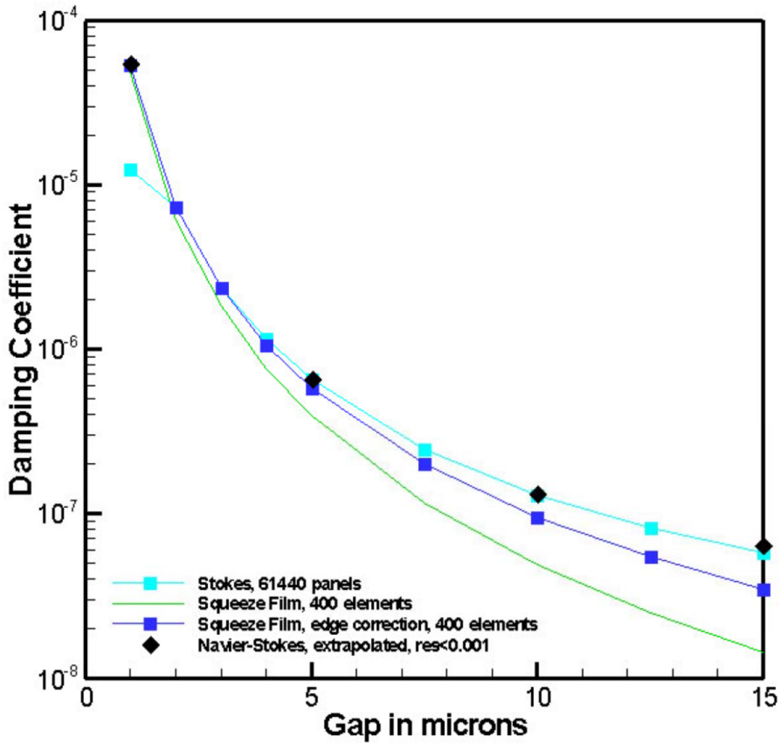
Comparison between full 3D models and Reynolds



fixed plate

Model problem for the study of Navier-Stokes, Stokes, and Reynolds solutions for damping coefficients.

A 50x50x4 plate (microns) oscillates above a 60x60x4 plate at 1atm, 300K.



Comparison of
Reynolds (1D)
Stokes (3D)
Navier-Stokes (3D)

STOKES PROBLEM BY BEM: MVT and slip BC

$$\nabla p(\mathbf{x}) - \eta \Delta \mathbf{u}(\mathbf{x}) = \mathbf{0} \qquad \nabla \cdot \mathbf{u}(\mathbf{x}) = 0 \qquad \text{in } \Omega$$

$$\mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) - c_t \mathbf{t}^S(\mathbf{x}) \qquad \text{on } S$$

t tractions, $\mathbf{t}^S$ tractions projected on surface

$$c_t := \frac{2-\sigma}{\sigma}\frac{\lambda}{\eta}$$

$$\mathbf{t}^S(\mathbf{x}) = [\mathbb{1} - \mathbf{n}(\mathbf{x}) \otimes \mathbf{n}(\mathbf{x})] \cdot \mathbf{t}(\mathbf{x})$$

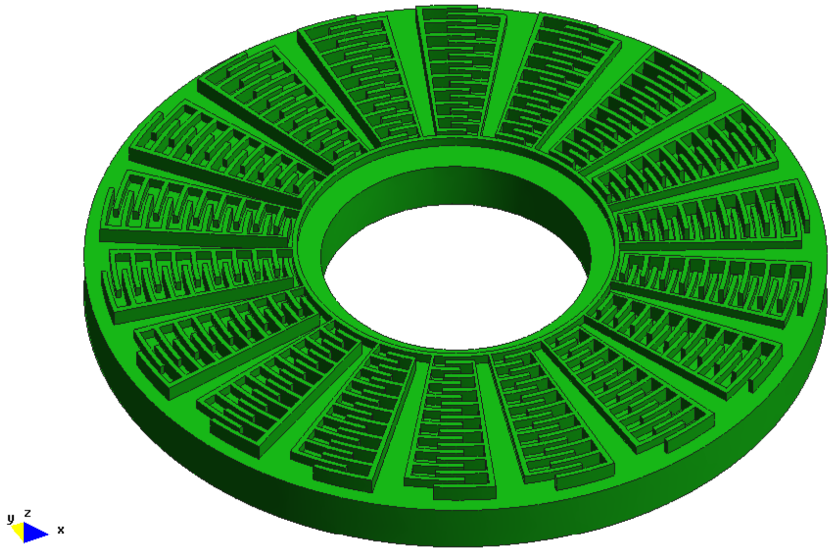
Bounday Integral Equation Approach

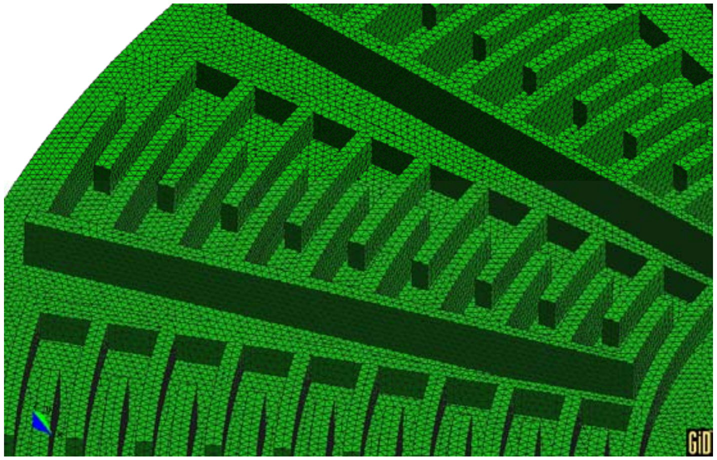
$$\mathbf{g}(\mathbf{x}) - \frac{c_t}{2}\mathbf{t}^S(\mathbf{x}) - \frac{\gamma}{\eta}\frac{1}{2}\mathbf{t}(\mathbf{x}) = \int_S \Big\{ \boldsymbol{\mathcal{V}}(\mathbf{r}) \cdot \mathbf{t}(\mathbf{y}) + c_t \left[\boldsymbol{\mathcal{K}}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{y})\right] \cdot \mathbf{t}^S(\mathbf{y})$$
$$+ \frac{\gamma}{\eta}\left([\mathbf{n}(\mathbf{x}) \cdot \boldsymbol{\mathcal{K}}(\mathbf{r})] \cdot \mathbf{t}(\mathbf{y}) - c_t \left[\mathbf{n}(\mathbf{x}) \cdot \boldsymbol{\mathcal{W}}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{y})\right] \cdot \mathbf{t}^S(\mathbf{y}) \right) \Big\} \mathrm{d}S_y$$

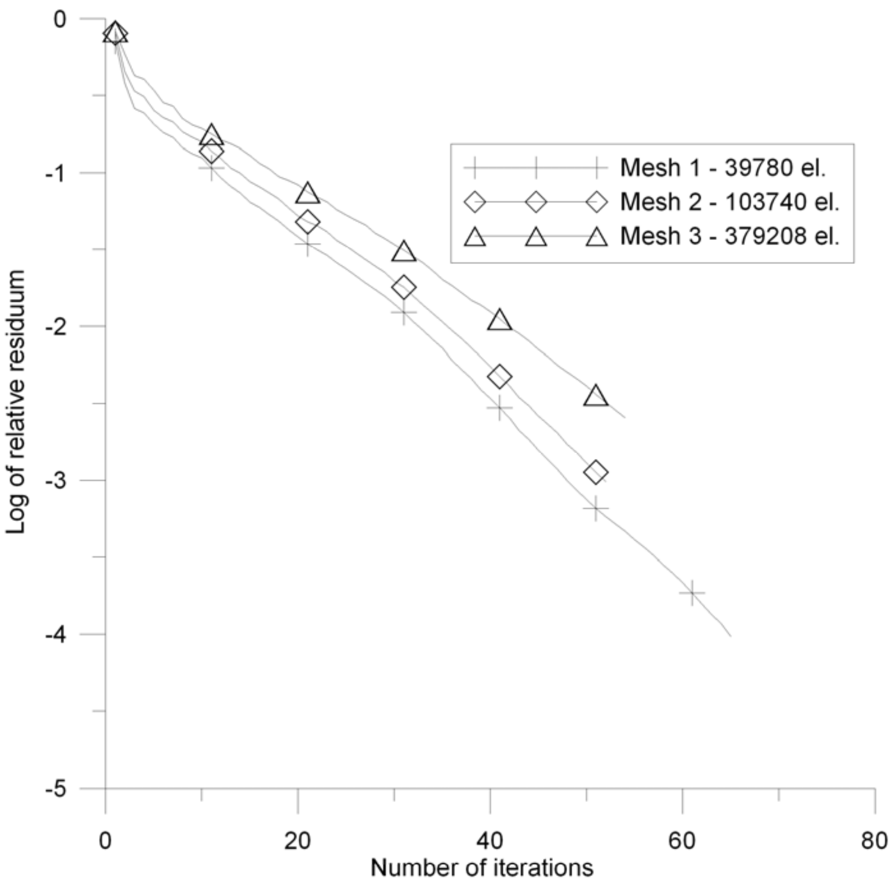
Code licensed to: COVENTOR 3D MEMS Software

•Frangi A., Tausch J., *Engineering Analysis with Boundary Elements*, 2005
•Frangi A., *Engineering Analysis with Boundary Elements*, 2005
•Frangi A., *Mechanics Research Communications*, 2006
•Frangi A. et al., *International J. Numerical Methods in Engineering,* 2006

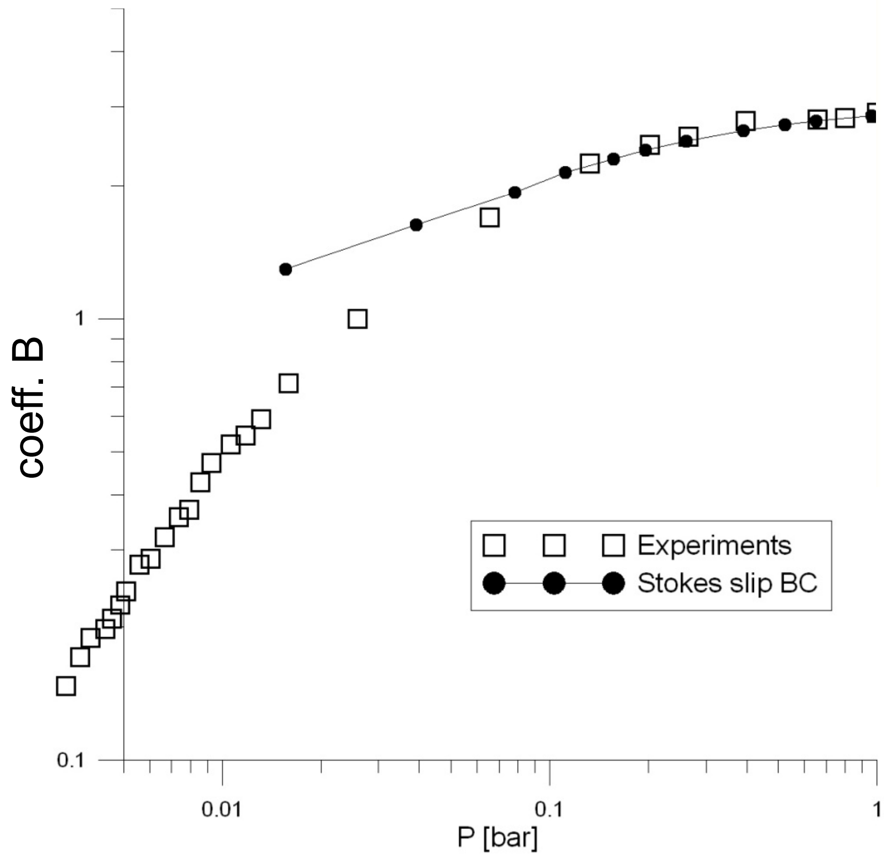
Examples of full scale analysis: rotational resonator

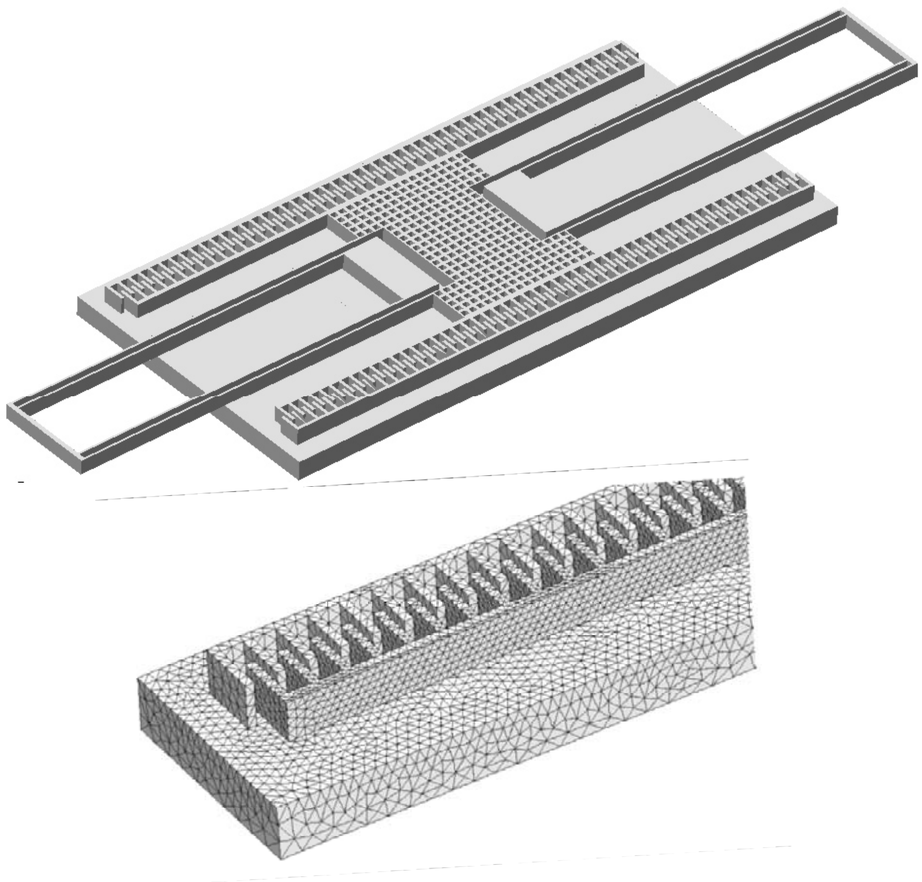






TANG RESONATOR



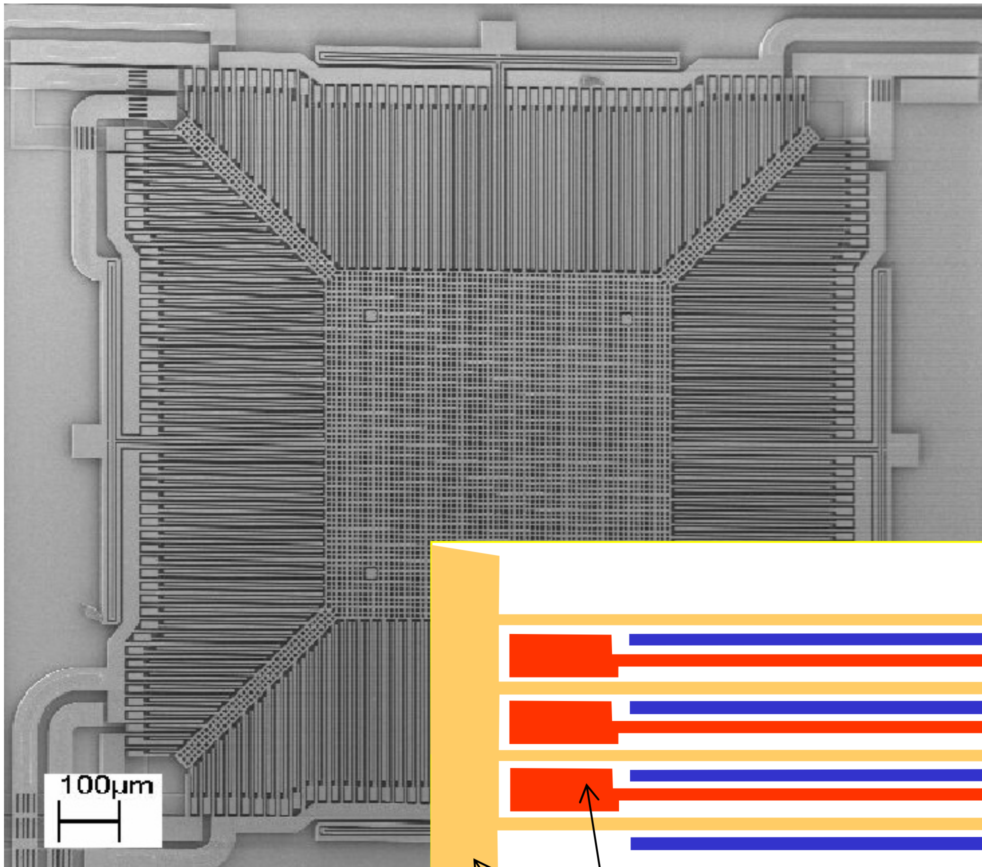


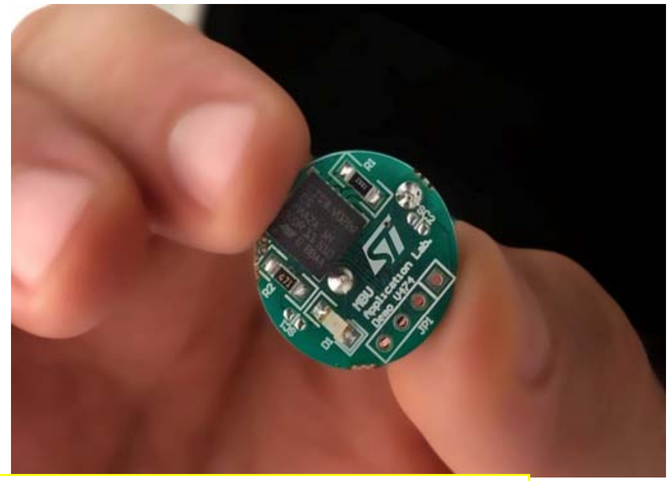
diverge at low pressure since continuum model fails

$$B = \int_{\partial\Omega} t_x(\mathbf{x})\, dS$$

$$M\ddot{U}(t) + B\dot{U}(t) + KU(t) = F(t)$$

BIAXIAL ACCELEROMETER





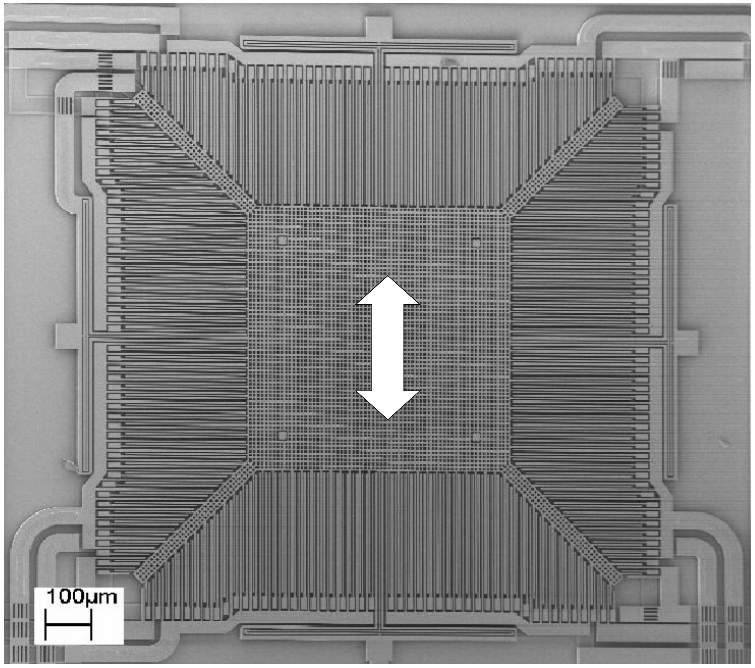
f = 4400Hz

stator (red and orange)
Fixed to the substrate

Rotor (blue)
Mobile

Mobile mass
(holed)

EXPERIMENTAL VALIDATION: LINEAR ACCELEROMETER AND SLIP B.C.

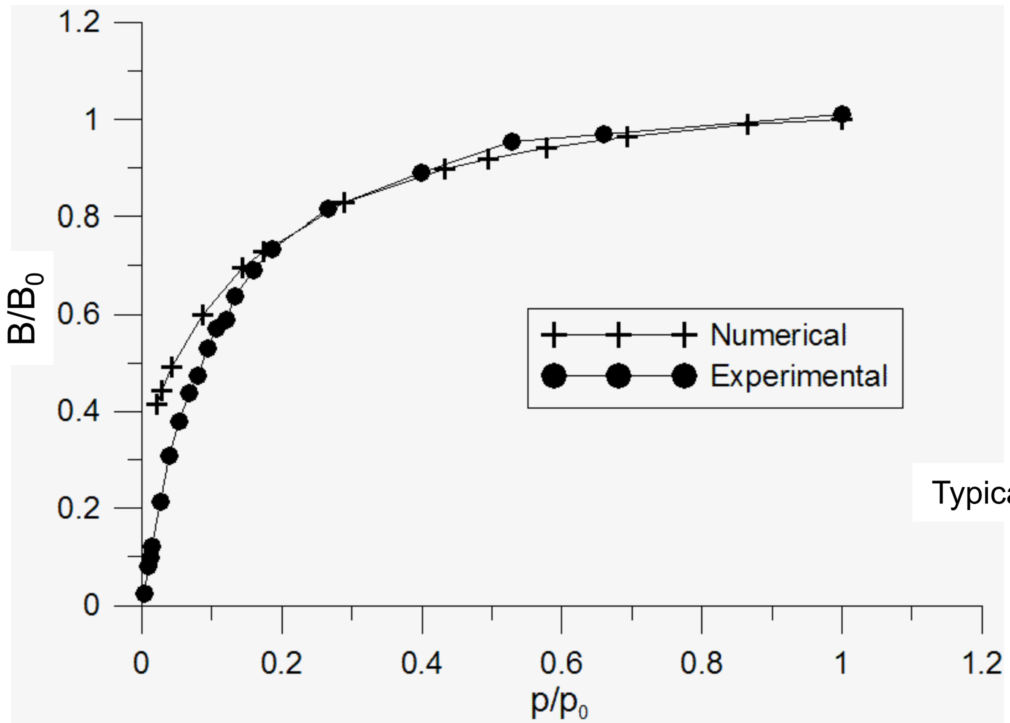


p = 1 bar

	Numerical "no slip"	Numerical "slip"	Experimental
Squeeze flow	$2.32\,10^{-4}$ N	$2.10\,10^{-4}$ N	-
Couette flow	$7.37\,10^{-6}$ N	$7.03\,10^{-6}$ N	-
Mass with holes	$2.10\,10^{-6}$ N	$1.94\,10^{-6}$ N	-
Total force	$2.41\,10^{-4}$ N	$2.19\,10^{-4}$ N	$2.21\,10^{-4}$ N

EXPERIMENTAL VALIDATION: LINEAR ACCELEROMETER AND SLIP B.C.

$$M\ddot{U}(t)+B\dot{U}(t)+KU(t)=F(t)$$



Typical comparison of simulations and results

excellent agreement at high pressure
diverge at low pressure
since continuum model fails

EXTENSION TO HIGHER WORKING FREQUENCIES

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \eta\Delta\mathbf{u} \qquad \frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{u} = 0$$

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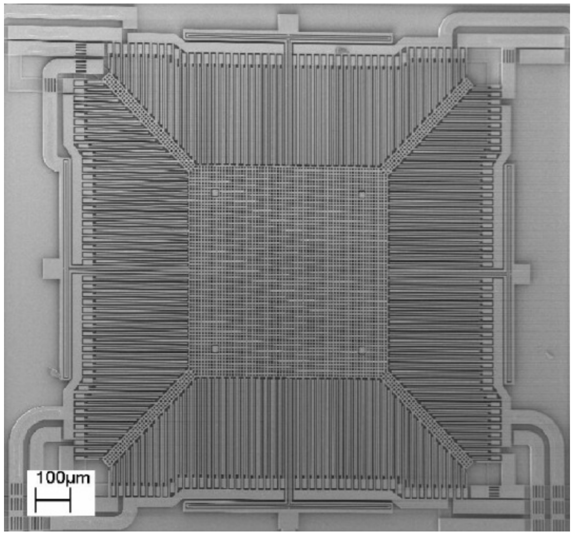

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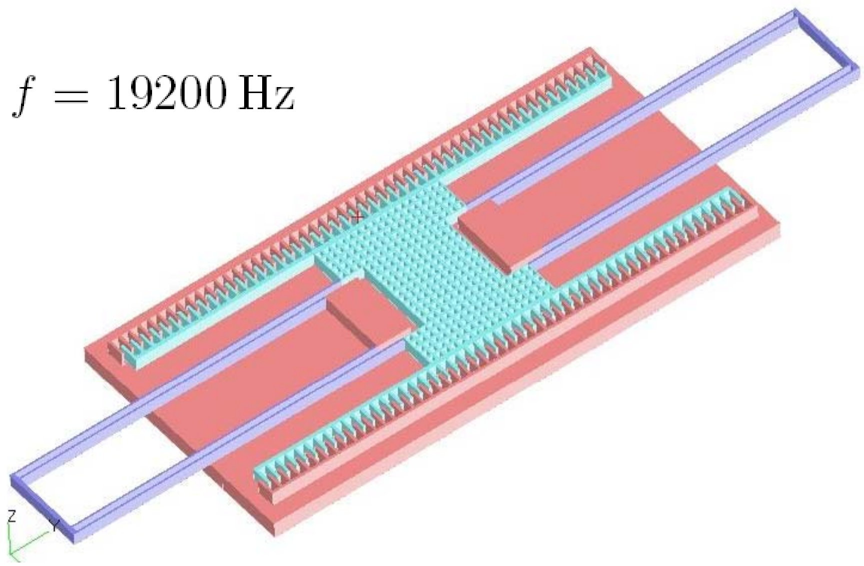
Incompressible frequency-domain Stokes formulation

EXAMPLE (J.WHITE and coll.,MIT)

$f = 19200\,\text{Hz}$



Finger gap	$2.88\mu m$
Finger length	$40.05\mu m$
Finger overlap	$19.44\mu m$
Center plate	$54.9 \times 19.26\mu m^2$
Side plate1 $\times 2$	$28.26 \times 89.6\mu m^2$
Side plate2 $\times 4$	$11.3 \times 40.5\mu m^2$
Thickness	$1.96\mu m$
Substrate gap	$2\mu m$
Truss length	$78\mu m$
Truss width	$13\mu m$

Drag force (pN)	Steady	Unsteady
Bottom	508.75	510.72
Side	284.84	294.50
Top	102.31	142.8
Total	895.9	948.02

$$L\sqrt{\frac{\omega\rho}{\eta}} \ll 1$$

•Frangi A. Bonnet M., *CMES*, 2010

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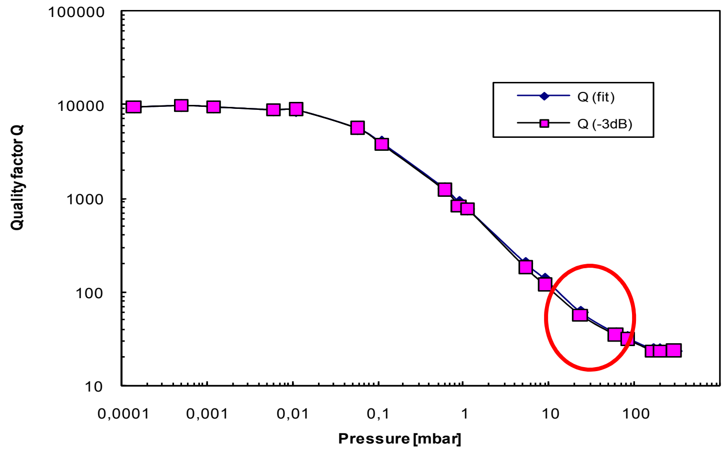

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$$\rho = \int_{\mathbb{R}^3} f \mathrm{d}\boldsymbol{\xi}$$

$$\rho \mathbf{v} = \int_{\mathbb{R}^3} f \boldsymbol{\xi} \mathrm{d}\boldsymbol{\xi}$$

$$T = \frac{1}{3\mathcal{R}\rho} \int_{\mathbb{R}^3} f \left|\boldsymbol{\xi} - \mathbf{v}\right|^2 \mathrm{d}\boldsymbol{\xi}$$

$$\boldsymbol{\sigma} = \int_{\mathbb{R}^3} f \; (\boldsymbol{\xi} - \boldsymbol{v}) \otimes (\boldsymbol{\xi} - \boldsymbol{v}) \;\; d\boldsymbol{\xi}$$

$f(x,\xi)$: mass density probability depends on location x and molecular velocity ξ

mass density probability for a gas at rest

$$f_0 = \frac{\rho_0}{(2\pi\mathcal{R}T_0)^{3/2}} \exp\left(-\frac{\left|\boldsymbol{\xi}\right|^2}{2\mathcal{R}T_0}\right)$$

rest Maxwellian

rhs accounting for molecule collisions

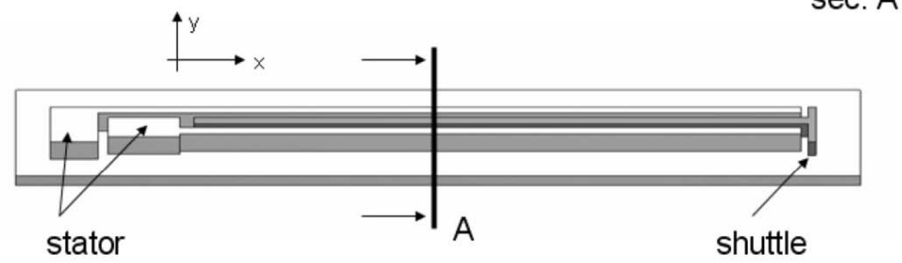
$$\boxed{\frac{\partial f}{\partial t} + \boldsymbol{\xi} \cdot \nabla f = \overbrace{\nu(\rho, T)(f_M - f)}}$$

BGK model
of Boltzmann equation

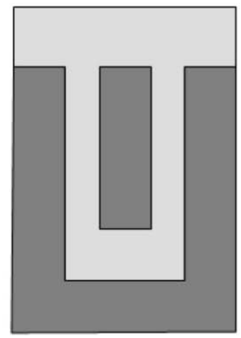
$$\boxed{f_M = \frac{\rho}{(2\pi\mathcal{R}T)^{3/2}} \exp\left(-\frac{\left|\boldsymbol{\xi} - \mathbf{v}\right|^2}{2\mathcal{R}T}\right)}$$

local Maxwellian

BGK MODEL BY FINITE VOLUMES



sec. A

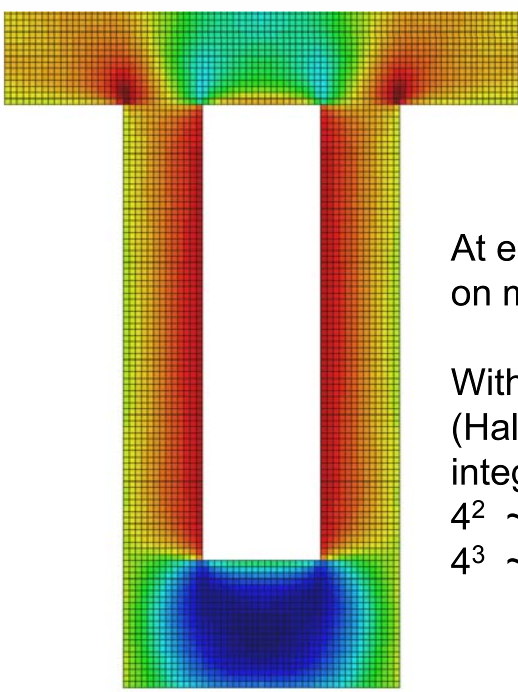


Cell size should be at most ~ 1/3 λ
(λ molecular free-path, λ=.065 μm at 1bar)

λ is proportional to 1/p

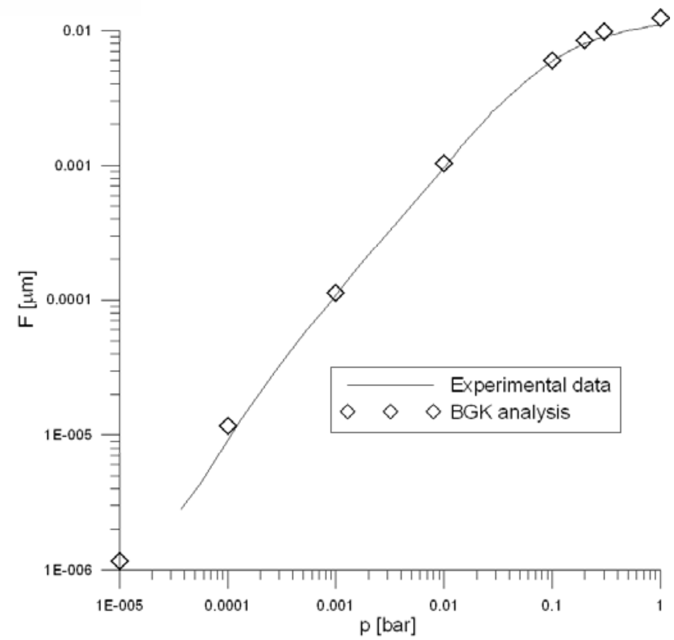
Very refined meshes are required at high pressure

Approach "practically" limited to p < .1 bar



At each grid cell f depends on molecular velocity **ξ**

With modern approaches (Half Range Gauss Hermite integrations), in each cell
$4^2 \sim 8^2$ unknowns in 2D
$4^3 \sim 8^3$ unknowns in 3D



•Lorenzani S., Frangi A., Cercignani C. et al., *Nanoscale and Microscale Therm. Engng,* 2007
•Frangi A., Frezzotti A., Lorenzani S., *Computer & Structures*, 85, 810–817, 2007
•Cercignani C., Frangi A., et al., *Sensors & Letters*, 6, 121-129, 2008
•Frangi A., Ghisi A., Frezzotti A., *Sensors & Letters*, 6, 57-68, 2008

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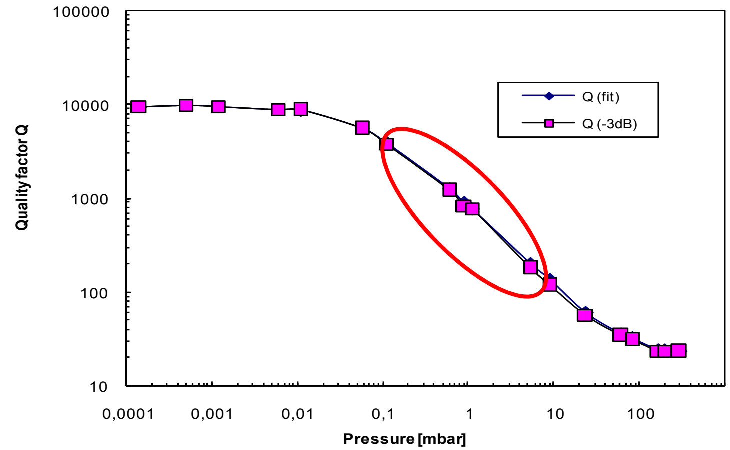

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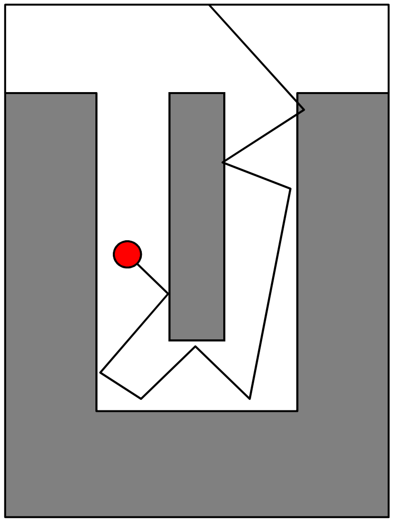
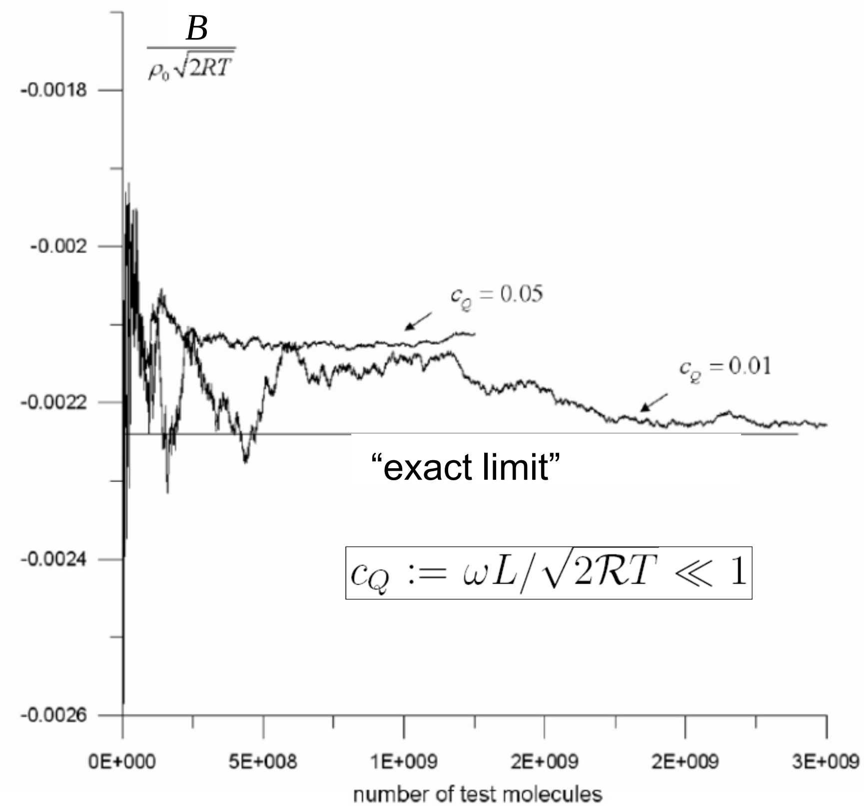
TEST PARTICLE MONTE CARLO METHOD

$\Delta\mathbf{q}$ denotes the linear momentum change of one molecule due to one collision and $\mathbf{w}$ the instant velocity of the shuttle,

The total dissipation induced by a single molecule before exiting the analysis domain through an in-flow surface is

$$d_i = \sum_c \Delta\mathbf{q} \cdot \mathbf{w}$$





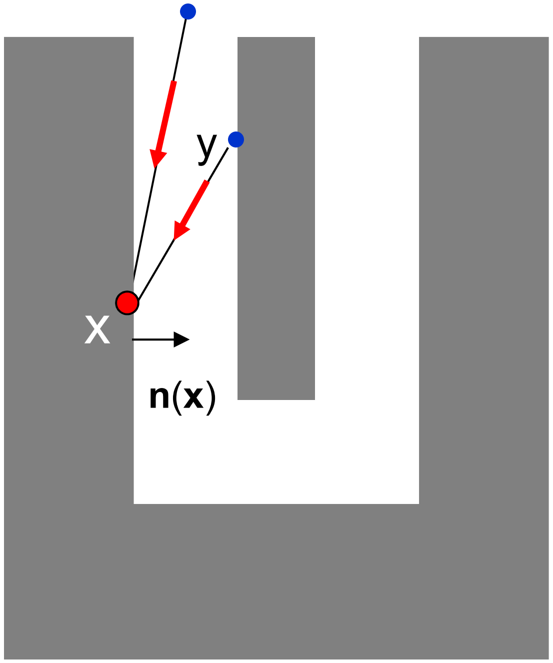
Typical velocity of shuttle never exceed fractions of m/s and is always
<< thermal velocity of molecules

FREE MOLECULE FLOW

$f(x,\xi)$: *mass density probability depends on location x and molecular velocity ξ*

$$\boxed{\frac{\partial f}{\partial t} + \boldsymbol{\xi} \cdot \nabla f = 0}$$

collisions between molecules neglected



for molecules coming from other MEMS surfaces:

$$f(\mathbf{x}, \boldsymbol{\xi}, t) = f(\mathbf{y}, \boldsymbol{\xi}, t - \frac{r}{\xi}) \qquad \mathbf{y} = \mathbf{x} + \mathbf{r}, \quad \mathbf{r} = -r\frac{\boldsymbol{\xi}}{\xi}$$

for molecules coming from far field region:

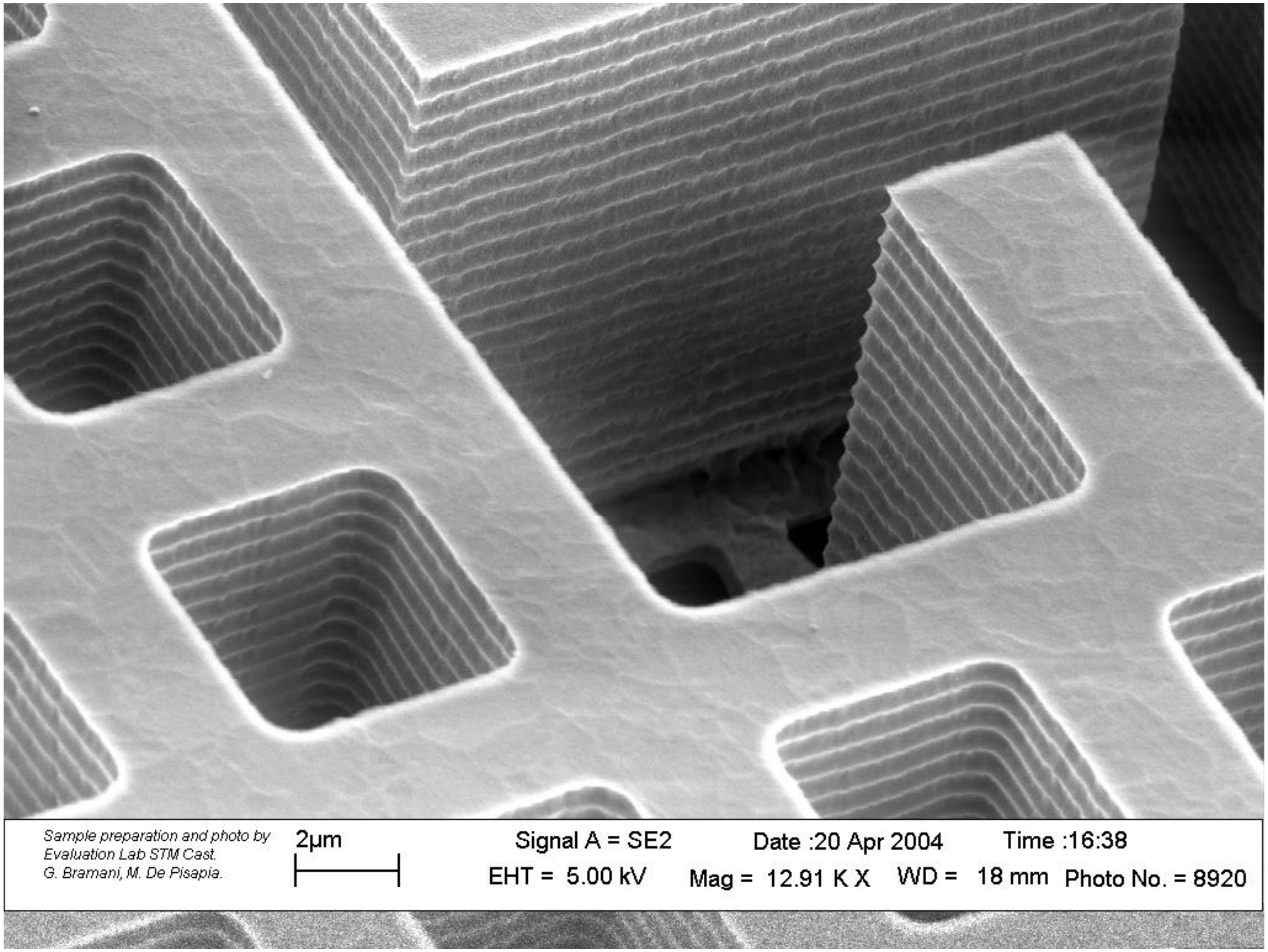
$$f(\mathbf{x}, \boldsymbol{\xi}) = f_0(\boldsymbol{\xi}) = \frac{\rho_0}{(2\pi \mathcal{R} T_0)^{3/2}} \exp\left(-\frac{|\boldsymbol{\xi}|^2}{2\mathcal{R} T_0}\right)$$

Diffuse model for molecules re-emitted from surfaces

$$f(\mathbf{x}, \boldsymbol{\xi}) = \frac{\rho_w(\mathbf{x})}{(2\pi \mathcal{R} T_w(\mathbf{x}))^{3/2}} \exp\left(-\frac{|\boldsymbol{\xi} - \mathbf{w}(\mathbf{x})|^2}{2\mathcal{R} T_w(\mathbf{x})}\right) \quad \text{for} \quad (\boldsymbol{\xi} - \mathbf{w}) \cdot \mathbf{n} > 0$$

$$\rho_w(\mathbf{x}) = \left(\frac{2\pi}{\mathcal{R} T_w(\mathbf{x})}\right)^{1/2} \int_{\mathbb{R}^3, (\boldsymbol{\xi} - \mathbf{w}) \cdot \mathbf{n} < 0} |(\boldsymbol{\xi} - \mathbf{w}(\mathbf{x})) \cdot \mathbf{n}(\mathbf{x})| \, f(\mathbf{x}, \boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}$$

ROUGHNESS OF SURFACES DUE TO ETCHING



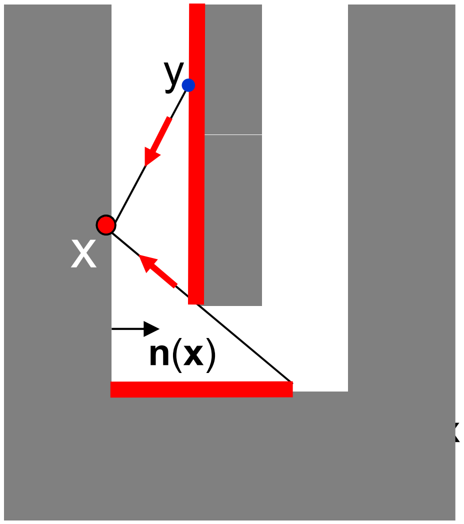
$$
\begin{aligned}
J(\mathbf{x},\omega) = & \sqrt{\pi}\tilde{\mathbf{g}}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x}) \\
& -\frac{2}{\pi}\int_{S+} J(\mathbf{y},\omega)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{x})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{y})\big)\frac{1}{r^4}T_3(i\tilde{\omega}r)\mathrm{d}S \\
& +\frac{4}{\pi}\int_{S+}\big(\mathbf{r}\cdot\tilde{\mathbf{g}}(\mathbf{y})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{x})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{y})\big)\frac{1}{r^5}T_4(i\tilde{\omega}r)\mathrm{d}S
\end{aligned}
$$

$$
T^{(3)}(0) = 1/2
$$

$$
T^{(4)}(0) = 3\sqrt{\pi}/8.
$$

Limit case: $\dfrac{\omega}{\sqrt{2\mathcal{R}T}}L \ll 1$

J(x): normalised flux of molecules at the wall



$$
\begin{aligned}
J(\mathbf{x}) = & \sqrt{\pi}\tilde{\mathbf{g}}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x}) \\
& -\frac{1}{\pi}\int_{S+} J(\mathbf{y})\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{x})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{y})\big)\frac{1}{r^4}\mathrm{d}S \\
& +\frac{3}{2}\frac{1}{\sqrt{\pi}}\int_{S+}\big(\mathbf{r}\cdot\tilde{\mathbf{g}}(\mathbf{y})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{x})\big)\big(\mathbf{r}\cdot\mathbf{n}(\mathbf{y})\big)\frac{1}{r^5}\mathrm{d}S
\end{aligned}
$$

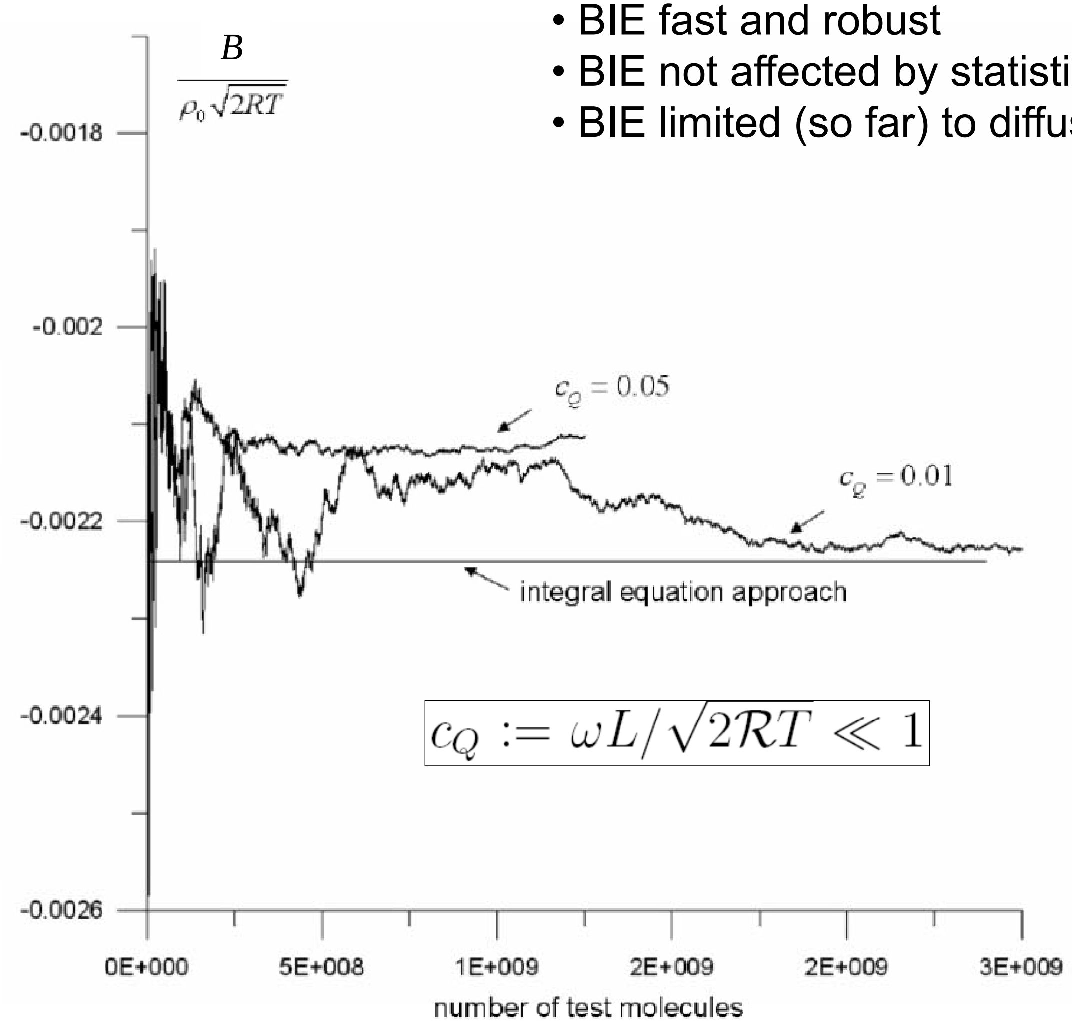
Quasi static approximation: radiosity equation

•Frangi A., *PAMM Proc. Appl. Math. Mech.*, 2008
•Frangi A., Ghisi A., Coronato L., *Sensor & Actuators*, 2009
•Frangi A., *Engineering Analysis with Boundary Elements*, 2009

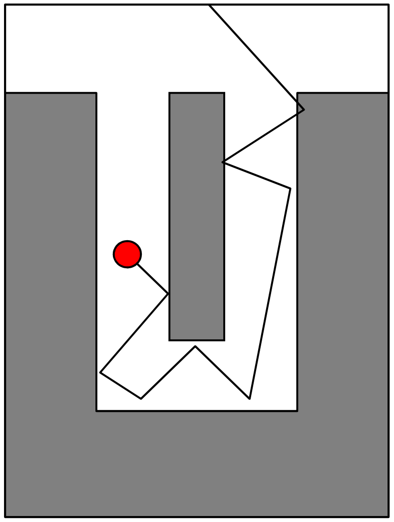
RADIOSITY COMPUTATION IN COMPUTER GRAPHICS

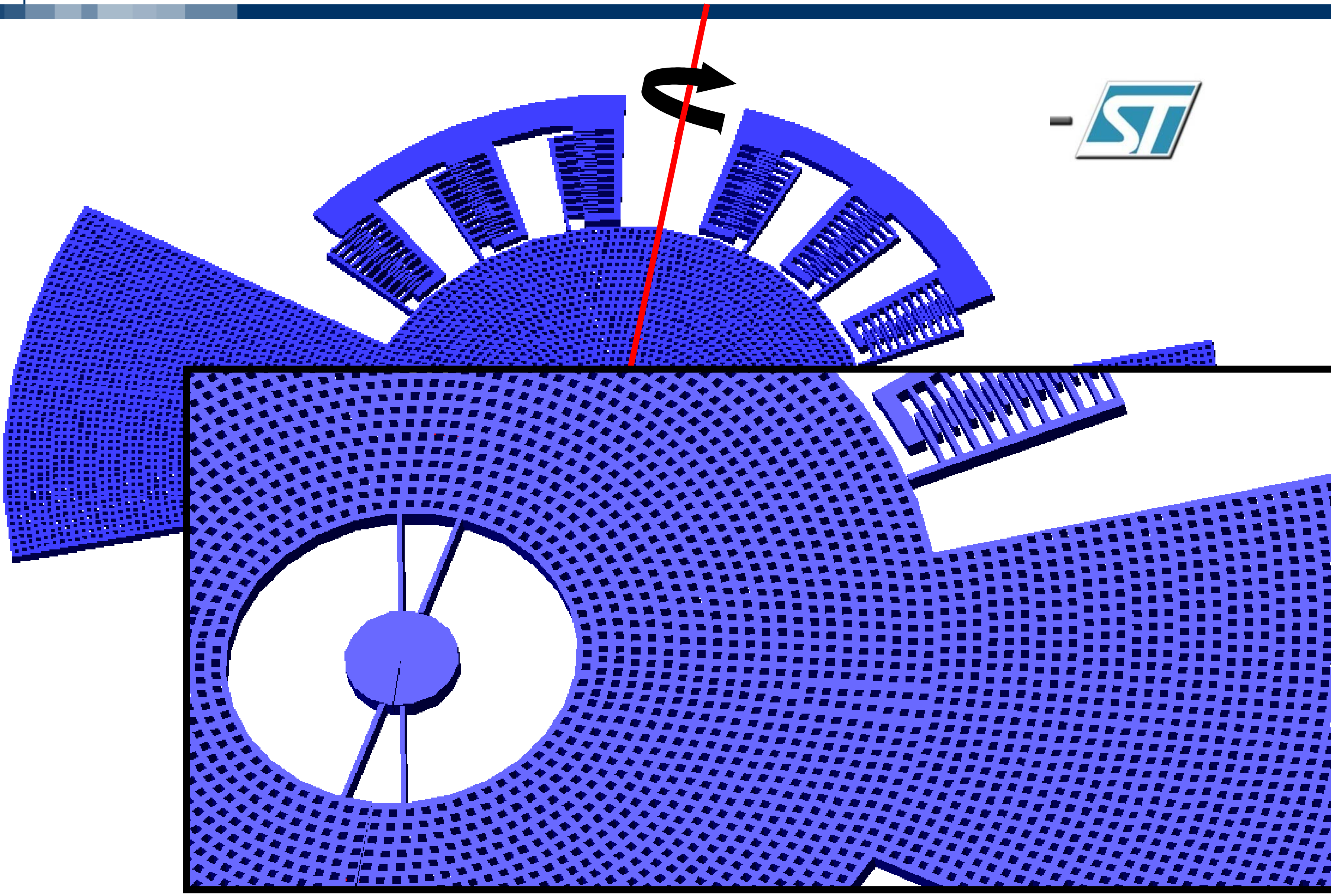
BIE vs TEST PARTICLE MONTE CARLO METHOD

- BIE independent of velocity profile imposed
- BIE fast and robust
- BIE not affected by statistical noise
- BIE limited (so far) to diffuse reflection boundary conditions

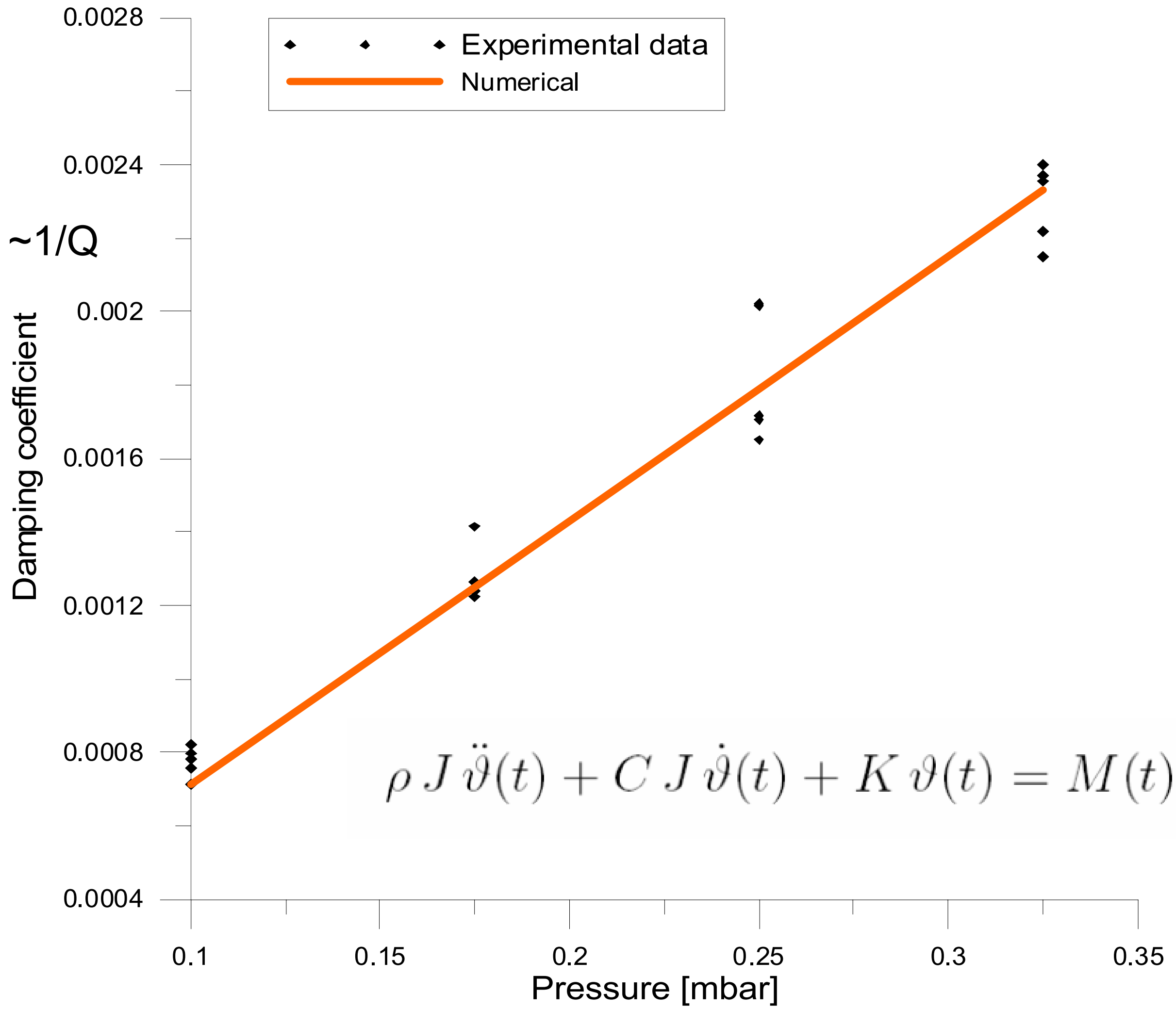


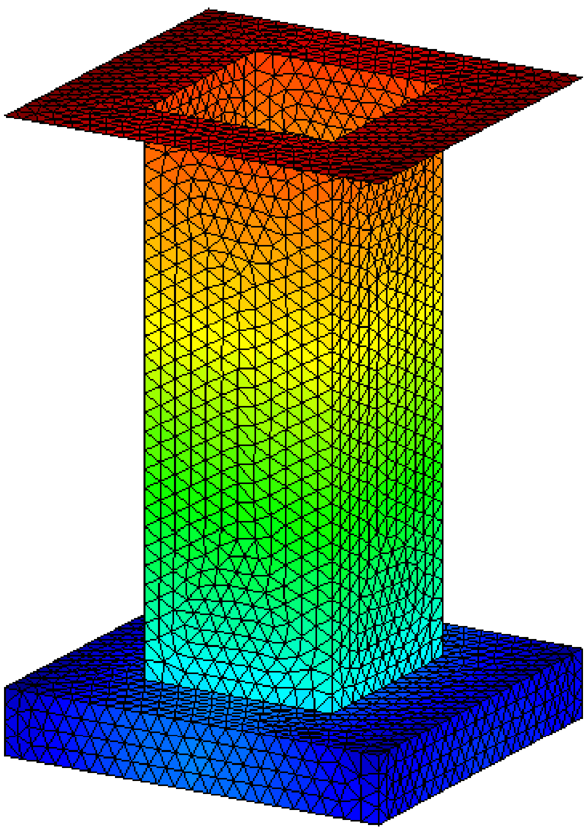
$$c_Q := \omega L/\sqrt{2\mathcal{R}T} \ll 1$$





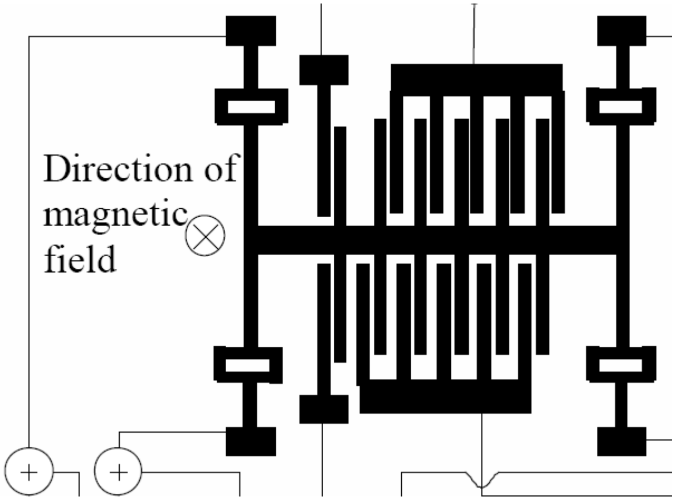
OUT-OF-PLANE ROTATIONAL RESONATOR

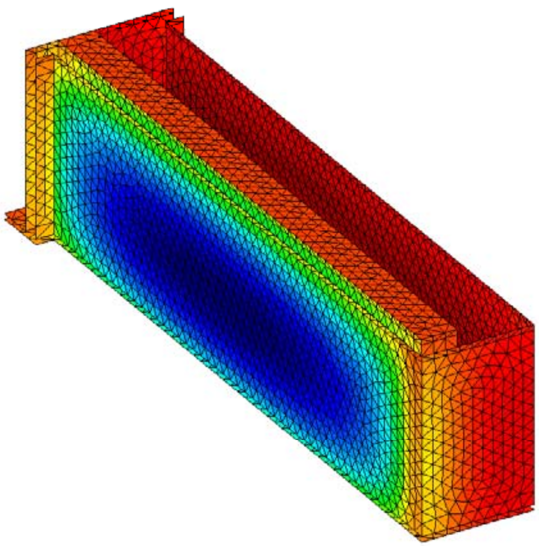


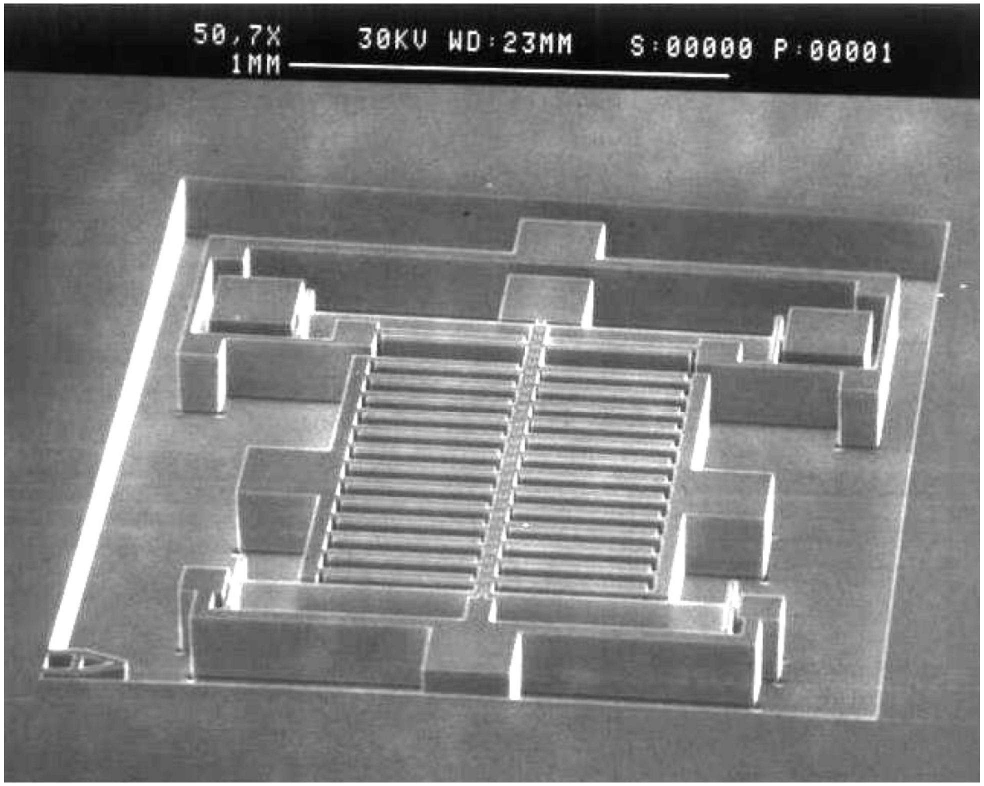


$$\rho\, J\, \ddot{\vartheta}(t) + C\, J\, \dot{\vartheta}(t) + K\, \vartheta(t) = M(t)$$

Qinetiq MAGNETOMETER







SEM of magnetometer structure

C. Cercignani: The Boltzmann equation and its applications, Springer, New York, 1988

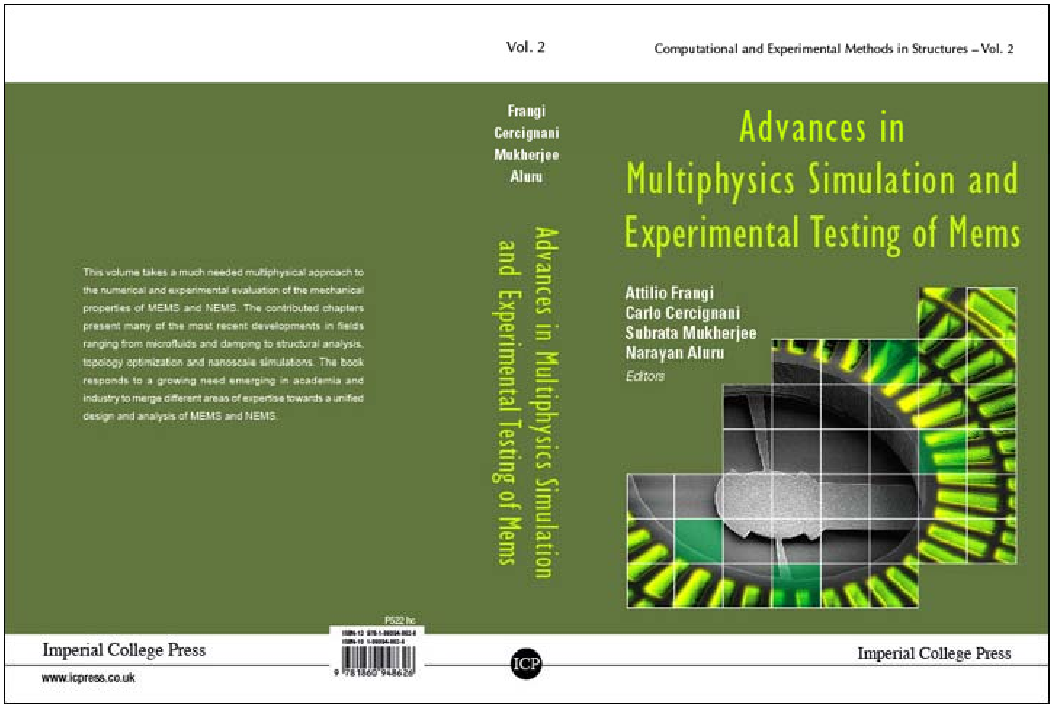
M. Gad-el-Hak: The fluid mechanics of microdevices - the Freeman scholar lecture, J. Fluids Eng.. 121, 5–33, (1999)

S. Chapman and T. G. Cowling, : The mathematical theory of non-uniform gases, Cambridge University Press, 1960

G. A. Bird, Molecular Gas Dynamics and the Direct Simulation of Gas Flows, Oxford Sicence Publication, (1994)

M. Gad-el-Hak, ed. : The MEMS handbook, CRC Press, , 2002

G. E. Karniadakis and A. Beskok, : Micro flows, fundamentals and simulation, Springer, New York, 2002

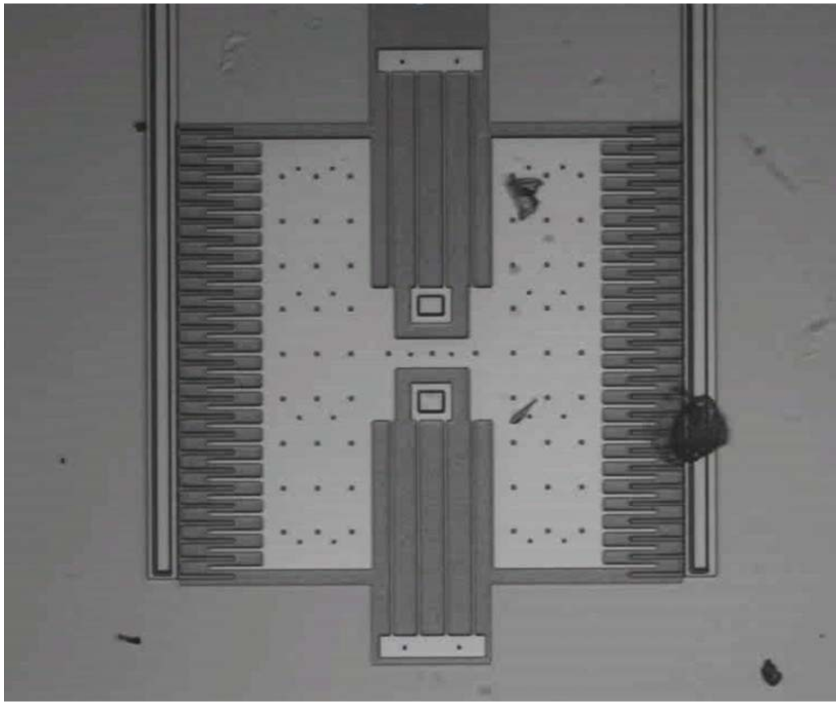


Overview

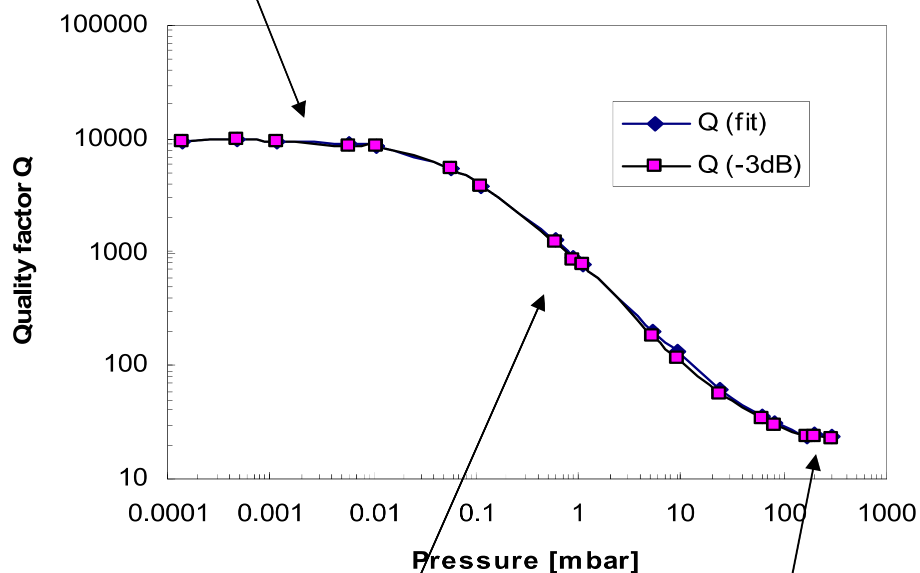
1. *Fluid* damping
    - Continuum regime
    - Rarefied regime

2. *Solid* damping
    - Thermoelastic dissipation
    - Anchor losses
    - Surface losses

Example: Tang resonator



solid/surface damping: internal friction, thermoelastic damping (resonators)



fluid damping: rarefied regime (gyroscopes)

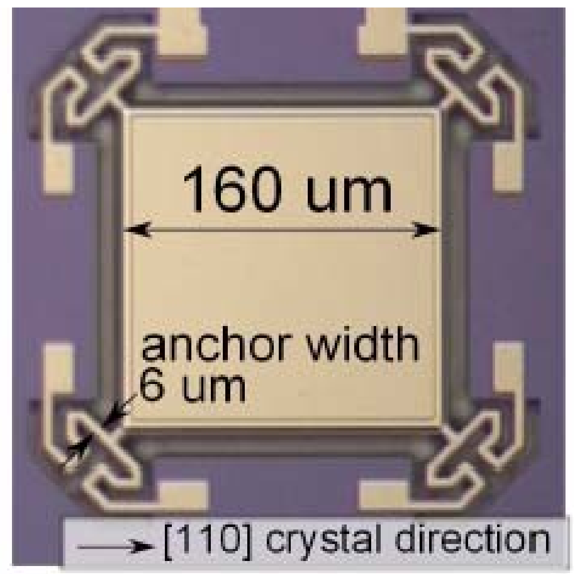
fluid damping: continuum regime (accelerometers)

Example

Go4-TIME

Global, Flexible, On demand and Resourceful Timing IC & MEMS Encapsulated System

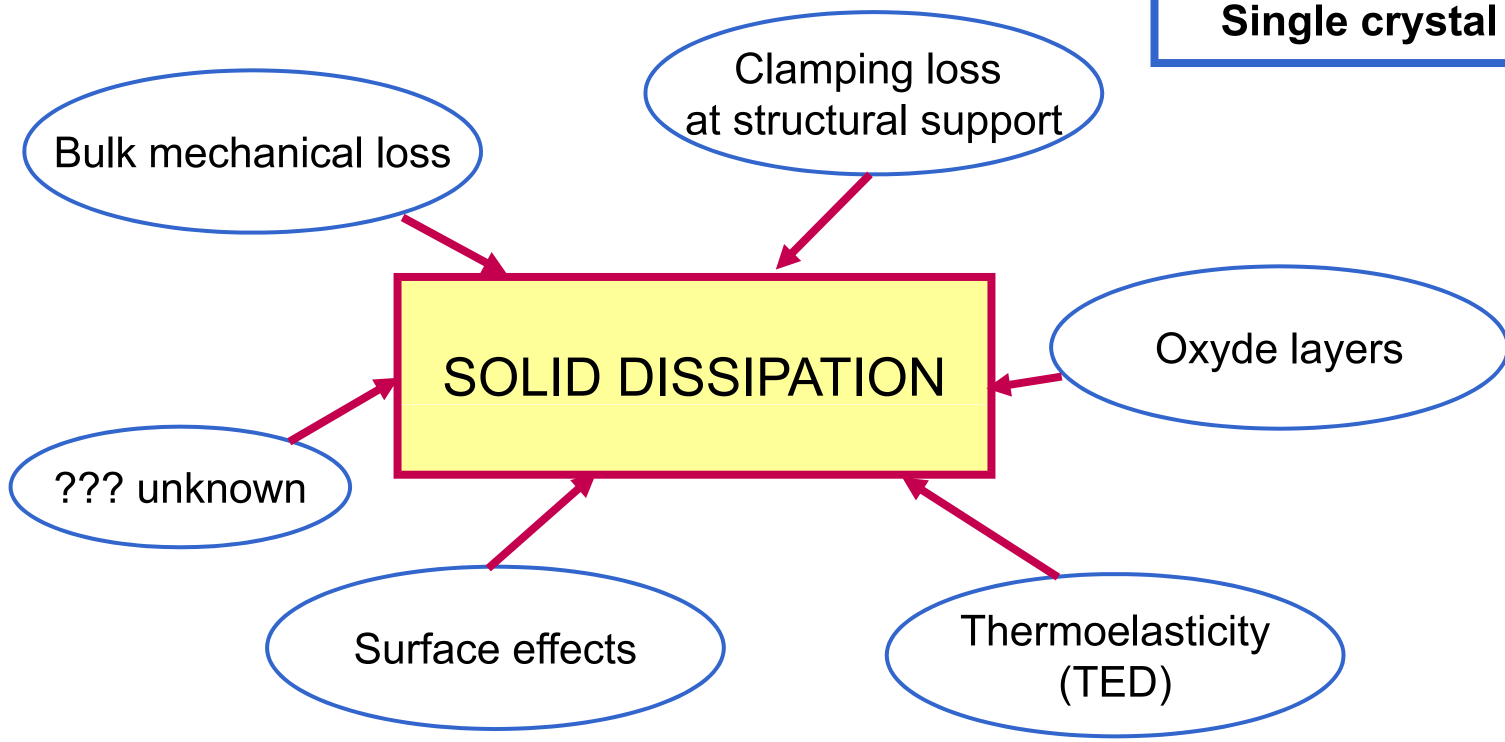
SEVENTH FRAMEWORK PROGRAMME



Go4Time will target the realization of a generic miniature timing module relying on the combination of an integrated circuit together with different MEMS resonators assembled hermetically in a single package

Solid damping mechanisms

Single crystal Si



Overall quality factor Q (inverse of dissipation):

$$\frac{1}{Q}=\frac{1}{Q_{\text{bulk}}}+\frac{1}{Q_{\text{support}}}+\frac{1}{Q_{\text{oxide}}}+\frac{1}{Q_{\text{TED}}}+\frac{1}{Q_{\text{surface}}}+\frac{1}{Q_{\text{unknown}}}$$

Overview

1. *Fluid* damping
   - Continuum regime
   - Rarefied regime

2. *Solid* damping
   - Thermoelastic dissipation
   - Anchor losses
   - Surface losses

Thermoelastic damping

Standard model

$$\sigma_{ij} = D_{ijhk}\varepsilon_{hk} - D_{ijhk}\delta_{hk}\underbrace{\left[\alpha\left(T - T_0\right)\right]}_{\text{thermal expansion}}$$

$$-q_{i,i} = C\rho\dot{T} + \underbrace{T_0\,\alpha\,3K\,\dot{\varepsilon}_{kk}}_{\text{heat source due to volume change}}$$

thermal expansion

heat source
due to volume change

Zener's solution for rectangular beams: 98.6% of dissipation occurs through the first bending mode

$$\boxed{Q_{TED} = \frac{C\rho}{\alpha^2 TE}\frac{f^2 + f_0^2}{f \cdot f_0}}$$

E: Young's modulus
f: resonance frequency

k_T : thermal conductivity
h: beam thickness

relaxation rate

$$f_0 = \frac{\pi}{2}\frac{k_T}{C\rho}\frac{1}{h^2}$$

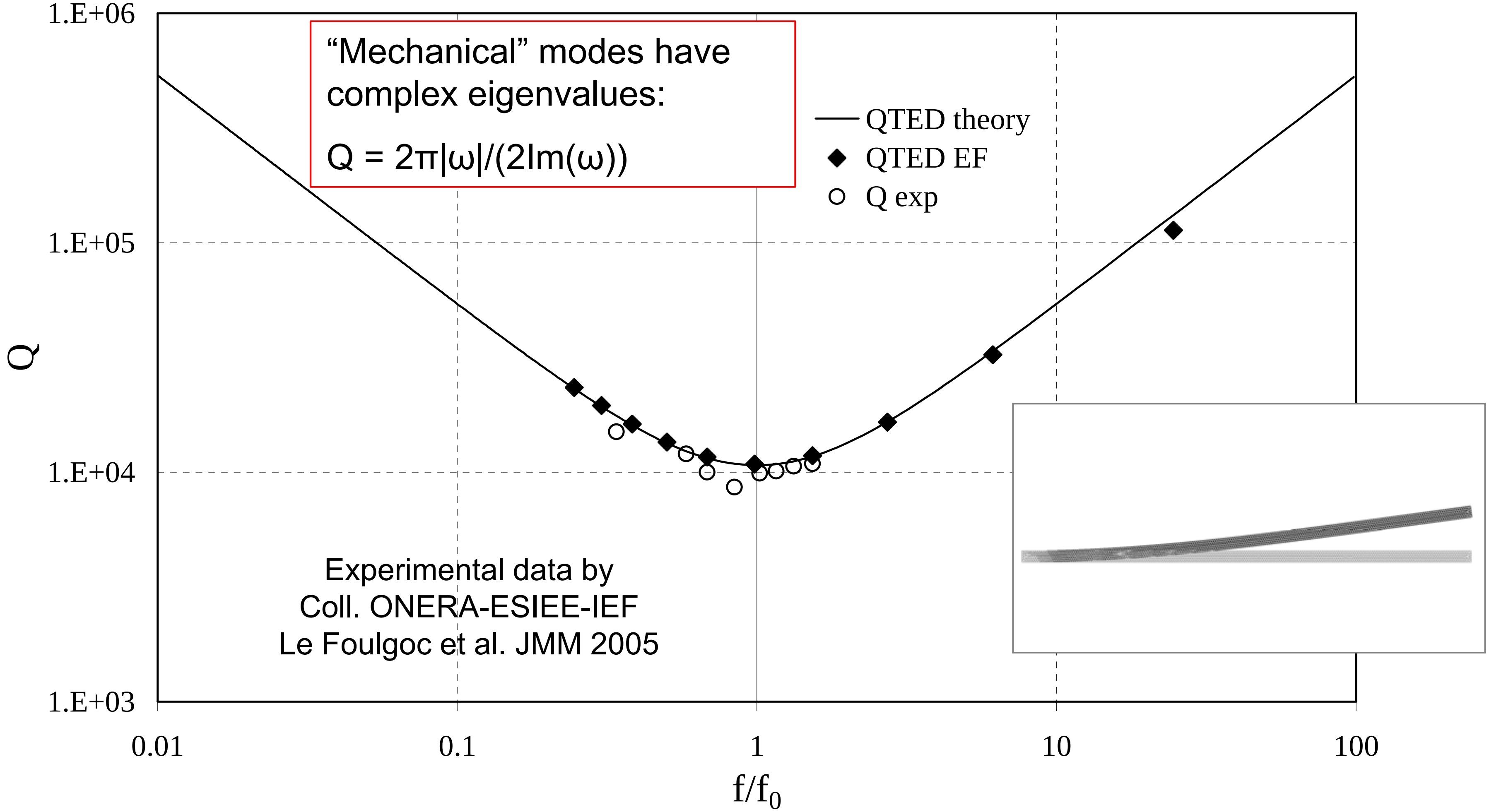
•Ardito R., Comi C., Corigliano A., Frangi A., *Meccanica*, 2008

Thermoelastic damping

cantilever beam ; thickness 30μm

first flexural mode

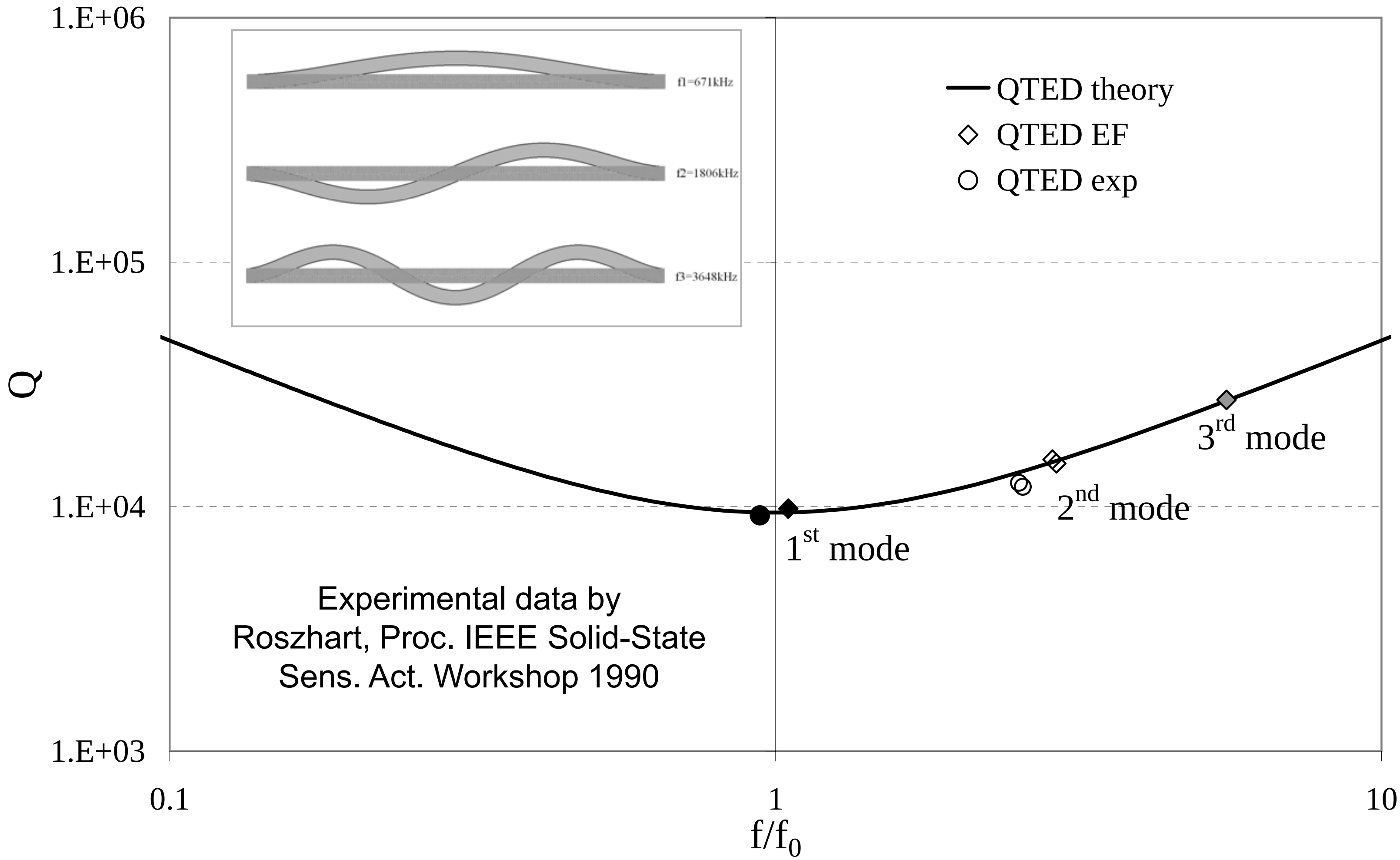
E	150GPa	ρ	2330 kg/m^3
ν	0.2	C	700 J/(kg · K)
α	$2.6\cdot10^{-6}$ 1/K	k	148 W/(m · K)



FEM-based discretization of thermoelastic problem

Silicon micro-beams (plane stress)
doubly clamped beam; thickness 15μm

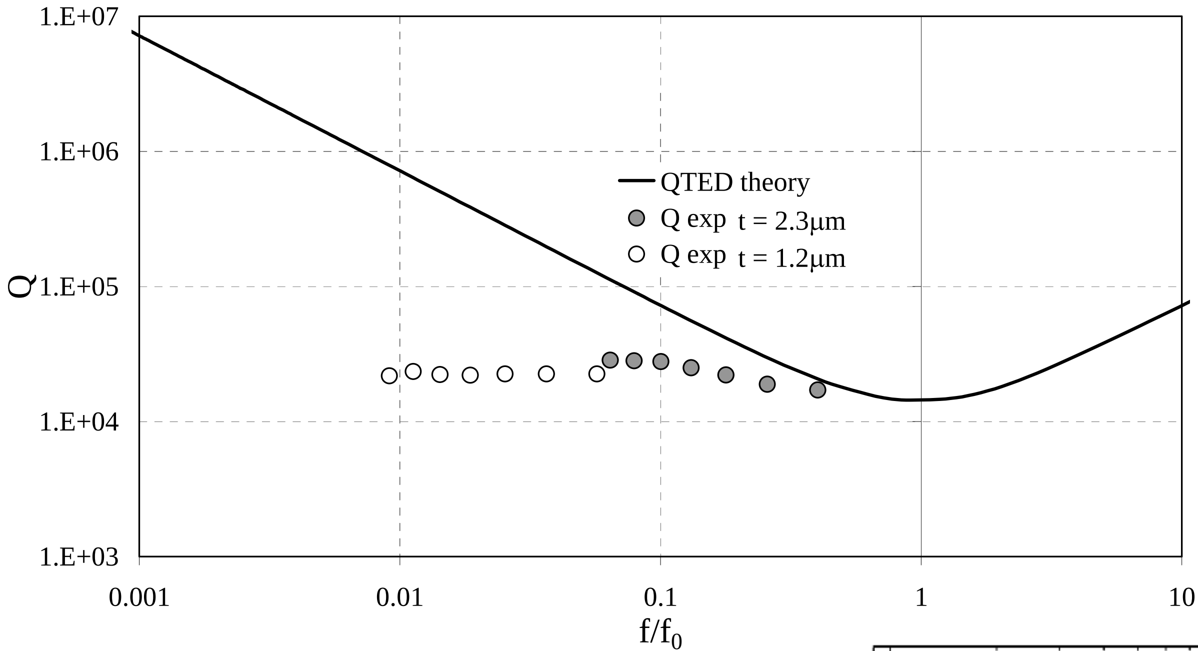
E	150GPa	ρ	2330 kg/m³
ν	0.2	C	700 J/(kg · K)
α	2.6·10⁻⁶ 1/K	k	148 W/(m · K)

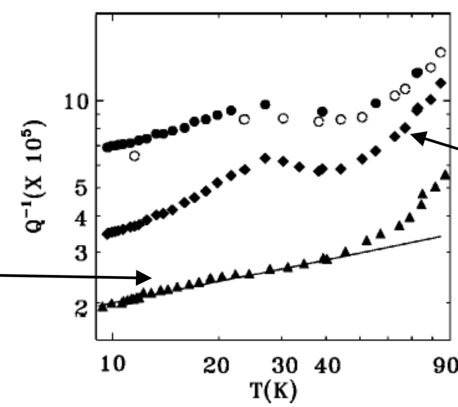


Thermoelastic damping

ultrathin cantilevers

from: Yasumura et al., JMEMS 2000





- no dissipation is provided for purely **deviatoric** modes (torsional modes) or **homogeneous** modes (pure extension)

from: Mohanty et al., Phys.Rev.B 2002

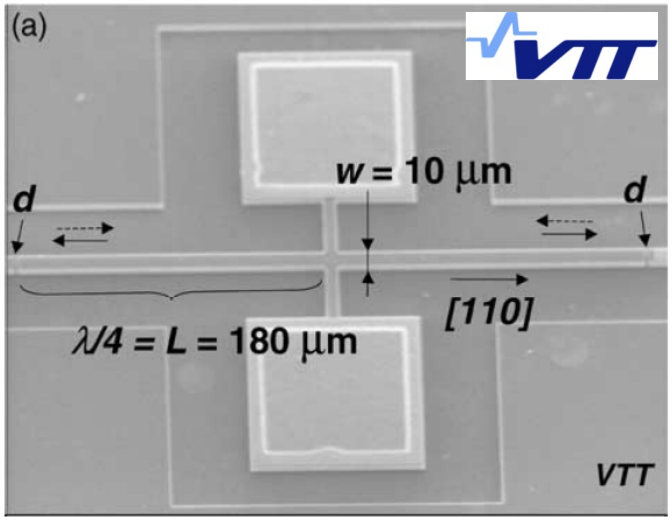
Overview

1. *Fluid* damping
    - Continuum regime
    - Rarefied regime

2. *Solid* damping
    - Thermoelastic dissipation
    - Anchor losses
    - Surface losses

Simulation of anchor losses

Case of study:
VTT extensional resonator
(resonates according to the first axial mode of the thin beam)



Assume that the elastic waves scattered through the anchor are dissipated in the support (which is generally much larger than the resonator itself)

How to simulate this "dispersion"?

Preferred strategy. Use FEM for the resonator, truncate at finite distance and use:

- ***Absorbing Boundary Conditions*** (ABC)
- ***Perfectly Matched Layers*** (PML)
- ***Coupling with Boundary Elements***

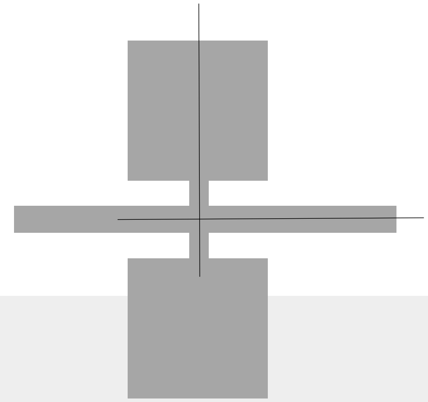
$Q = 2\pi|\omega|/(2\mathrm{Im}(\omega))$

where ω is the (complex) frequency of the mode corresponding to the resonating mode of the MEMS with no substrate

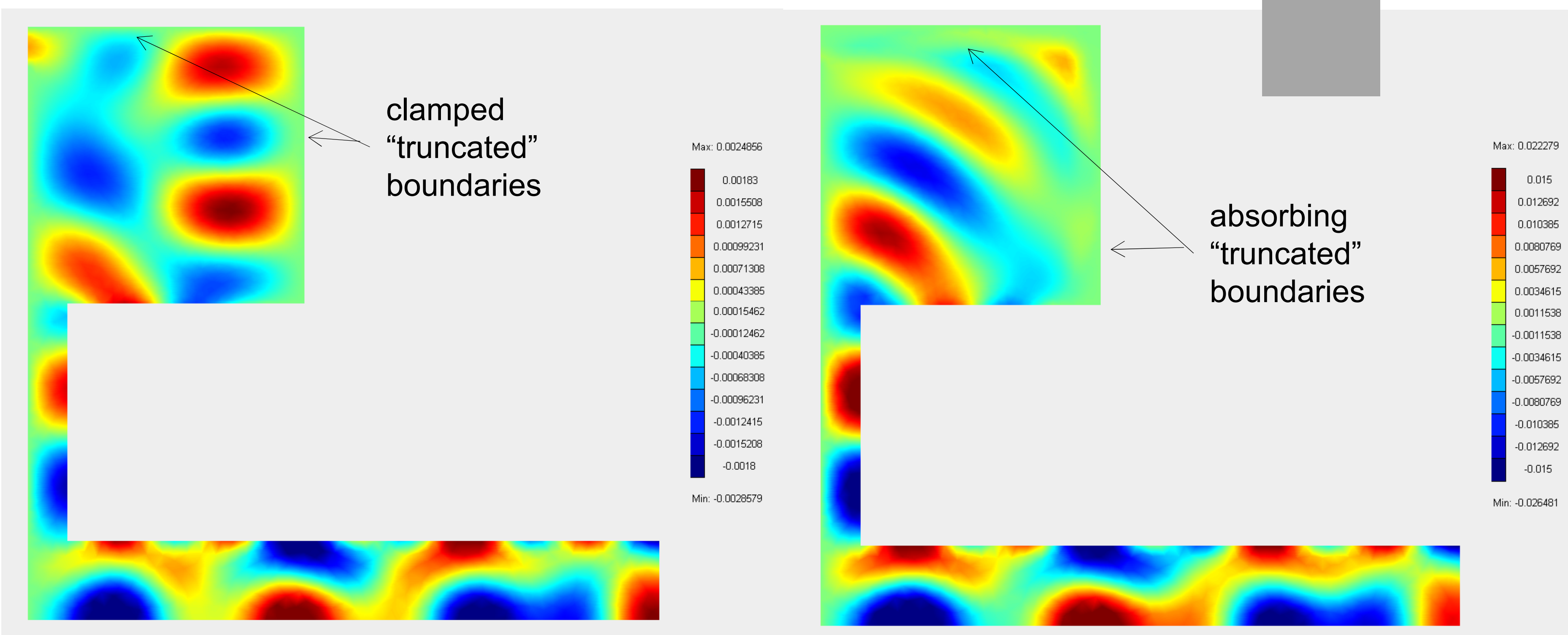
David S. Bindel and Sanjay Govindjee, *Int. J. Numer. Meth. Engng 2005;*

Simulation of anchor losses

Problem is truly 3D but assume it is 2D for simplicity

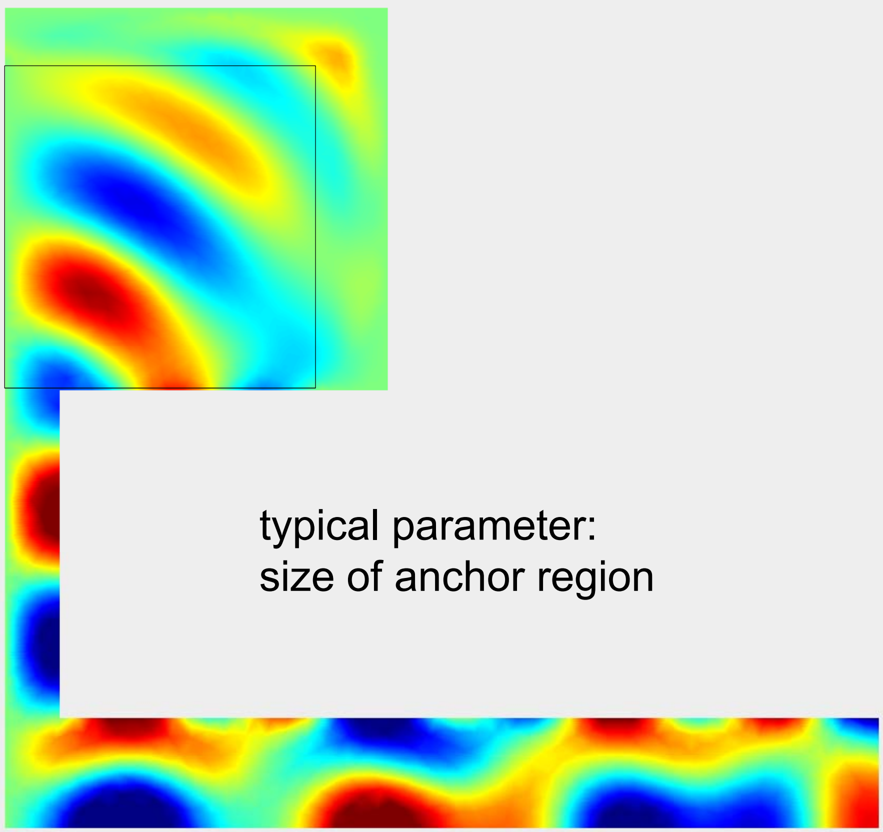


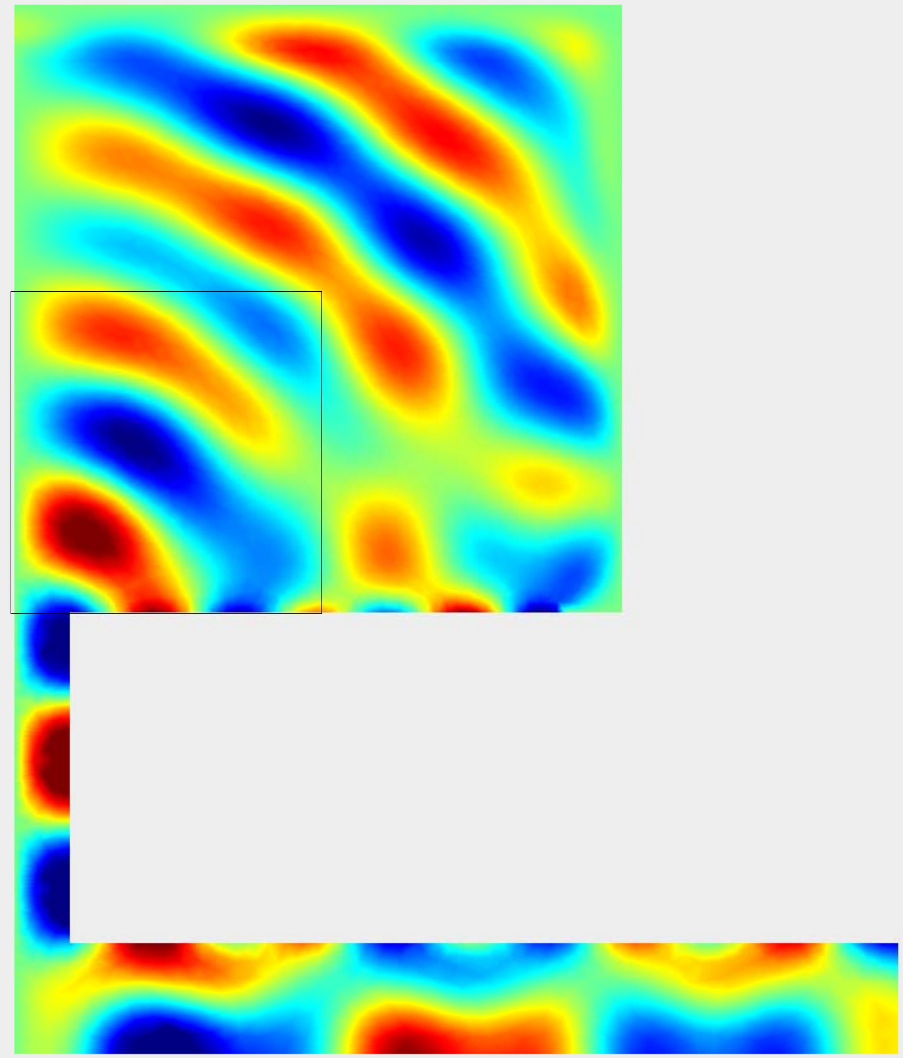
mode corresponding to an high frequency axial mode of the resonator



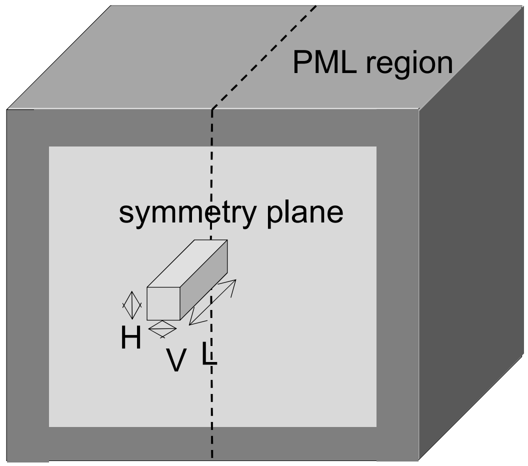
Simulation of anchor losses

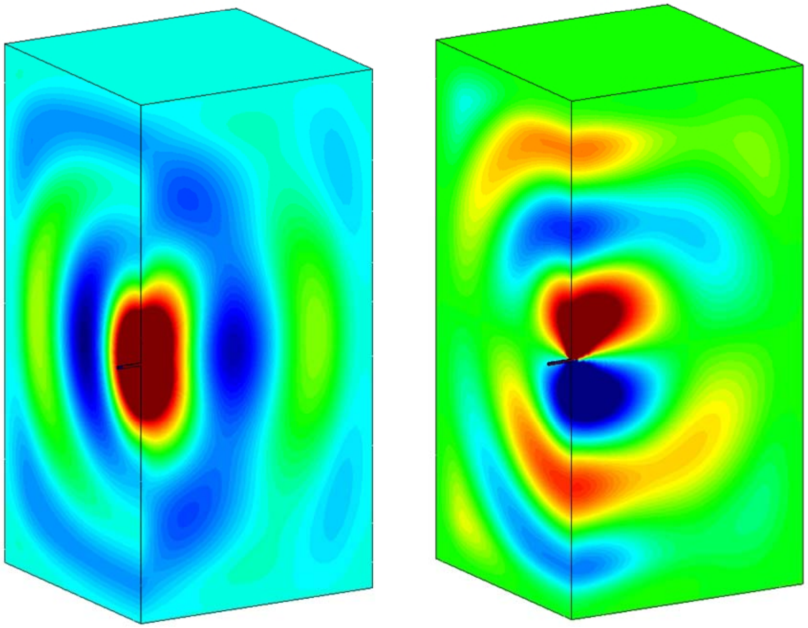
Challenges:
•migrate to 3D
•correct choice (calibration) of parameters





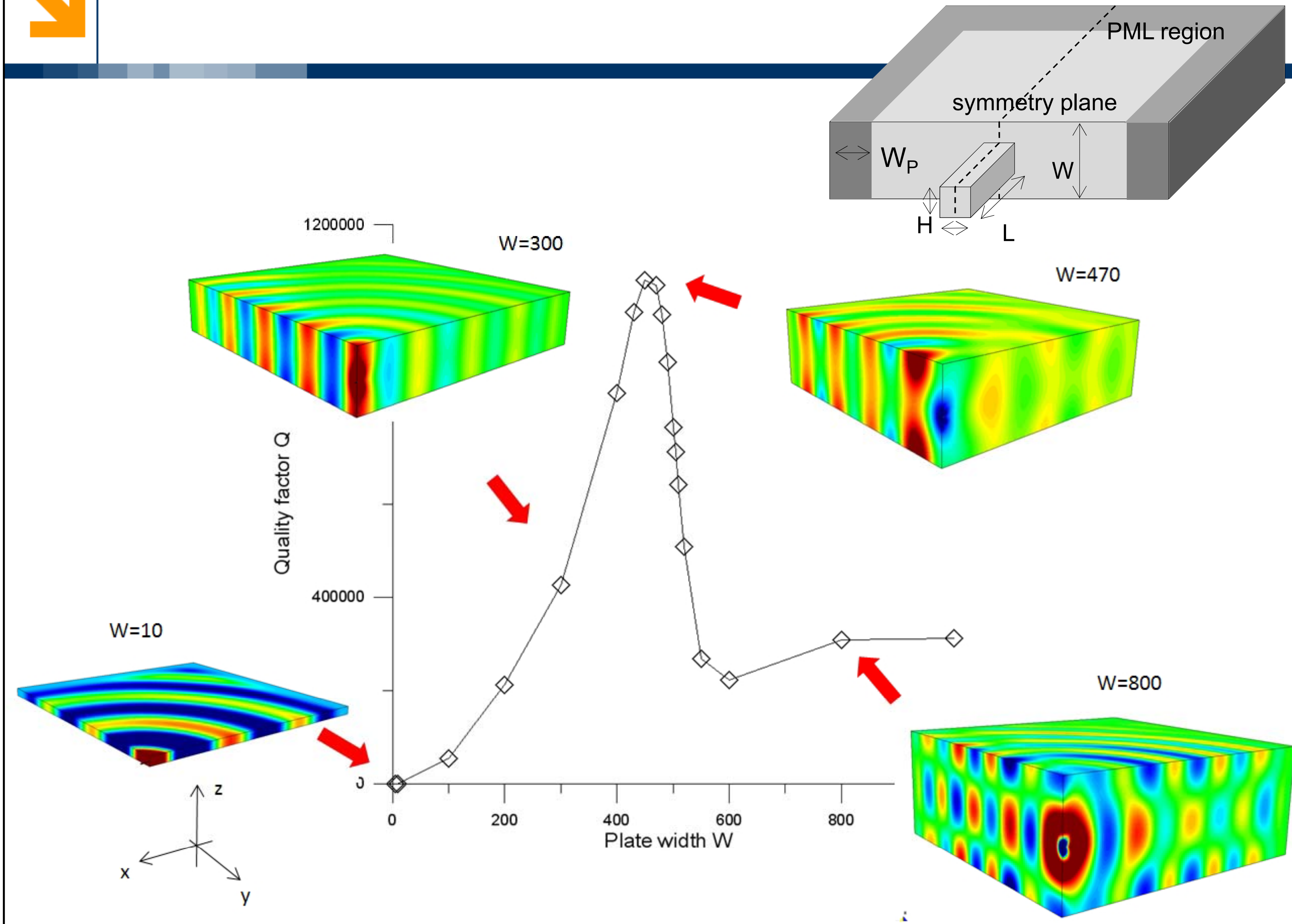
typical parameter:
size of anchor region

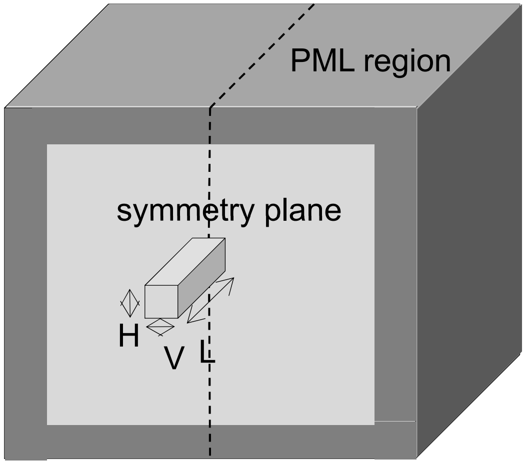




Dissipation in bending modes

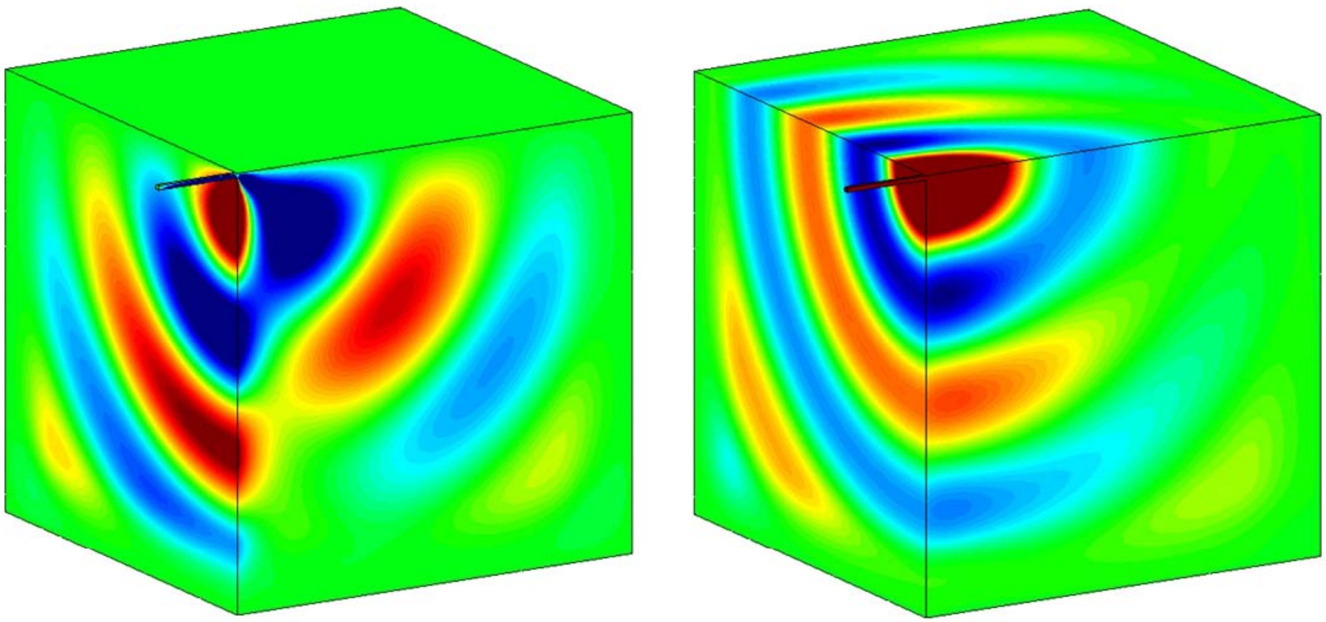
β	3	4	5	ref.[14]	ref.[11]
$Q\ (W_\infty = 700,\ N_{el} = 57453)$	324323	318715	316061	317500	296448
$Q\ (W_\infty = 700,\ N_{el} = 118785)$	325034	319381	316712	317500	296448
$Q\ (W_\infty = 700,\ N_{el} = 143118)$	325702	320011	317328	317500	296448
$Q\ (W_\infty = 800)$	318016	316103	315439	317500	296448


PML region
symmetry plane
W_P
W
H
L
W=300
W=470
W=10
W=800
1200000
400000
0
Quality factor Q
0
200
400
600
800
Plate width W
z
x
y



Dissipation in axial modes

β	1	2	3	ref.[11]
Q	68.17	65.72	65.58	56.02



Piezotransduced Single-Crystal Silicon BAW Resonators

Antti Jaakkola, Piia Rosenberg, Arto Nurmela, Tuomas Pensala, Tommi Riekkinen, James Dekker, Tomi Mattila, and Ari Alastalo
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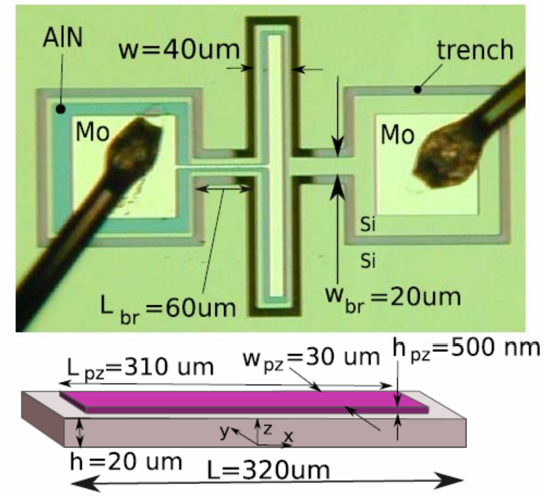

Figure 1. Micrograph of a wire-bonded resonator. The beam dimensions are $L \times w \times h$, and the piezolayer size is $L_{pz} \times w_{pz} \times h_{pz}$. L_{br} and w_{br} denote the anchor dimensions.

experimental values:

ν=0.06 Q=20000

ν=0.28 Q=3500

Overview

1. *Fluid* damping
   - Continuum regime
   - Rarefied regime

2. *Solid* damping
   - Thermoelastic dissipation
   - Anchor losses
   - Surface losses …interfaces?

Experimental evidence

IEEE ELECTRON DEVICE LETTERS, VOL. 25, NO. 4, APRIL 2004 173

Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications

Ville Kaajakari, Tomi Mattila, *Member, IEEE*, Aarne Oja, Jyrki Kiihamäki, and Heikki Seppä

capacitive transduction

Q=130000

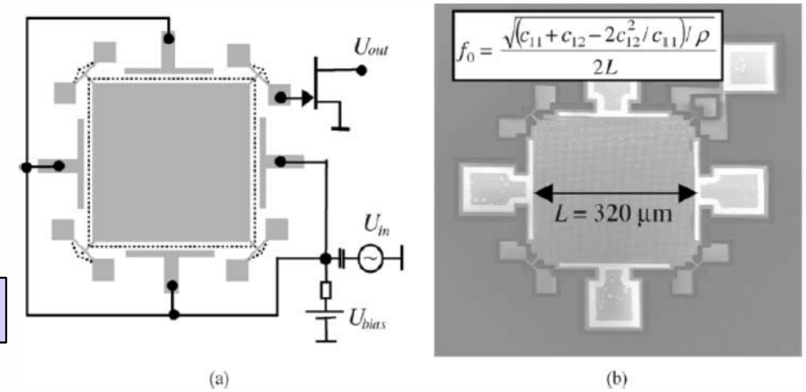

Fig. 1. Square-extensional microresonator (f_0 = 13.1 MHz and Q = 130 000). (a) Schematic of the resonator showing the vibration mode in the expanded shape and biasing and driving setup. (b) SEM image of the resonator.

piezoelectric transduction (very thin AlN layer)

Q=18000

Q=51000

Piezoelectrically transduced Single-Crystal-Silicon Plate Resonators

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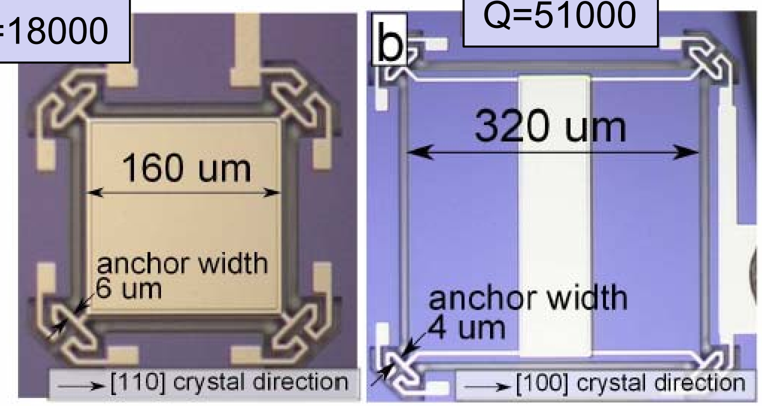
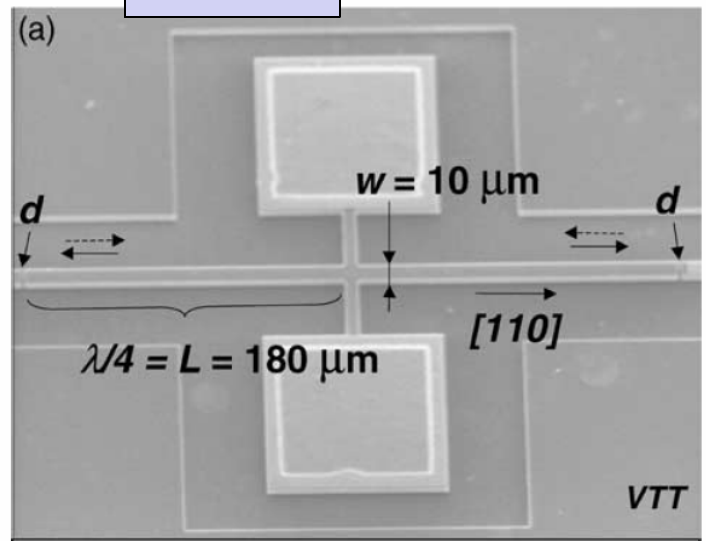


Figure 1. Micrographs of resonators A and B. The thickness of the resonators is 20 μm.

Experimental evidence on a simpler problem

Q=180000



A 12 MHz micromechanical bulk acoustic mode oscillator

T. Mattila[a,*], J. Kiihamäki[b], T. Lamminmäki[c], O. Jaakkola[a], P. Rantakari[c], A. Oja[a], H. Seppä[a], H. Kattelus[b], I. Tittonen[c]

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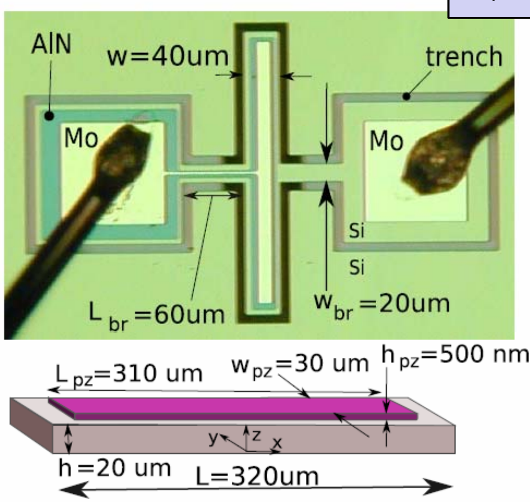
[c]*Metrology Research Institute, Helsinki University of Technology, P.O. Box 3000, 02015 Espoo, Finland*

Received 12 February 2002; received in revised form 19 June 2002; accepted 19 June 2002

Q=20000



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Thank you!