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Topology Optimization with Isogeometric Analysis in a Phase Field Approach

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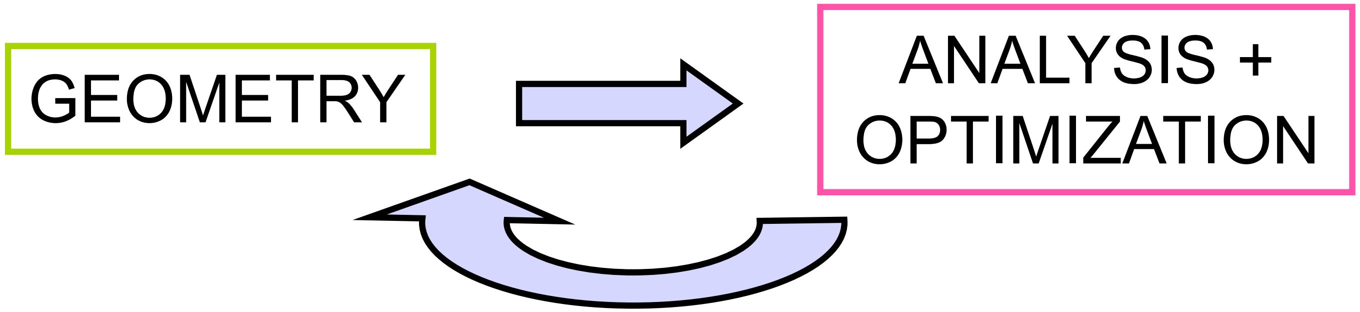
Design, Analysis, Optimization

Design processes often involve or require some optimization of the geometry (CAD).



- **Shape Optimization:**

find the optimal shape of a body;

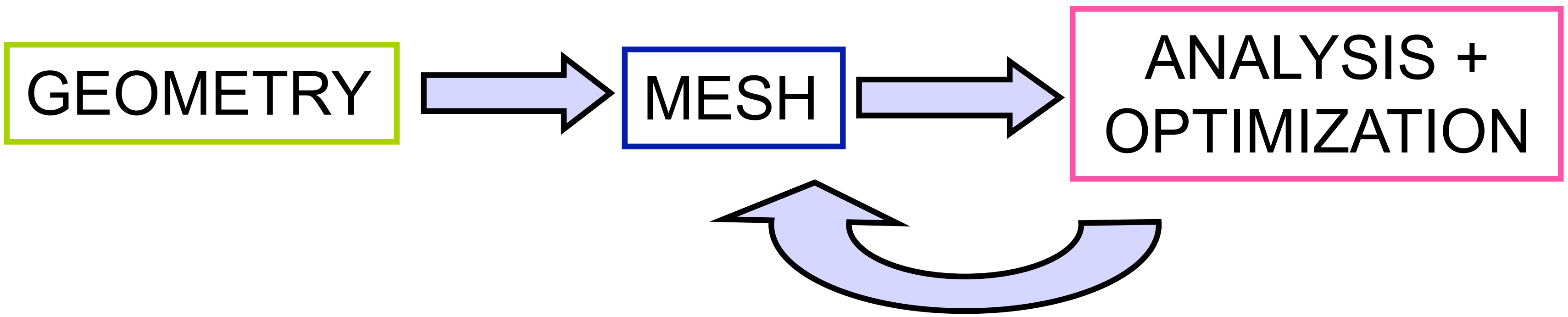
- **Topology Optimization**:

find the optimal distribution of the material in a domain or component.

Design, Analysis, Optimization

Current **Engineering procedures** based on the **Finite Element Analysis** do <u>not</u> allow a straightforward use of optimization tools.

This is principally due to the <u>CAD geometry-mesh mapping</u>.



Design, Analysis, Optimization

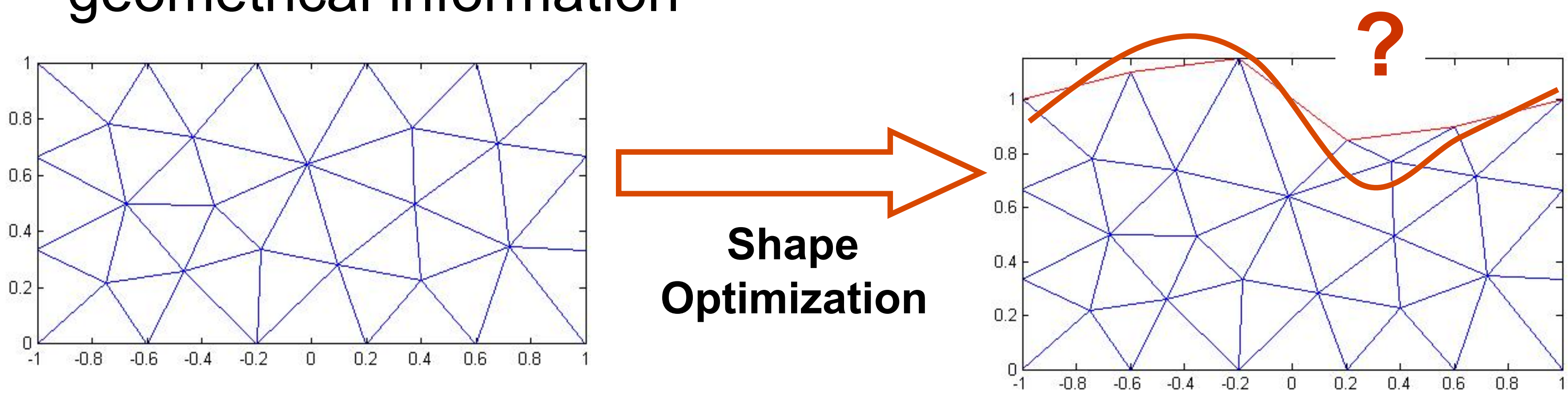
Example: Shape Optimization

$$\boxed{\begin{array}{l}\text{find } \partial\Omega \text{ s.t. } J(u;\partial\Omega) \text{ is miminum, with:}\\ L(u)u = f \text{ in } \Omega \ \ \& \text{ BC}s.\end{array}}$$

[Jamenson, Mohammadi, Pironneau, Sokolowski, Svanberg, Zolesio, …]

Difficulties:

- nonlinearities/differentiation/optimization tech.
- re-meshing
- geometrical information



Design, Analysis, Optimization

Difficulties in geometry optimization are inherited by drawbacks in <u>Finite Element Analysis</u>:

- Computational geometry and analysis are separate fields
  - Finite Element Method (FEM), started in 1950's
  - Computer Aided Design (CAD) and Computational Geometry (CG), started in 1970's
- Geometry is a foundation of CAD
- Geometry is a foundation of computational analysis
- <u>CAD and FEM use different representations of geometry</u>
- Mesh generation is a *bottleneck* in Design through Analysis.

Design, Analysis, Optimization

Encapsulate the exact CAD geometry in:

- Analysis
- Design
- <u>Optimization:</u>
    - Shape Optimization
    - Topology Optimization

⇨ <u>Isogeometric Analysis</u>

[Hughes, Cottrell, Bazilevs, 2005]

Isogeometric Analysis

- Analysis framework built on the primitives (basis functions) of **CAD** and Computational Geometry
  - Original instantiation based on Non-Uniform Rational B-Splines (**NURBS**)
  - Framework extended to more advanced discretizations (e.g., **T-splines**, Subdivision)

- Generalizes and improves on Finite Element Analysis
  - Encapsulates "**exact geometry**" and its parameterization at the coarsest level of discretization
  - Allows for *smooth* basis functions
  - Allows for *h*-, *p*- and *k*-refinement
  - Geometry and its parameterization unchanged during refinement

Objects of B-spline geometry

Linear combination of the spline basis in $\hat{\Omega}=(0,1)^{\alpha}$ $\alpha=1,\ldots,d$

and objects in $\mathbb{R}^d$

$$\mathbf{\Omega}=\mathbf{F}(\boldsymbol{\xi})=\langle \boldsymbol{C},\boldsymbol{N}\rangle(\boldsymbol{\xi})=\sum_{i=1}^{n}\boldsymbol{C}_i N_i(\boldsymbol{\xi}) \qquad \forall \boldsymbol{\xi}\in\hat{\mathbf{\Omega}}=(0,1)^{\alpha}$$

α =1 gives rise to a *B-spline curve* in $\mathbb{R}^d$

α =2 gives rise to a *B-spline surface* in $\mathbb{R}^d$

α =3 gives rise to a *B-spline solid* in $\mathbb{R}^d$

Control points.
Their multi-linear interpolation forms
Control mesh

Cannot represent conic sections (i.e. circles, ellipses) exactly.
Need NURBS.

Univariate (1-D) splines

Knot vector on $\hat{\Omega}$ and *p-order*

$$\Xi = \{\xi_1, \xi_2, \xi_3, \ldots, \xi_{n+p+1}\}$$

n = number of basis functions

Knots ξ_i with multiplicity m_i

B-spline basis on $\hat{\Omega}$ by recursion:

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases}$$

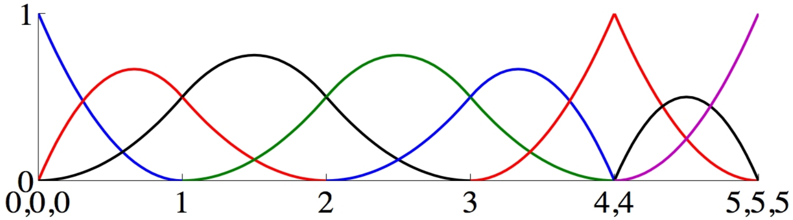
Start with piece-wise constants

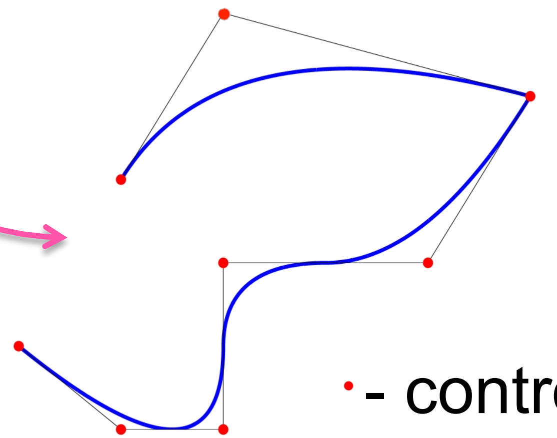
$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

Bootstrap recursively to *p*

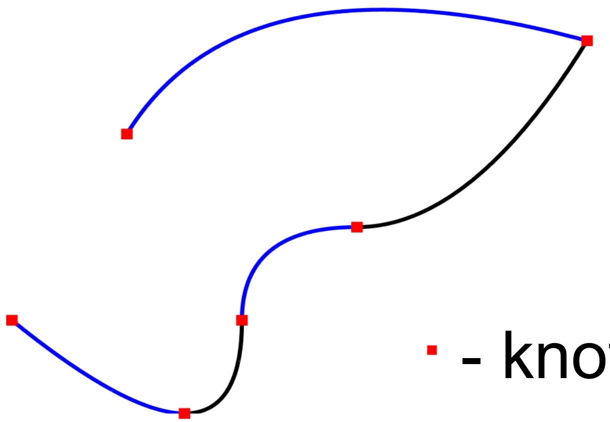
Univariate (1-D) splines

Knot vector on $\hat{\Omega}$ and *order* 2





- control points



- knots

Objects of NURBS geometry

Linear combination of the spline basis in $\hat{\Omega}=(0,1)^\alpha$ $\alpha = 1,\ldots,d$ and objects in $\mathbb{R}^{d+1}$ projected back into $\mathbb{R}^d$ by a *projective transformation* :

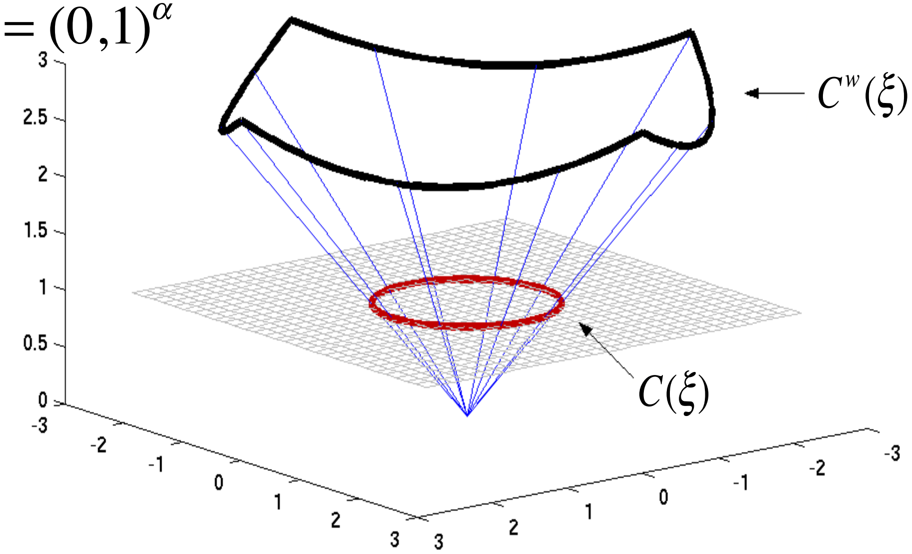
$$\boxed{\boldsymbol{\Omega} = \mathbf{F}(\xi)} = \Pi\left(\sum_{i=1}^{n}\{\boldsymbol{C}_i, w_i\}N_i(\xi)\right) =$$

$\boldsymbol{C}_i, w_i$

Part of geometrical map.
NURBS basis is geometry-specific.

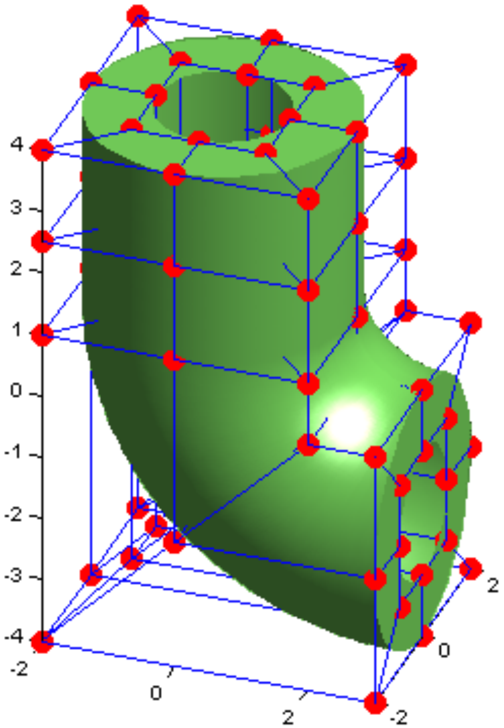
$$\sum_{i=1}^{n}(\boldsymbol{C}_i / w_i)\left(w_i N_i(\xi) / \left(\sum_{j=1}^{n} w_j N_j(\xi)\right)\right) =$$

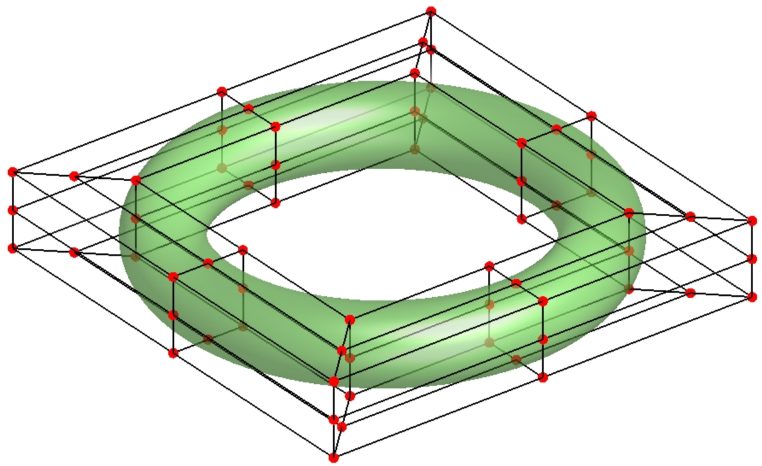
$$\boxed{\sum_{i=1}^{n}\hat{\boldsymbol{C}}_i \frac{w_i N_i(\xi)}{w} = \sum_{i=1}^{n}\hat{\boldsymbol{C}}_i R_i(\xi)} \quad \forall \xi \in \hat{\Omega} = (0,1)^\alpha$$

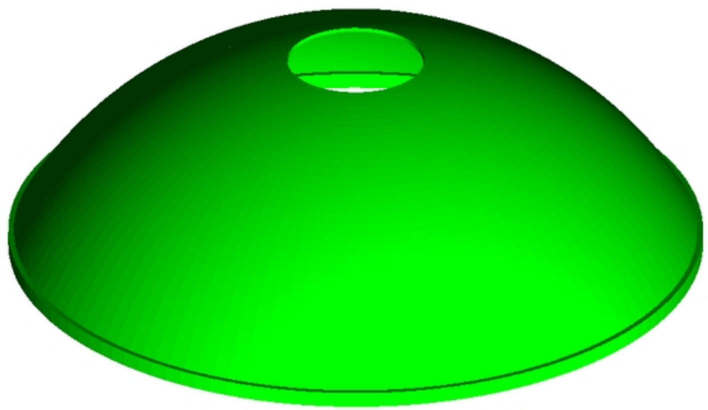


Conic sections represented exactly.

Objects of NURBS geometry







From NURBS to Isogeometric Analysis

$$\Omega = \left\{ \boldsymbol{x} \in \mathbb{R}^d, \boldsymbol{x} = \mathbf{F}(\boldsymbol{\xi}) = \sum_{i=1}^{n} \boldsymbol{C}_i R_i(\boldsymbol{\xi}), \forall \boldsymbol{\xi} \in \hat{\Omega} = (0,1)^\alpha \right\}$$

Physical domain

Parametric domain

$$\boxed{u \in V \quad \text{solution of PDE}}$$

Isogeometric Analysis

$$\boxed{u_h \in V_h = \text{span}\left\{ R_i \circ \mathbf{F}^{-1} \right\}_{i=1,\ldots,n} \subset V}$$

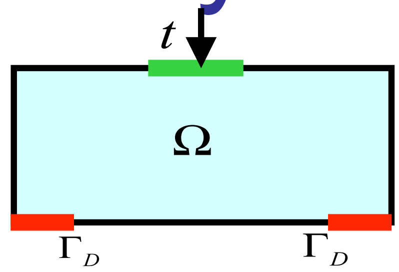
Basis (Isoparametric construction)

NURBS approximation space over physical domain

$$u_h = \hat{u}_h \circ \mathbf{F}^{-1} \qquad \hat{u}_h(\xi) = \sum_{i=1}^{n} u_i R_i(\xi) \quad \text{in } \hat{\Omega}$$

Optimization with Isogeometric Analysis

Initial CAD Geometry



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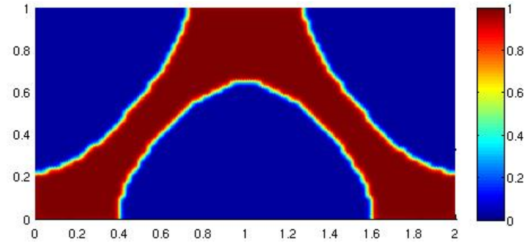
Topology Optimization



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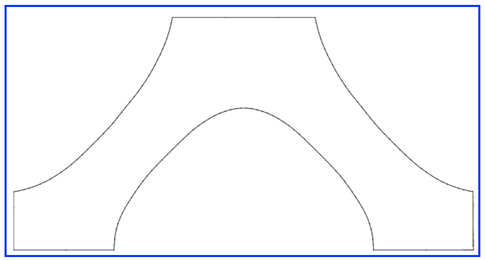
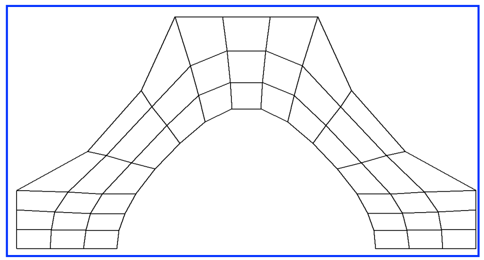
Geometry generation

CAD (NURBS or T-splines)

[J.Zhang]



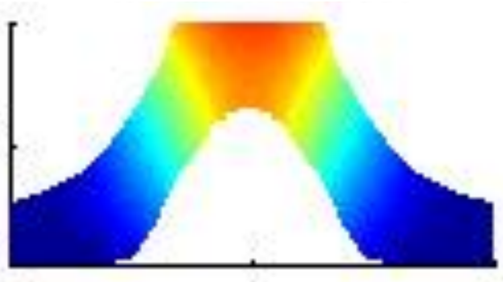


⇩

Shape Optimization

⇩

Optimal CAD Geometry



Topology Optimization

Applications:

- Beams/Trusses/Bridges [Bendsoe, Kikuchi, Sigmund, …]
- Aeronautical structures [Bendsoe]
- Crashworthiness design [Pedersen]
- MEMS devices / Piezoelectric micro-tools
  [Buhl, Carbonari, Sigmund, Silva, Paulino,…]
- Dynamical systems [Jensen]
- Acoustics / Photonic / Thermal / Fluid problems
  [Bendsoe, Gersborg-Hansen, Jensen, Sigmund, …]

▪ 2D/3D problems

▪ Single or multi-material

Topology Optimization

[Bendsoe, Kikuchi, Sigmund, Stolpe, Svanberg, …]

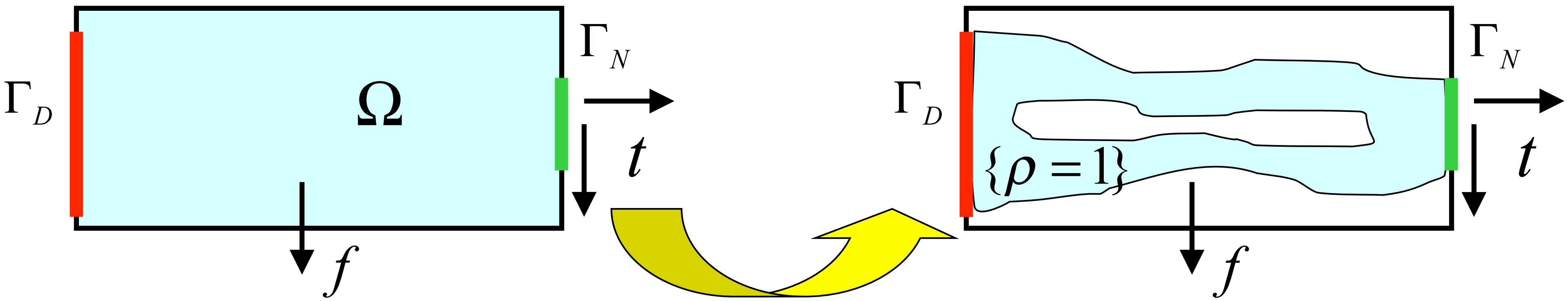
find $\rho = \{0,1\}$ in $\Omega \subset R^d$ s.t. $J_E(\rho,u)$ minimum, with:

$$
\begin{cases}
-div\,\sigma(u) = f \ \text{ in } \{\rho = 1\} \subseteq \Omega \\
u = 0 \ \text{ on } \Gamma_D, \\
\sigma(u)\cdot n = t \text{ on } \Gamma_N, \\
\sigma(u)\cdot n = 0 \text{ on } \partial\Omega \setminus (\Gamma_D \cup \Gamma_N)
\end{cases}
$$

& inequality constraints

Structural Topology Optimization: Linear Elasticity

($u = u(\rho)$ and $\sigma(\mathrm{u})=\sigma(u(\rho))$)



Minimum Compliance

[Bendsoe, Kikuchi, Sigmund, Pedersen]

Find the optimal distribution of ρ in the domain Ω
in order to minimize the compliance $J_E(\rho)$ of the system
under a volume constraint V (equality or inequality)

$$J_E(\rho) = \int_\Omega f \cdot u(\rho) + \oint_{\Gamma_N} t \cdot u(\rho)$$

$$\int_\Omega \rho \leq V < |\Omega|$$

(Alternative criterion:
minimize weight of the structure
under stress constraints)

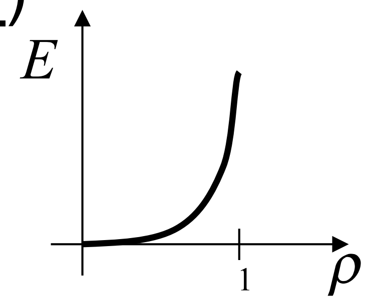
SIMP Approach

- $\boxed{\rho = \{0,1\} \quad \rightarrow \quad 0 \le \rho \le 1}$ (with inequality constraints)

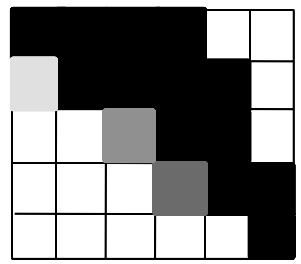
- Solid Isotropic Materal with Penalization (<u>SIMP</u>)

$$\boxed{E(\rho) = \rho^P E_0, \quad P \ge 3}$$

(Young modulus depends on density)



- Finite Element approximation; Low Order
  ρ piecewise constant over mesh elements

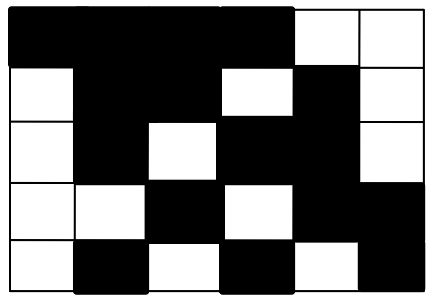


- Constrained Optimization procedure;
  e.g.: MMA (Method of Moving Asymptotes), [Svanberg]

SIMP Approach

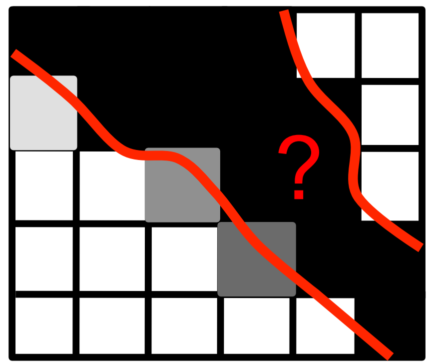
Advantages:
- simplicity & flexibility
- low number of DOFs

Drawbacks:
- instabilities & check-board phenomenon



→ *Required: regularization techniques, filters, sensitivity filters, perimeter limitation*

- high number of inequality constraints
- non-convex optimization



- ability to provide geometric information
- manufacturability - integration with CAD

Multiphase Approach

[Bourdin, Burger, Chambolle, Stainko, Wang, Zhou]

$$\rho = \{0,1\} \in L^{\infty}(\Omega) \quad \rightarrow \quad \rho \in H^{1}(\Omega) \bigcap L^{\infty}(\Omega)$$

<u>Sharp interfaces approximated by thin layers</u>

Add to cost functional a **total free energy** term (<u>Cahn-Hilliard</u> type):

$$J(\rho) = J_{E}(\rho) + \frac{1}{\varepsilon} J_{BLK}(\rho) + \varepsilon J_{INT}(\rho)$$

$$J_{BLK}(\rho) = \int_{\Omega} \rho(1-\rho)\, d\Omega$$

“Filtering” & perimeter limitation

$$J_{INT}(\rho) = \frac{1}{2} \int_{\Omega} |\nabla \rho|^{2}\, d\Omega$$

$$\varepsilon > 0$$

Finite Element approximation: order ≥ 1

Multiphase Approach

<u>Advantages</u>:

- sharp & smooth interfaces
- geometrical information
- filtering and perimeter limitation embedded in $J_{INT}(\rho)$
- possibility to remove inequality constraints $0 \leq \rho \leq 1$ by choosing $J_{BLK}(\rho)$

<u>Drawbacks</u>:

- non-convex optimization
  → *strong dependence on the optimization solver used*
- dependence on parameters

Multiphase Approach

Continuation method

- Quasi-Newton optimization method

- Continuation method (progressive reduction of $\varepsilon = \chi^L \varepsilon_0$);
  the final solution is obtained as a sequence of optimal states
  for increasing values of L

$$\boxed{J(\rho) = J_E(\rho) + \frac{1}{\varepsilon} J_{BLK}(\rho) + \varepsilon J_{INT}(\rho)}$$

- Isogeometric Analysis, order 3

Linear Elasticity, Plane Stress

$E_0 = 1, \quad \nu = 0.3, \quad f = 0, \quad t = -0.5\hat{y}$

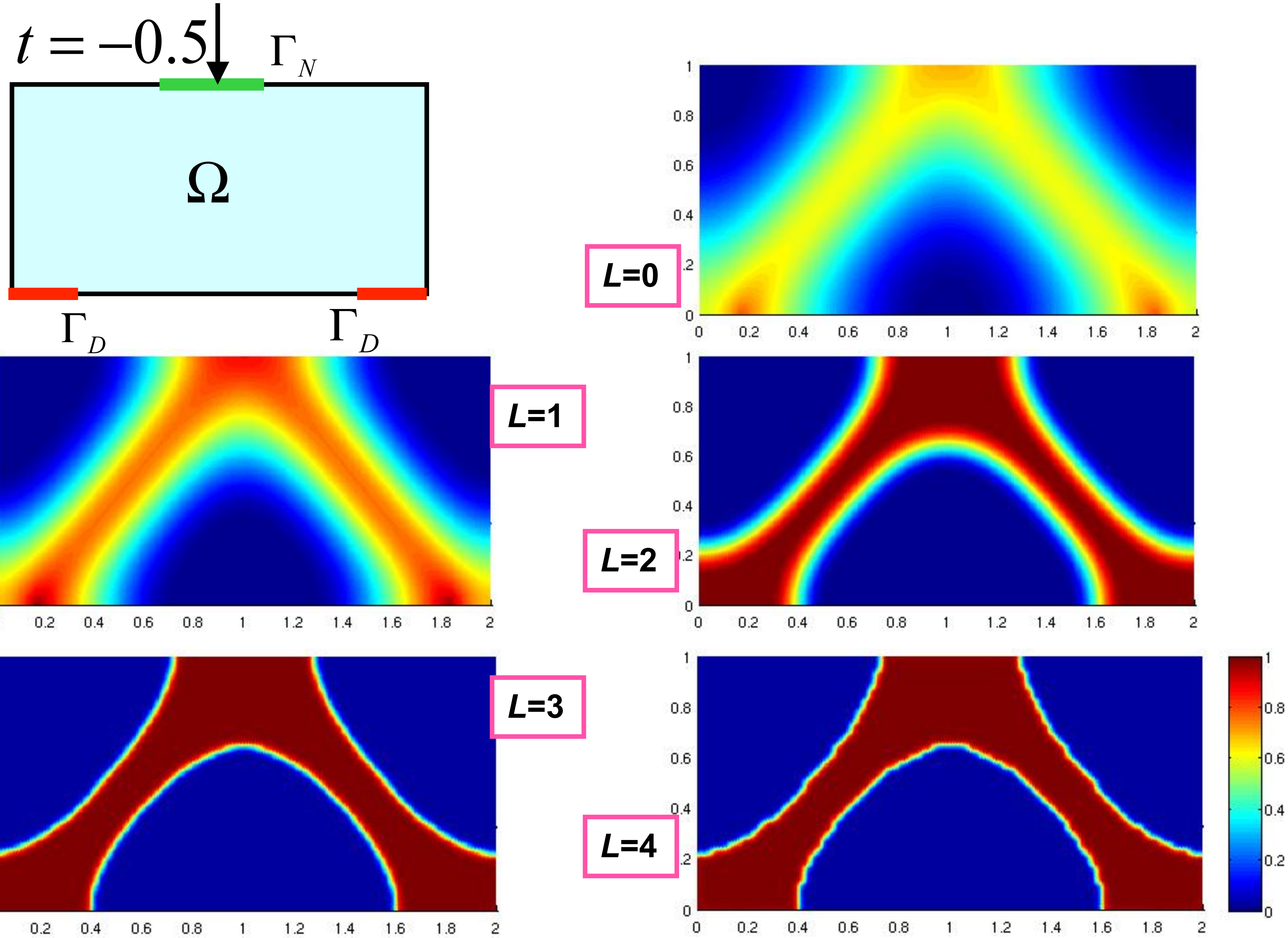
$\Omega = (0,2)\times(0,1), \quad \dfrac{\int_\Omega \rho}{|\Omega|} \le 0.35$

$E(\rho) = \rho^P E_0, \quad P = 5$

$\varepsilon_0 = 1.5, \quad \chi = 0.25$

$\#\,\text{d.o.f.} = 3960\ (IsoG.)$

Multiphase Approach



Topology Optimization & Cahn-Hilliard Equation

The Multiphase approach in Topology Optimization shows analogies with Multiphase problems, in particular with the:

<u>Cahn-Hilliard equation</u> (1957)

which describe the transition of two phases from a mixed status to a fully separated configuration.

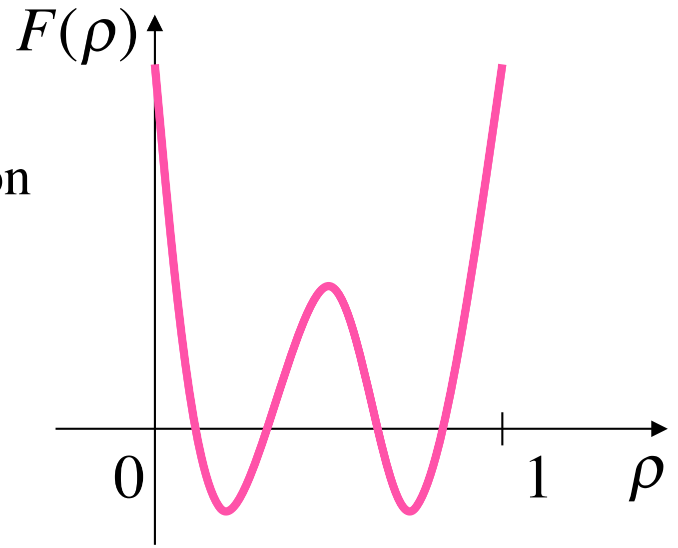
→ The CH eq. is a 4th order nonlinear parabolic PDE

Cahn - Hilliard Equation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = \nabla \cdot \left(M(\rho) \nabla \mu_{CH}(\rho) \right) & \text{in } \Omega \times (0,T) \\ \nabla \rho \cdot n = 0 & \text{on } \partial\Omega \times (0,T) \\ M(\rho) \nabla \mu_{CH}(\rho) \cdot n = 0 & \text{on } \partial\Omega \times (0,T) \\ \rho = \rho_0 & \text{in } \Omega \times \{t = 0\} \end{cases}$$

(or periodic conditions)

$$\boxed{J_{CH}(\rho) = \int_\Omega F(\rho)\, d\Omega + \frac{1}{2}\lambda \int_\Omega \nabla\rho \cdot \nabla\rho\, d\Omega \qquad \lambda > 0}$$



$$\mu_{CH}(\rho) \; : \; < \mu_{CH}(\rho), \varphi > = \frac{dJ_{CH}}{d\rho}(\rho)[\varphi] \quad \forall \varphi \text{ test function}$$

$$\mu_{CH}(\rho) = \frac{dF}{d\rho}(\rho) - \lambda \Delta \rho$$

$$V = \int_\Omega \rho_0\, d\Omega \equiv \int_\Omega \rho(t)\, d\Omega \quad \forall t \in (0,T)$$

$$\mathrm{M}(\rho) = D\rho(1-\rho) \quad \text{mobility}$$

Cahn - Hilliard Equation

<u>Time approximation</u>:

- generalized α-method (fully implicit, second order accurate)
- adaptive time-scheme (based on comparison of Backward Euler and α-method)

<u>Spatial approximation</u>:

- IsoGeometric Analysis, order ≥ 2

$$\lambda = \tau h^2$$

Interface thickness
Depending on resolution

$F(\rho)$ logarithmic, $\theta = 1.5$, $M = D\rho(1-\rho)$, $D = 1$,

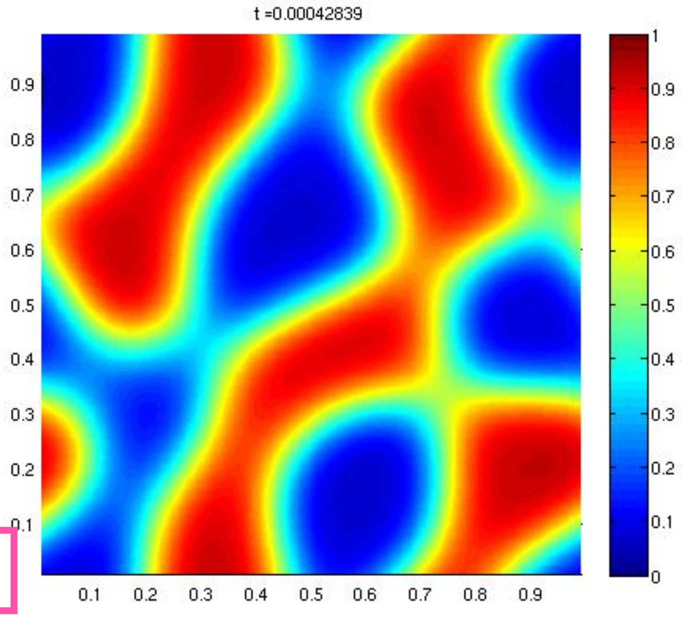
Initial random distribution: $Volume \pm 0.50\ \%$

IGA: order $p, q = 2$, Gauss pt.s $= 3$, $h = L_0 / 45$

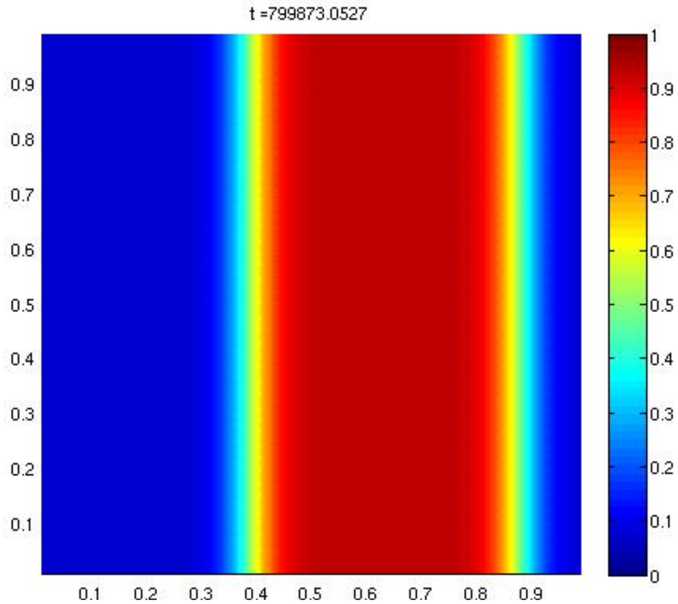
[Gomez, Calo, Bazilevs, Hughes, 2007]

Cahn - Hilliard Equation

Volume = 0.50, Periodic BCs

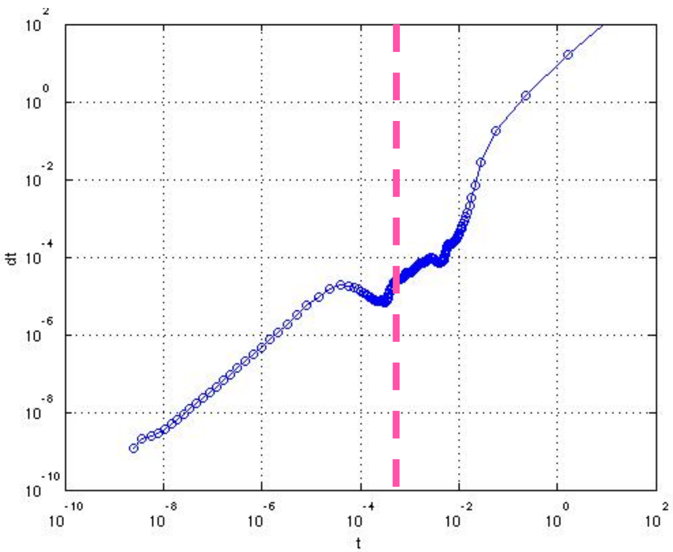


t = 4e-4



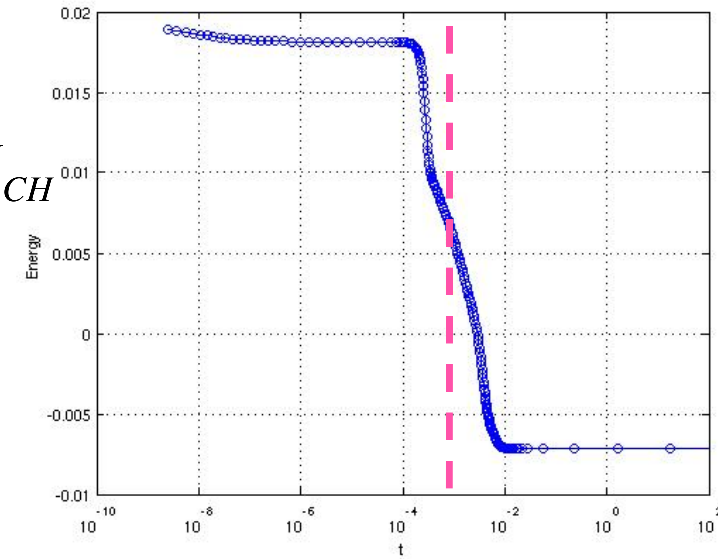
Steady state

dt



t

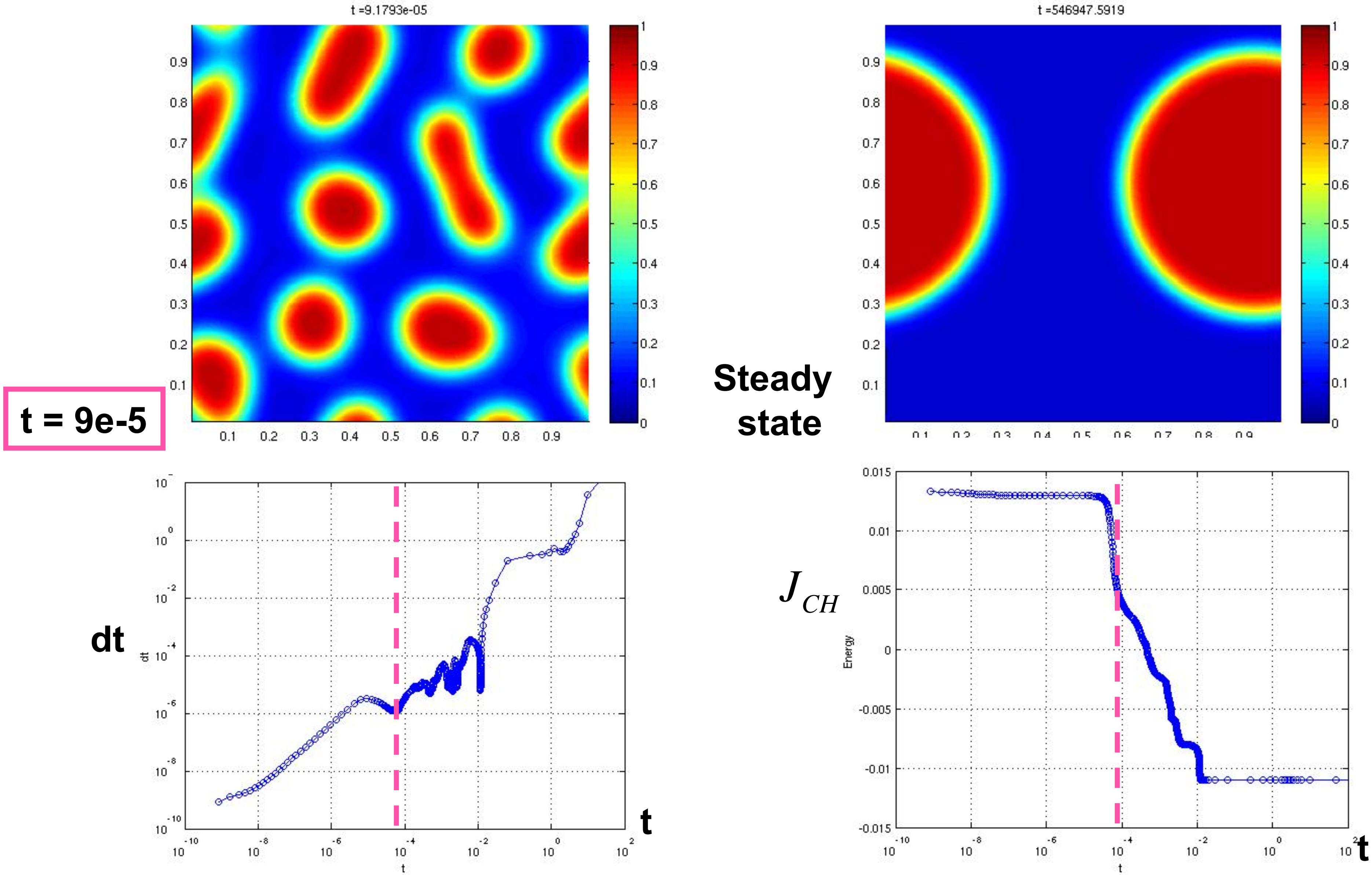
J_{CH}



t

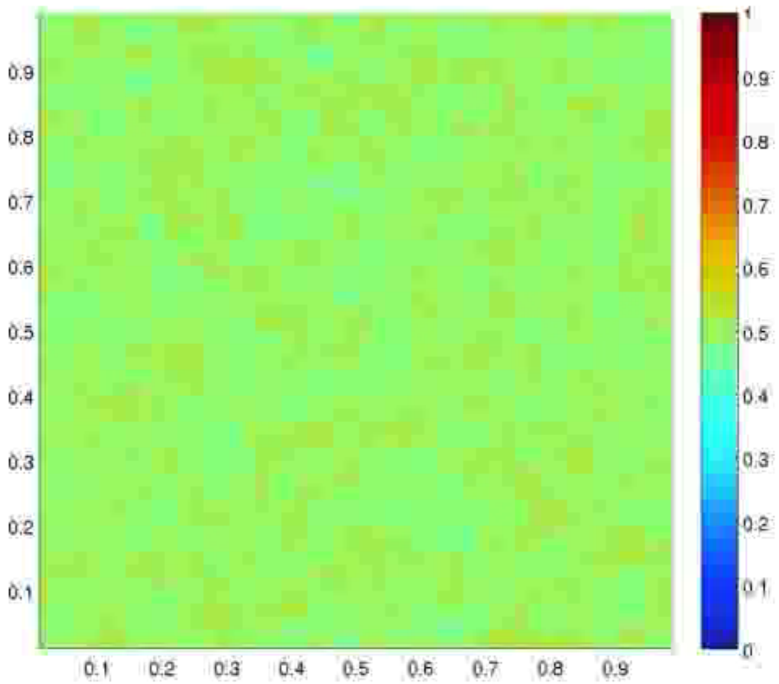
Cahn - Hilliard Equation

Volume = 0.37, <u>Periodic BCs</u>

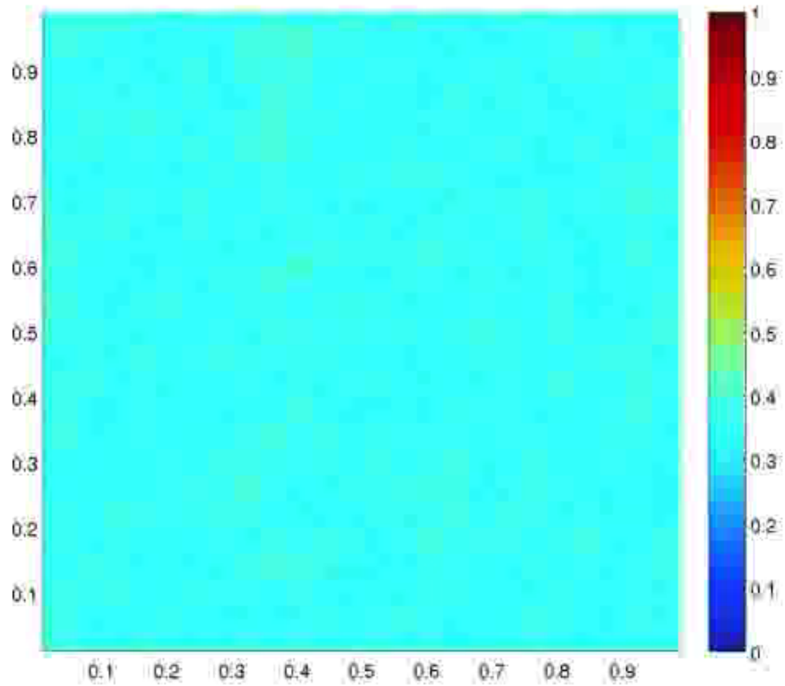


Cahn - Hilliard Equation

Volume = 0.50



Volume = 0.37



Topology Opt. with Phase Field

Generalized Cahn-Hilliard equations

$$\boxed{J(\rho) = J_{CH}(\rho) + \gamma J_E(u(\rho))}$$

$$\boxed{J_{CH}(\rho) = \int_\Omega F(\rho)\, d\Omega + \frac{1}{2}\lambda \int_\Omega \nabla\rho \bullet \nabla\rho\, d\Omega \qquad \lambda > 0}$$

$$\boxed{J_E(u(\rho)) = \oint_{\Gamma_N} t \bullet u(\rho)\, d\Gamma_N}$$

$$\mu(\rho, u(\rho)) = \mu(\rho) \ : \ < \mu(\rho), \varphi > = \frac{dJ}{d\rho}(\rho)[\varphi] \quad \forall \varphi \text{ test function}$$

$$\mu(\rho) = \mu_{CH}(\rho) + \mu_E(\rho, u(\rho)) = \frac{dF}{d\rho}(\rho) - \lambda \Delta\rho - \gamma P \rho^{P-1} \tilde{\sigma}(u(\rho)) : \varepsilon(u(\rho))$$

$$\sigma(\rho, u) = \rho^P \tilde{\sigma}(u)$$

$$V = \int_\Omega \rho_0\, d\Omega \equiv \int_\Omega \rho(t)\, d\Omega \quad \forall t \in (0, T)$$

$$\mathrm{M}(\rho) = D\rho(1-\rho) \quad \text{mobility}$$

Topology Opt. with Phase Field

Generalized Cahn-Hilliard equations

$$
\left\{
\begin{array}{ll}
\dfrac{\partial \rho}{\partial t} = \nabla \bullet \left(M(\rho) \nabla \mu(\rho, u) \right) & \text{in } \Omega \times (0,T) \\
\nabla \rho \bullet n = 0 & \text{on } \partial\Omega \times (0,T) \\
M(\rho) \nabla \mu(\rho, u) \bullet n = 0 & \text{on } \partial\Omega \times (0,T) \\
\rho = \rho_0 & \text{in } \Omega \times \{t = 0\}
\end{array}
\right.
$$

+

$$
\left\{
\begin{array}{ll}
-div\, \sigma(\rho, u) = f & \text{in } \Omega \times (0,T) \\
u = 0 & \text{on } \Gamma_D \times (0,T) \\
\sigma(\rho, u) \cdot n = t & \text{on } \Gamma_N \times (0,T) \\
\sigma(\rho, u) \cdot n = 0 & \text{on } \partial\Omega \setminus (\Gamma_D \cup \Gamma_N) \times (0,T)
\end{array}
\right.
$$

Topology Opt. with Phase Field

Generalized Cahn-Hilliard equations

- Mass/volume conservative ➔ topology optimization is volume constrained

$$\int_\Omega \rho = \int_\Omega \rho_0 = V \qquad \forall t \geq 0$$

- The cost functional $J(\rho)$ corresponds to the energy of the generalized CH eqs. and it is a Liapunov functional

$$\frac{dJ}{dt}(\rho) = -\int_\Omega M(\rho)\left|\nabla \mu(\rho)\right|^2 \leq 0 \qquad \forall t \geq 0$$

- The steady state of the generalized CH eqs. corresponds to the minimum of the energy and hence to the minimum of the cost functional

Topology Opt. with Phase Field

Generalized Cahn-Hilliard equations

Advantages:

- optimal topology is obtained as steady state of CH eqs.
- no optimization procedure
- provide geometrical information
- no use of “filtering” techniques

Drawbacks:

- set of 4th order nonlinear parabolic PDEs
- ability to capture sharp interfaces → resolution
- computational expensive

Topology Opt. with Phase Field

[Wang, Zhou, 2006-2007]: topology optimization with CH eq.
Low order FE approximation, multigrid method, fixed time step

⇨ **Isogeometric Analysis**

- $C^q(\Omega), q \geq 1,$ continuous basis
- adaptive time stepping method + implicit solver, α-method
- accurate and stable results capturing thin layers
- NO geometrical approximation of CAD geometries
- perform topology opt. in regions and components of existing structures

Topology Opt. with Phase Field

The choice of the parameters λ and γ

$$\boxed{J(\rho) = J_{CH}(\rho) + \gamma J_E(u(\rho))}$$

$$\boxed{J_{CH}(\rho) = \int_\Omega F(\rho)\, d\Omega + \frac{1}{2}\lambda \int_\Omega \nabla\rho \bullet \nabla\rho\, d\Omega \qquad \lambda > 0}$$

$$\boxed{J_E(u(\rho)) = \oint_{\Gamma_N} t \bullet u(\rho)\, d\Gamma_N}$$

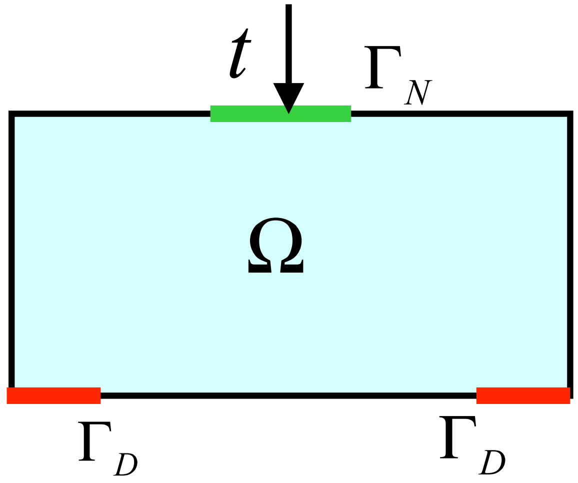
$$\lambda = \tilde{\lambda} h^2, \qquad \gamma = \tilde{\gamma}\gamma_E$$

- To balance the compliance and the CH parts of the energy, we choose:

$$\gamma_E = J_{CH}(\rho_0) / J_E(\rho_0)$$

$\tilde{\lambda}, \tilde{\gamma} =$ dimensionless, chosen by user (depend on each other, load case, volume fraction, penalization P, …)

Topology Opt. with Phase Field



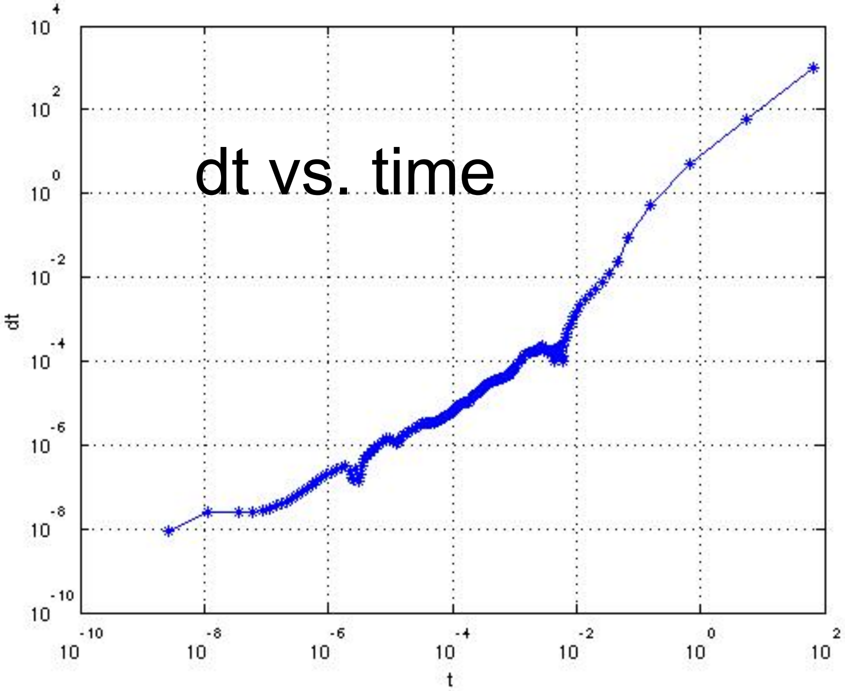
$\Omega = (0, 2.0m) \times (0, 1.0\,m), \quad V = 0.50\,|\Omega|, \quad \rho_0 = 0.5$

plane stress, $\quad E_0 = 200GPa, \quad \nu = 0.3, \quad |t| = 200\,MPa$

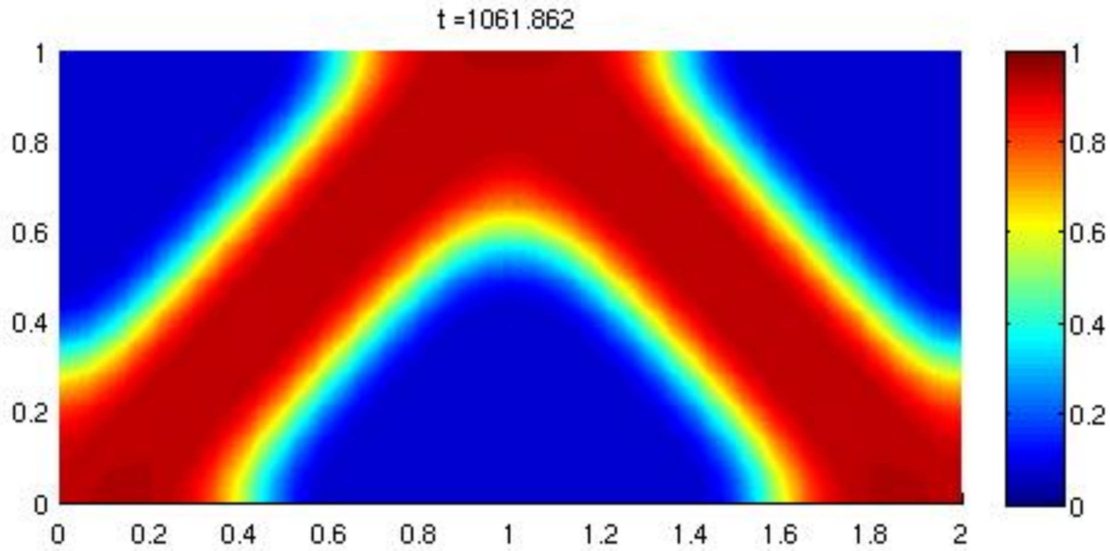
$P = 5, \quad$ Gauss points: $\;5 \times 5$

Order p=q=2, # DOF = 840

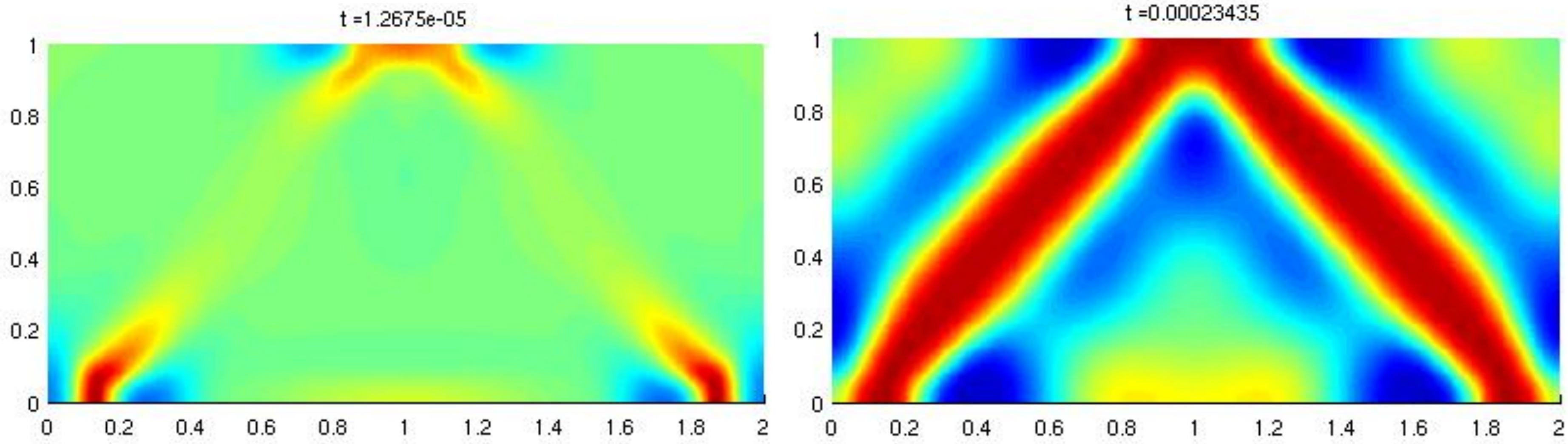
$\boxed{\tilde{\lambda} = 2.5 \qquad \tilde{\gamma} = 5.0}$



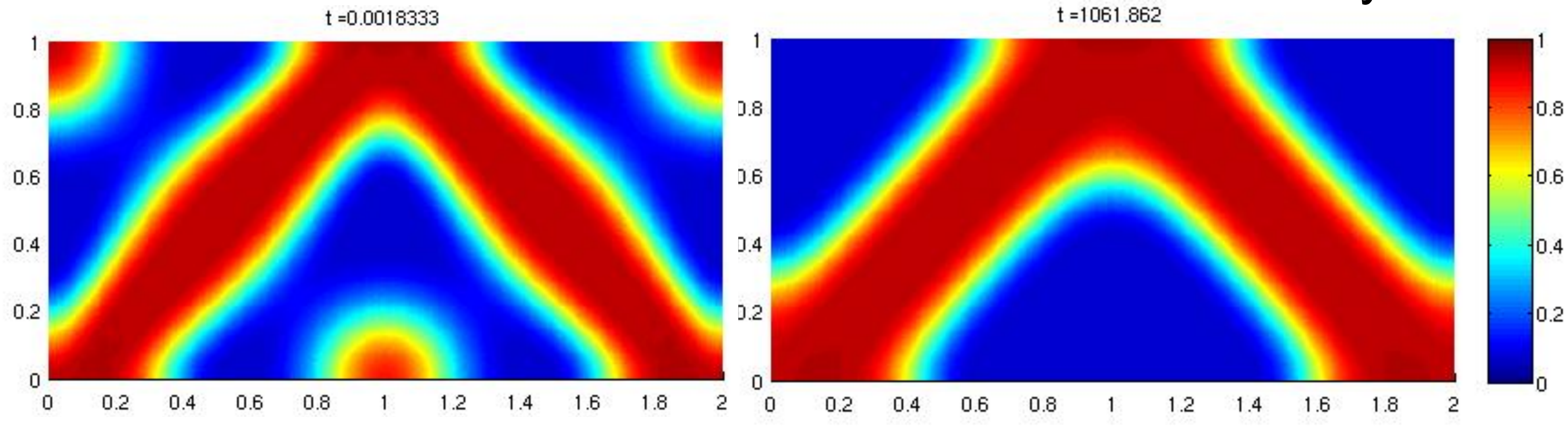
Steady state



Topology Opt. with Phase Field

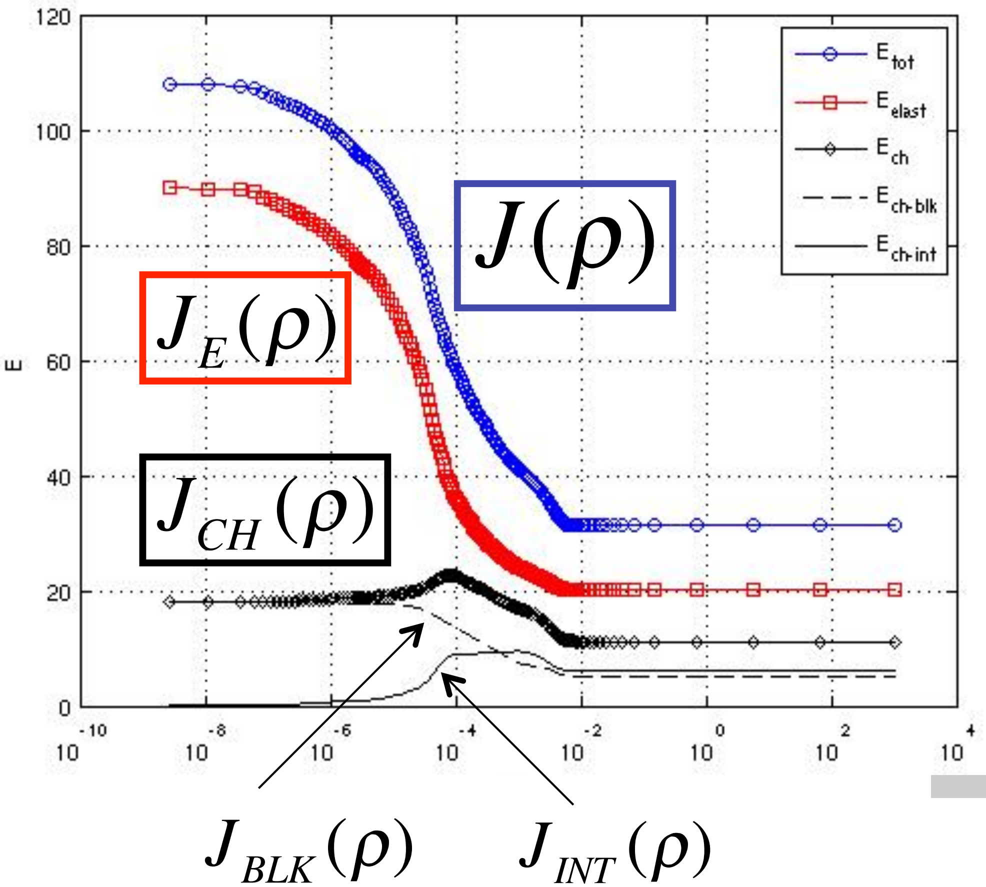


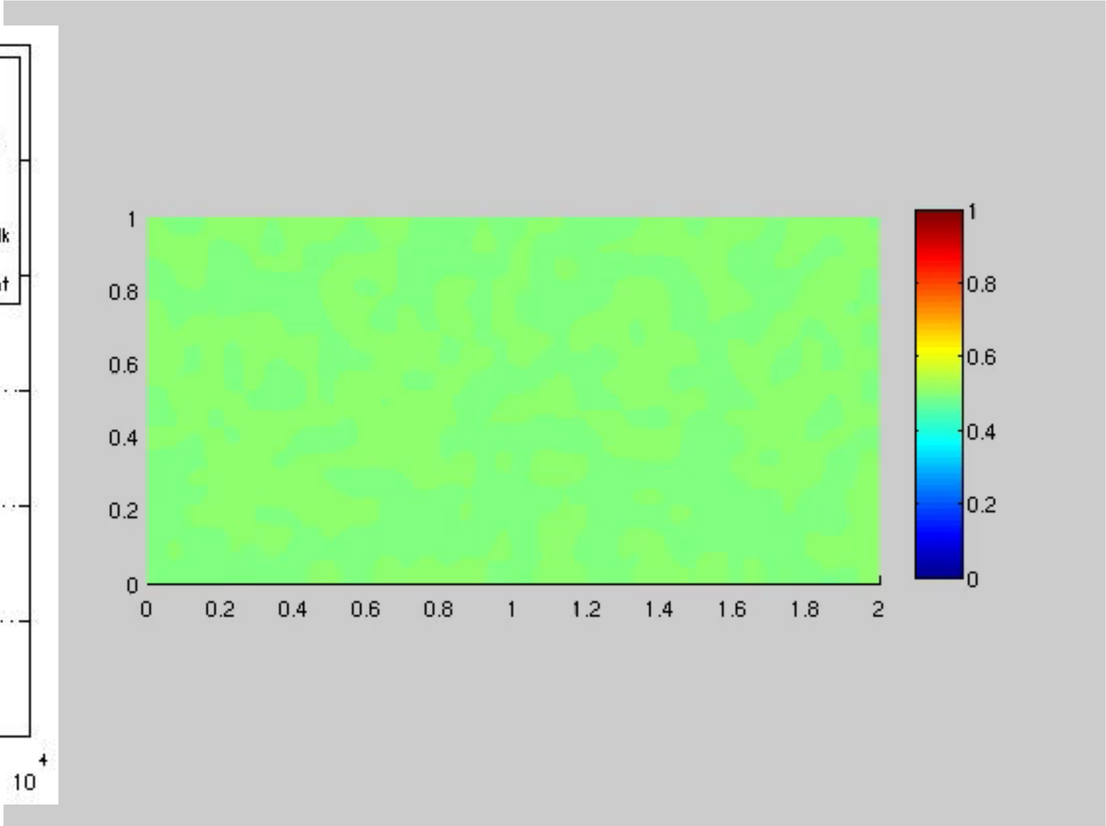
Steady state



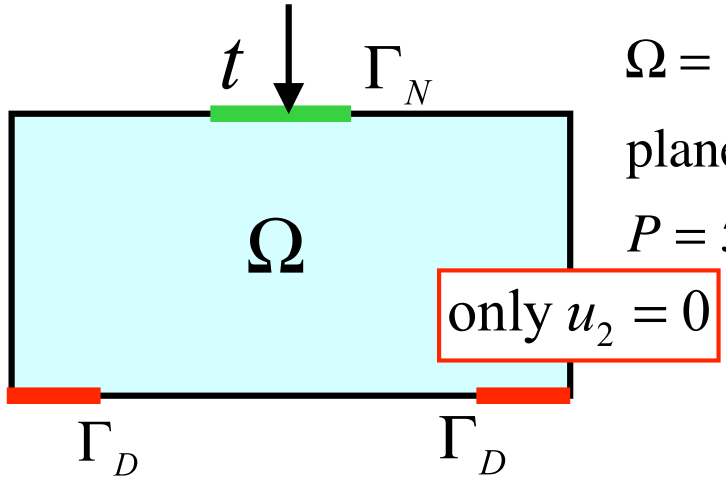
Topology Opt. with Phase Field

Energy vs. time
(dimensionless)





Topology Opt. with Phase Field



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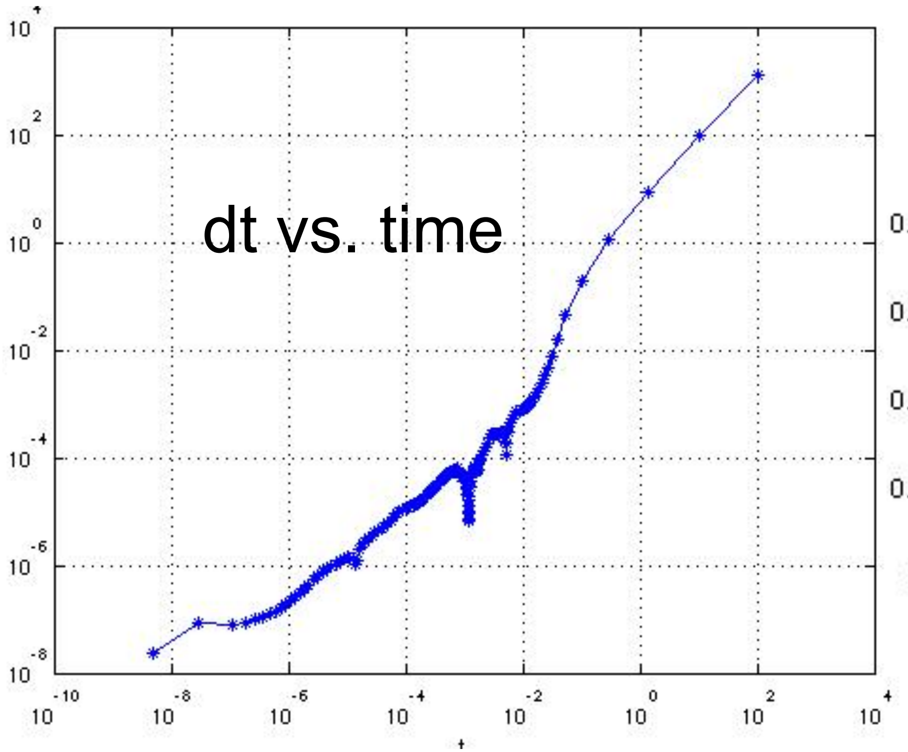
plane stress, $E_0 = 200GPa, \quad \nu = 0.3, \quad |t| = 200\,MPa$

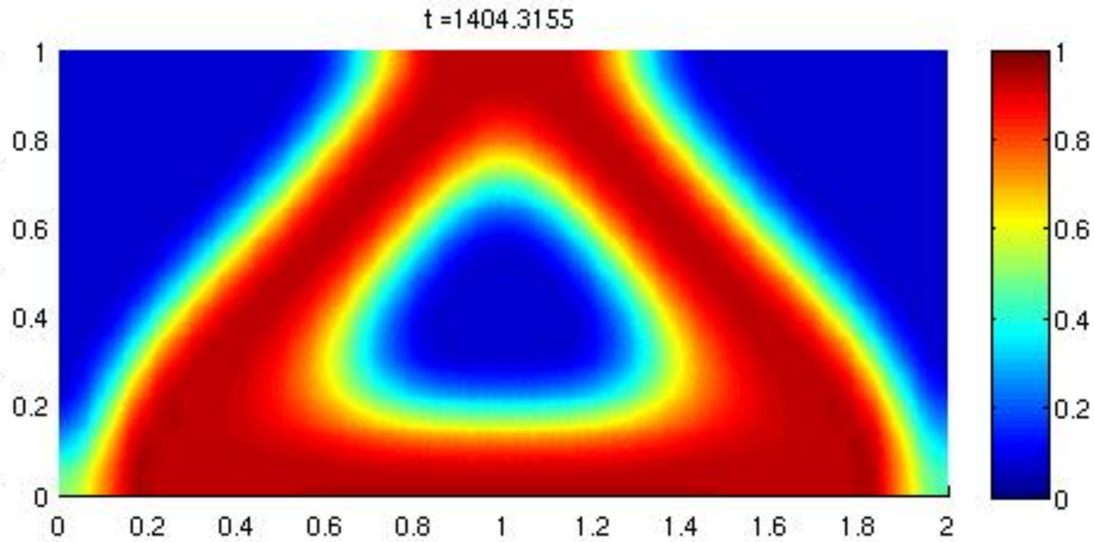
$P = 5, \quad$ Gauss points: 5×5

Order p=q=2, # DOF = 680

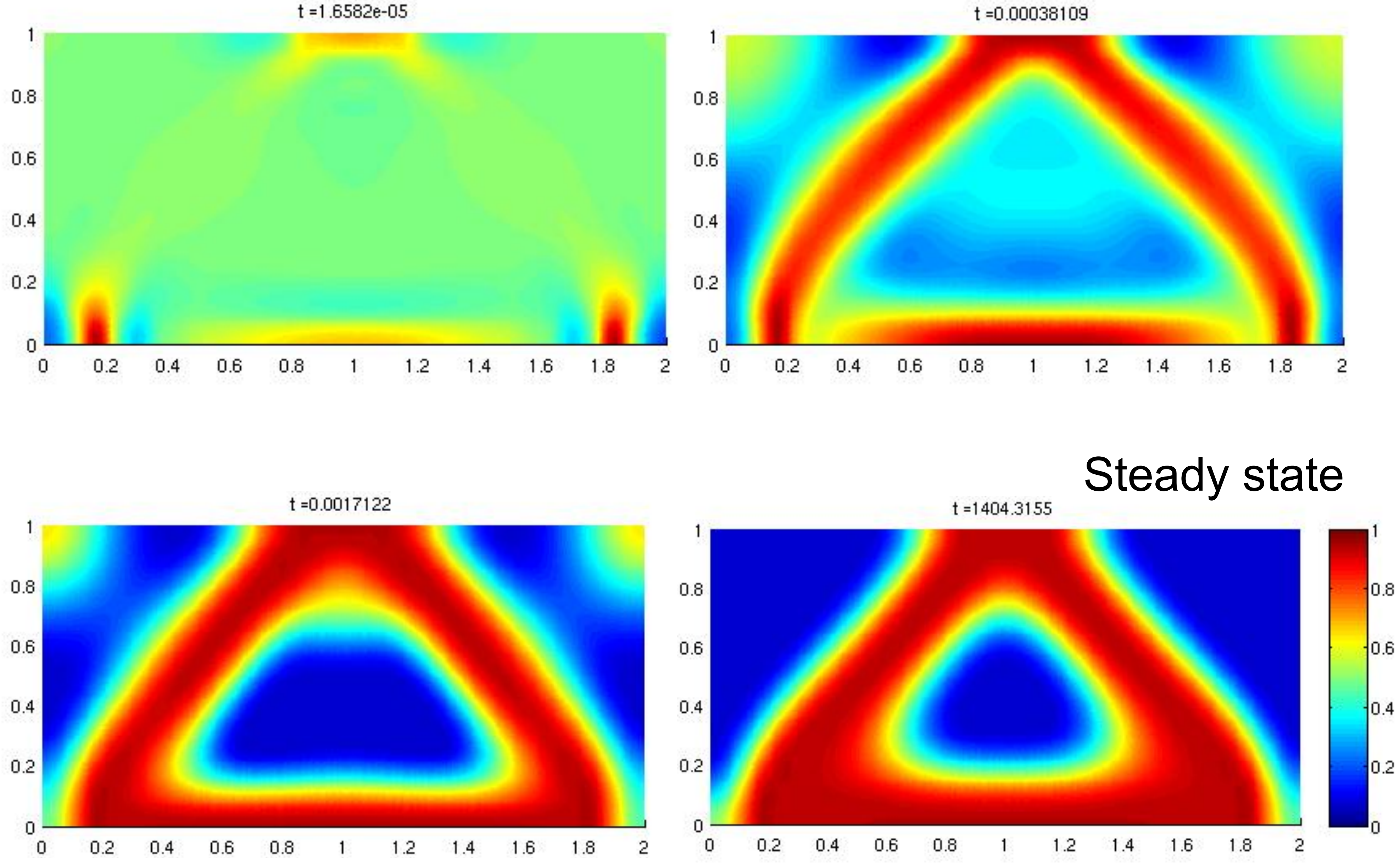
$\boxed{\tilde{\lambda} = 2.5 \qquad \tilde{\gamma} = 5.0}$



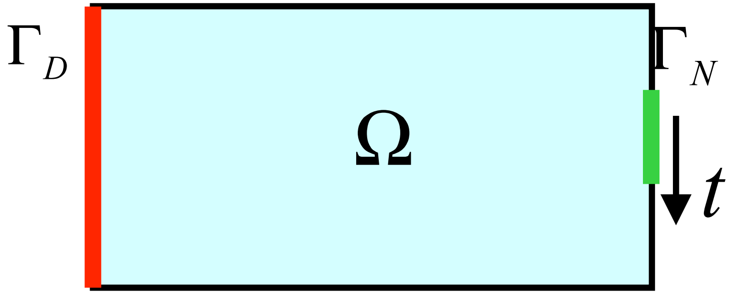
Steady state



Topology Opt. with Phase Field



Topology Opt. with Phase Field



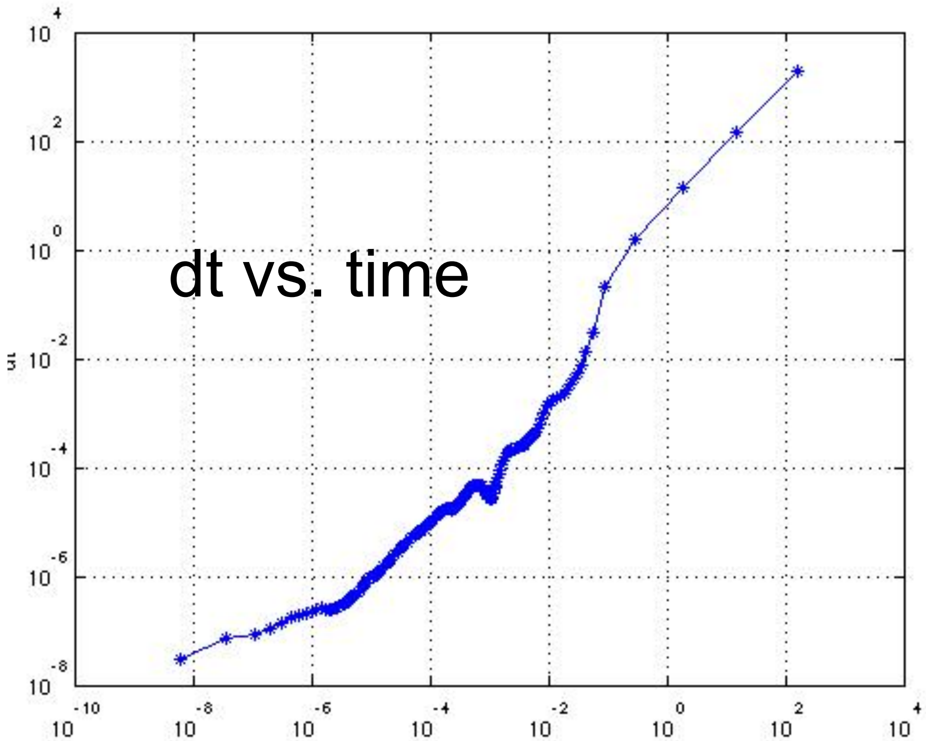
$\Omega = (0, 2.0m) \times (0, 1.0\,m), \quad V = 0.50\,|\Omega|, \quad \rho_0 = 0.5$

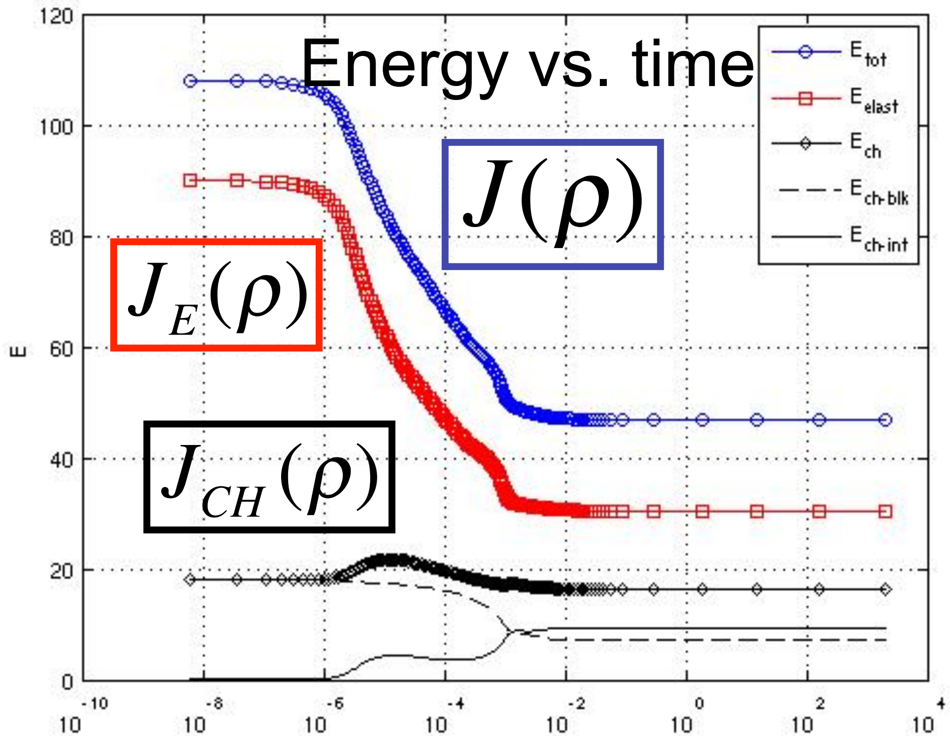
plane stress, $E_0 = 200GPa, \quad \nu = 0.3, \quad |t| = 200\,MPa$

$P = 5,$ Gauss points: 5×5

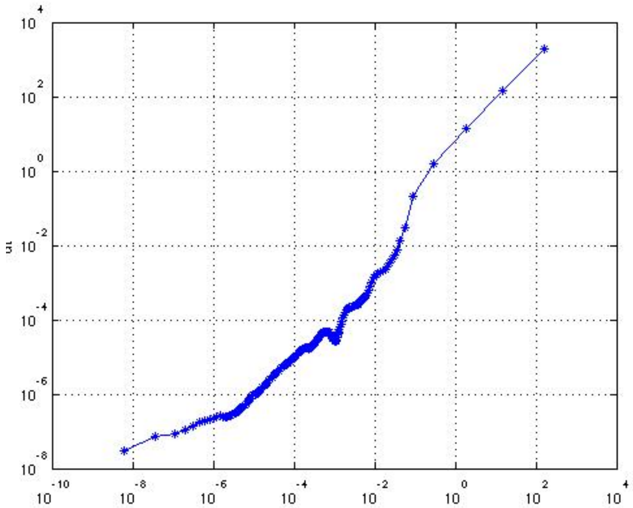
Order p=q=2, # DOF = 840

$\tilde{\lambda} = 2.5 \qquad \tilde{\gamma} = 5.0$

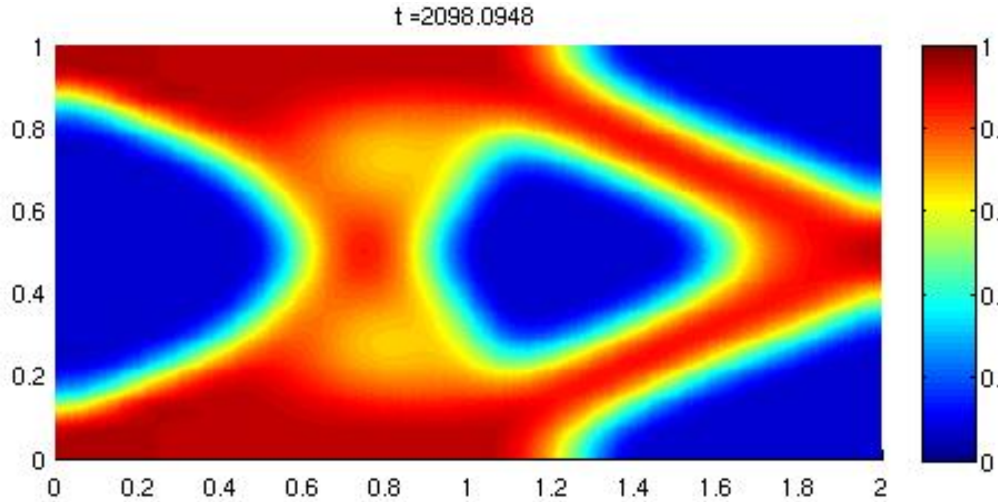




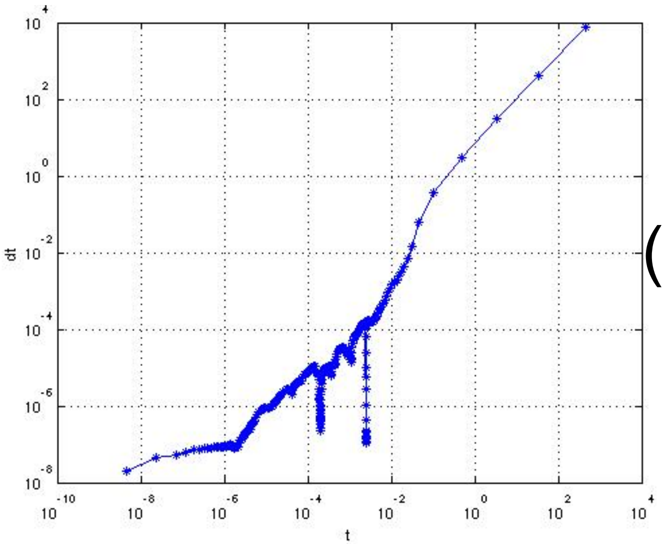
Topology Opt. with Phase Field



$\tilde{\lambda} = 2.5$

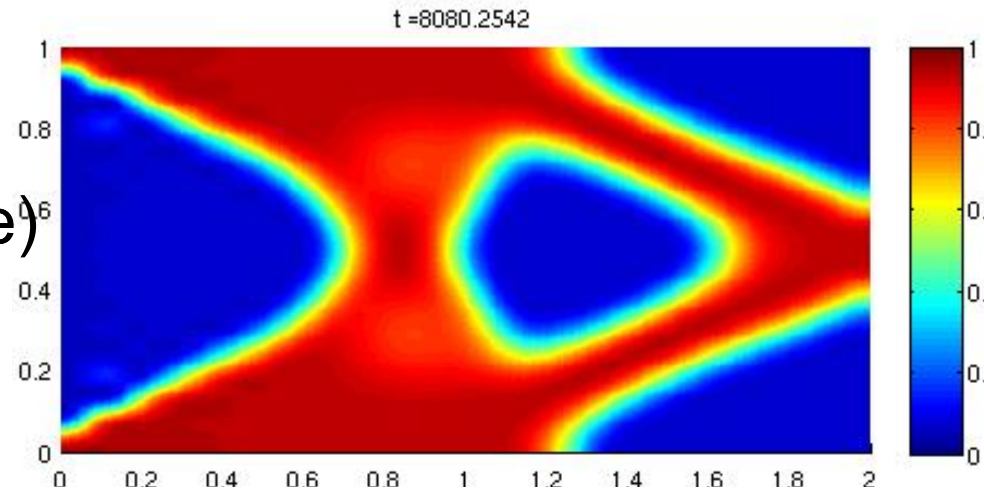


dt vs. time

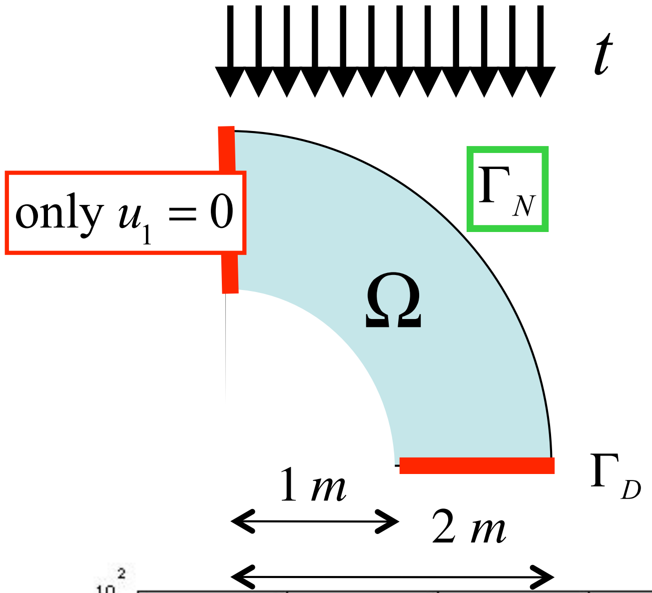


$\tilde{\lambda} = 1.375$

(limit value)



Topology Opt. with Phase Field



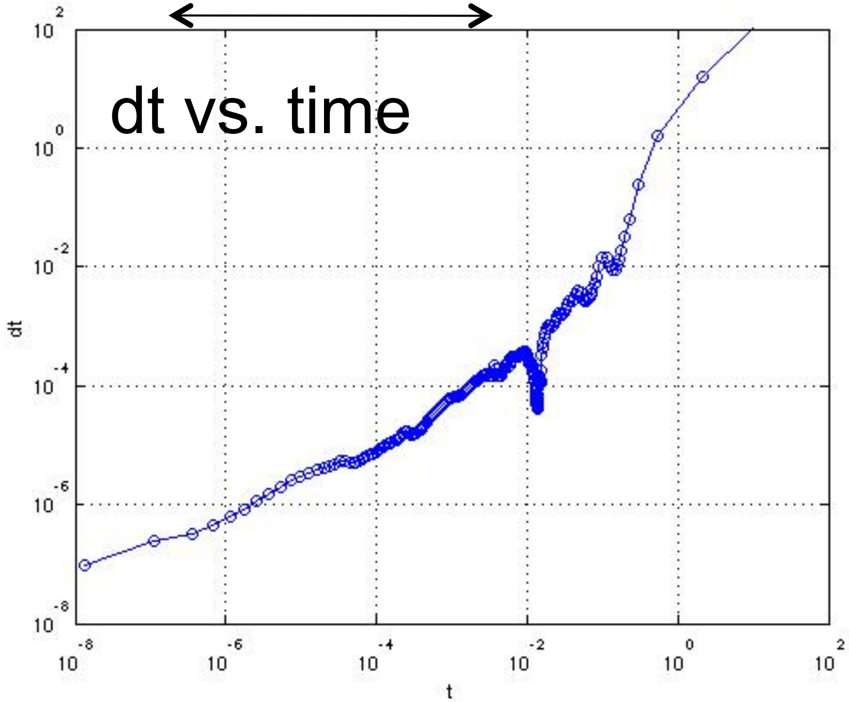
$\Omega = (0, 2.0m) \times (0, 1.0\,m), \quad V = 0.50\,|\Omega|, \quad \rho_0 = 0.5$

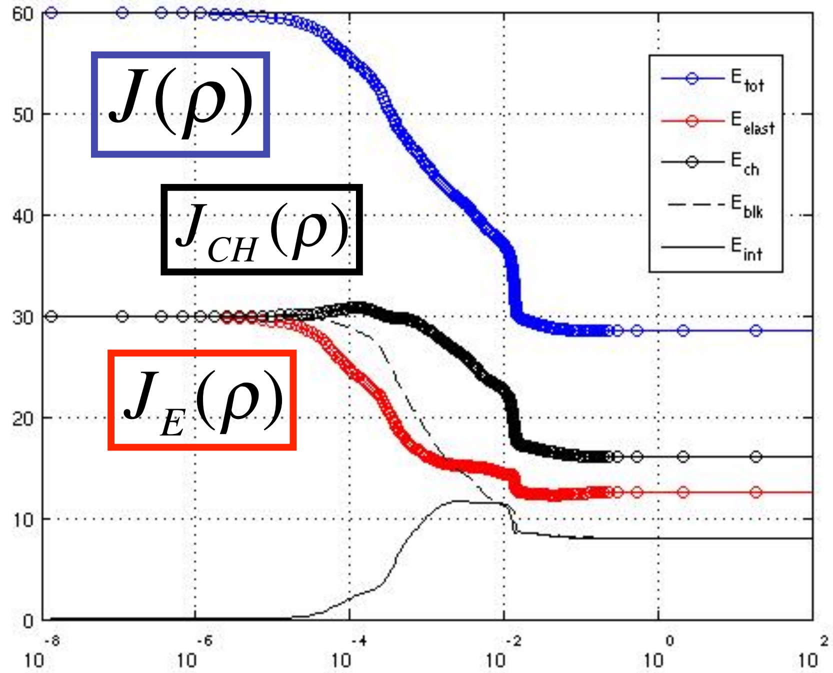
plane stress, $\quad E_0 = 200GPa, \quad \nu = 0.3, \quad |t| = 200\,MPa$

$P = 5, \quad$ Gauss points: $\; 5 \times 5$

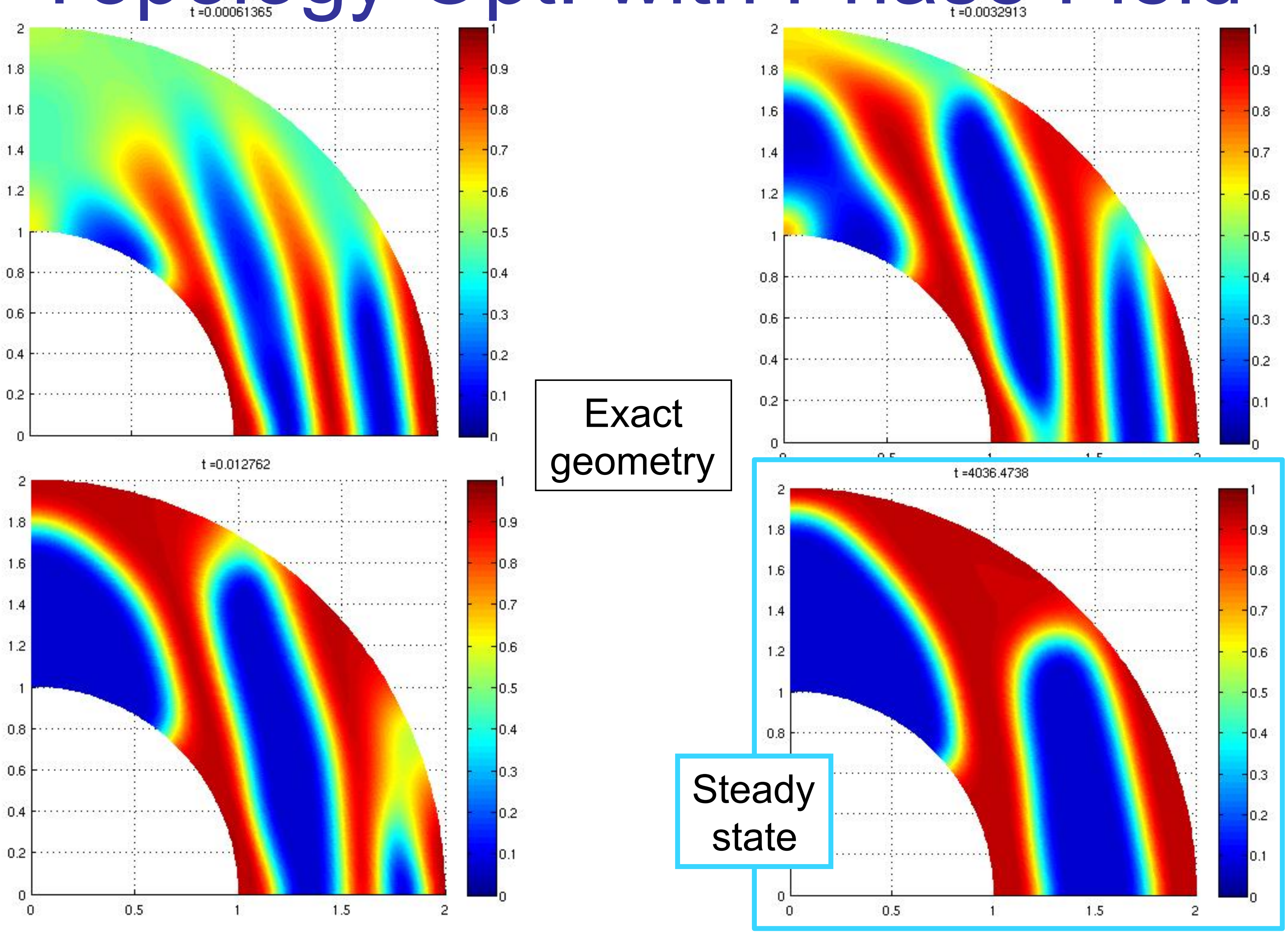
Order p=q=2, # DOF = 840

$\tilde{\lambda} = 1.5 \qquad \tilde{\gamma} = 1.0$





Topology Opt. with Phase Field



Shape Optimization

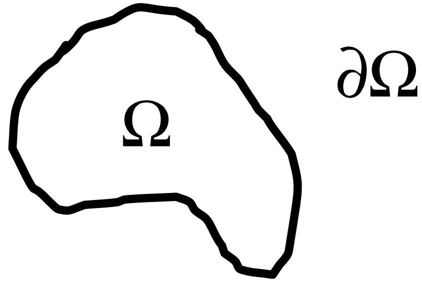
- The NURBS map turns the infinite dimensional problem:

$$\min_{\partial\Omega} J(\partial\Omega)$$

$$\text{s.t. } g_i(\partial\Omega) \geq 0 \quad i = 1, 2, \ldots, n_{ineq}$$

$$h_j(\partial\Omega) = 0 \quad j = 1, 2, \ldots, n_{eq}$$

$$\text{state eq. with b.c.'s}$$



into a finite dimensional one:

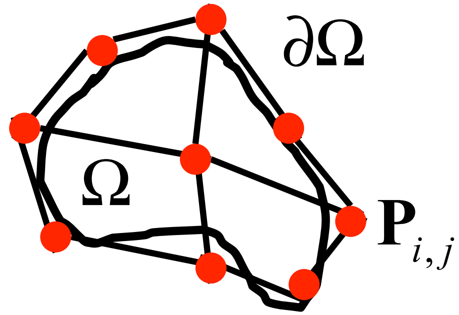
$$\min_{\mathbf{P}_{i,j}} J(\partial\Omega(\mathbf{P}_{i,j}))$$

$$\text{s.t. } g_i(\partial\Omega(\mathbf{P}_{i,j})) \geq 0 \quad i = 1, 2, \ldots, n_{ineq}$$

$$h_j(\partial\Omega(\mathbf{P}_{i,j})) = 0 \quad j = 1, 2, \ldots, n_{eq}$$

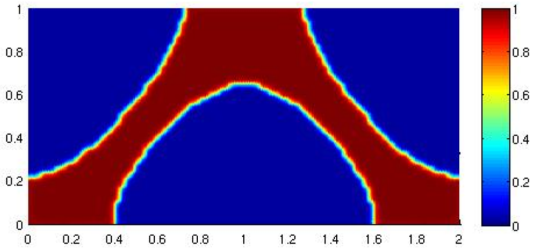
$$\text{state eq. with b.c.'s}$$

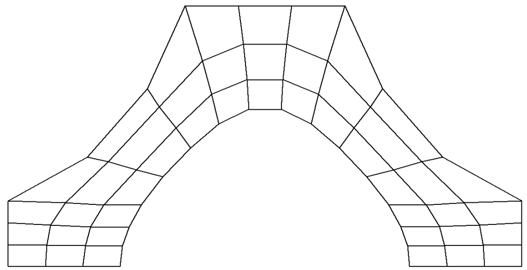
$$\boxed{\Omega = \Omega(\mathbf{P}_{i,j}) = \sum_{i,j} \mathbf{P}_{i,j} R_{i,j}}$$



Isogeometric Analysis $\Rightarrow$ Exact CAD geometries

Shape Optimization

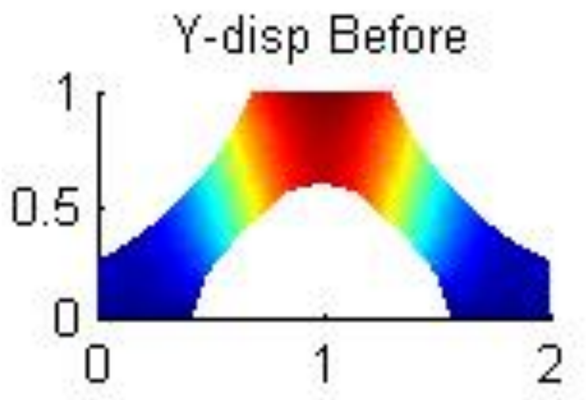




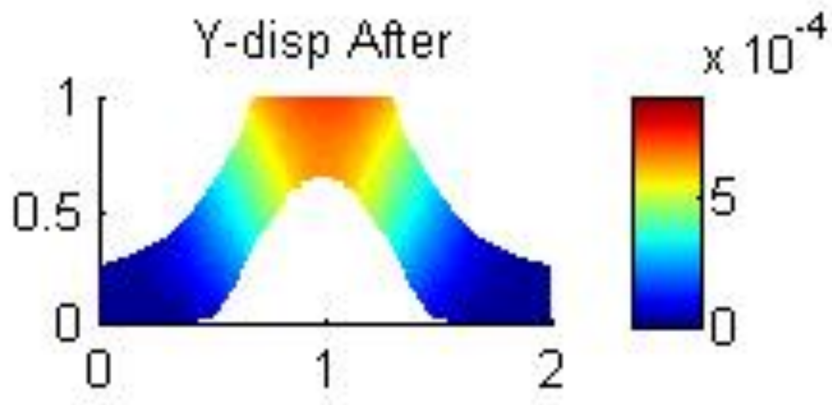
Topology optimization

NURBS geometry
(sweeping technique, J.Zhang)

<u>Shape Optimization</u>



$J = 2.6148 \cdot 10^{-4}$



$J = 2.1473 \cdot 10^{-4}$

Conclusions & Future Developments

- We developed a pipeline for geometry optimization with Isogeometric Analysis encapsulating exact CAD geometries.

- We solved topology optimization problems in a phase field approach based on the generalized Cahn-Hilliard equations

- Improve resolution for 2D problems (sharp interfaces)

- 3D problems