

DIGEP

**Department of Management
and Production Engineering**

**POLITECNICO
DI TORINO**



Crossing and Veering phenomena in a crank-mechanism

Marco Brino

Outline

Aim

- Investigate the dynamic behaviour of structures with close or even coincident eigenvalues

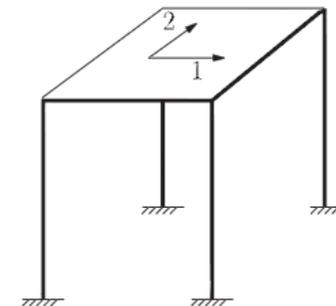
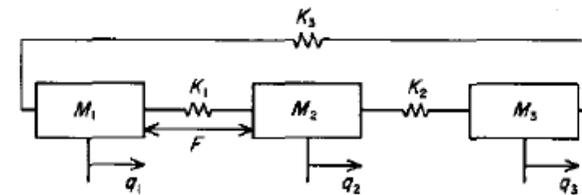
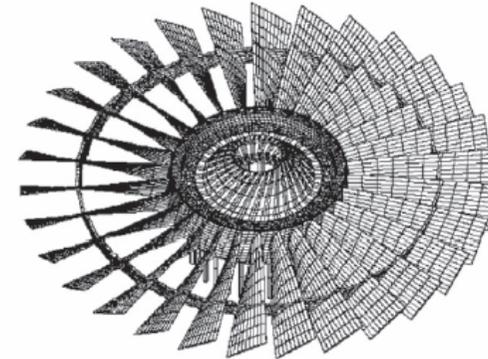
Dynamics with close eigenvalues

- Cyclic / symmetric structures
- Non-cyclic / non-symmetric structures
- Effects of boundary conditions
- From component to system dynamics

State of the art

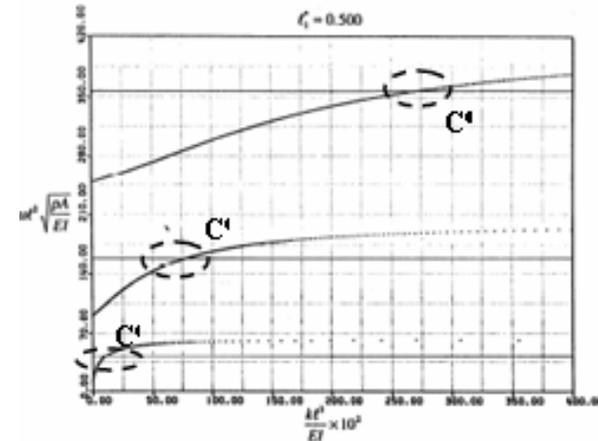
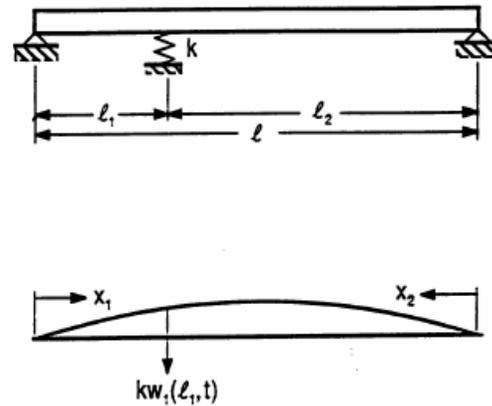
➤ Coincident or close modes structures:

- Symmetric structures
symmetric shells, bells,
bladed disk, ...
- Cyclic structures
lumped systems,
bladed disk (again), ...
- Uncoupled or slightly coupled systems
symmetric beams,
bladed disk (again), ...

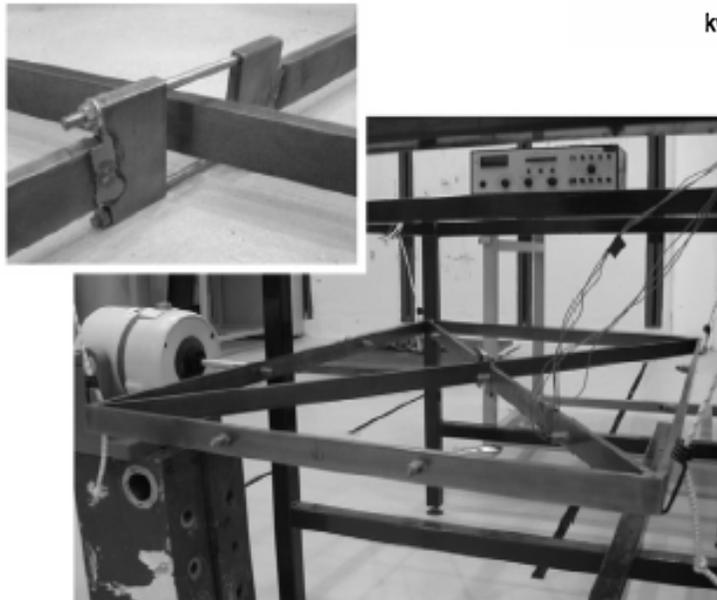


State of the art

Many references on theoretical dynamic systems with coincident or close eigenvalues



Young, Hwang, Chinese Proceed. 2007



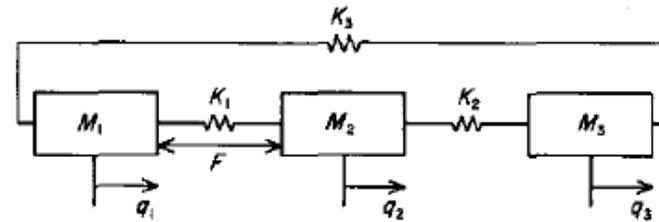
Few references on experimental test-rigs

No experimental test-rigs with NO-SYMMETRY or NO-CYCLIC, with structures “near” to wing-shape

Adhikari, duBois, Lieven, JSV 2009

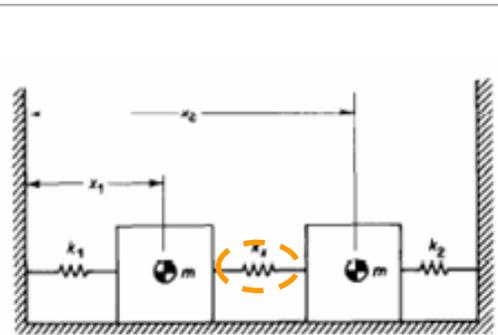
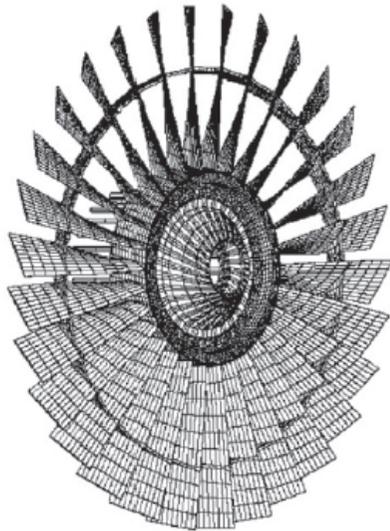
State of the art

Most of them presents SYMMETRY and/or CYCLIC characteristics, or some UNCOUPLING parameter

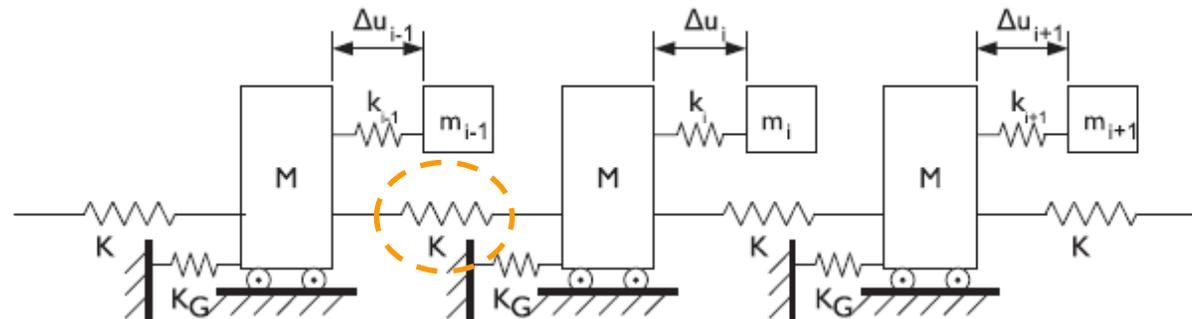
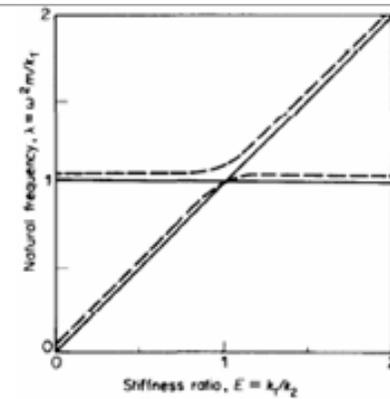


Balmes, JSV 1993

Chan, Ewins, MSSP 2010



Perkins, Mote, JSV 1986



Modal Analysis framework

Basic assumptions:

Equations of motion in matrix form: $\mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{f}$

Considering the undamped system: $\mathbf{M} \ddot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{f}$

Solving eigenproblem: $\det(\mathbf{K} - \omega^2 \mathbf{M}) = 0$

Solution: ω_r^2 eigenvalues
 $\Phi^{(r)}$ eigenvectors

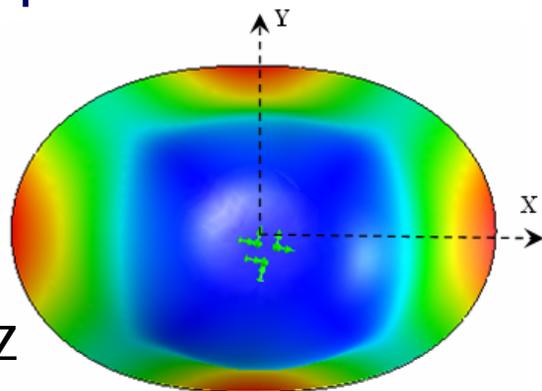
Coincident eigenvalues

Structures with symmetry:

➤ Example: BELL

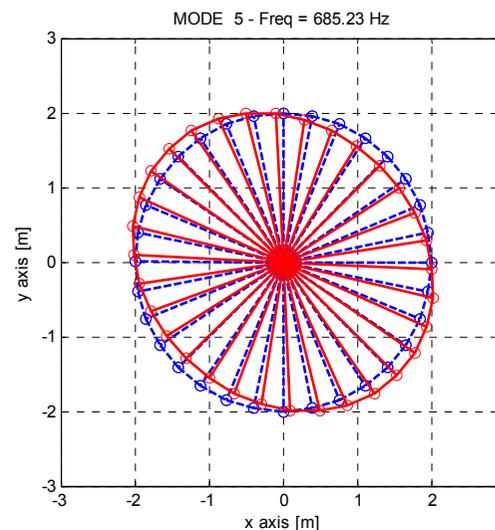
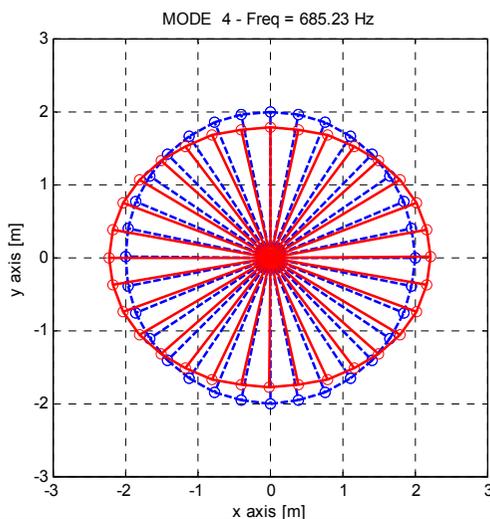
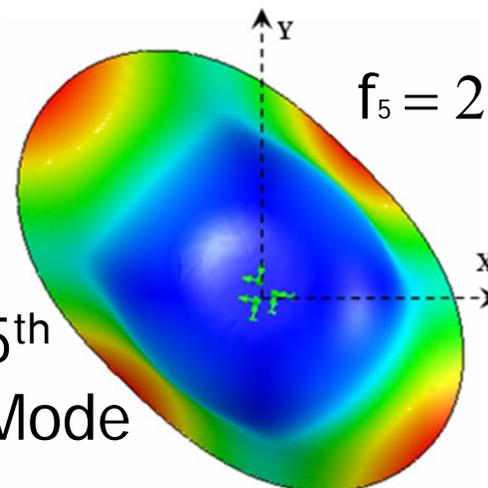
4th
Mode

$f_4 = 2860$ Hz



5th
Mode

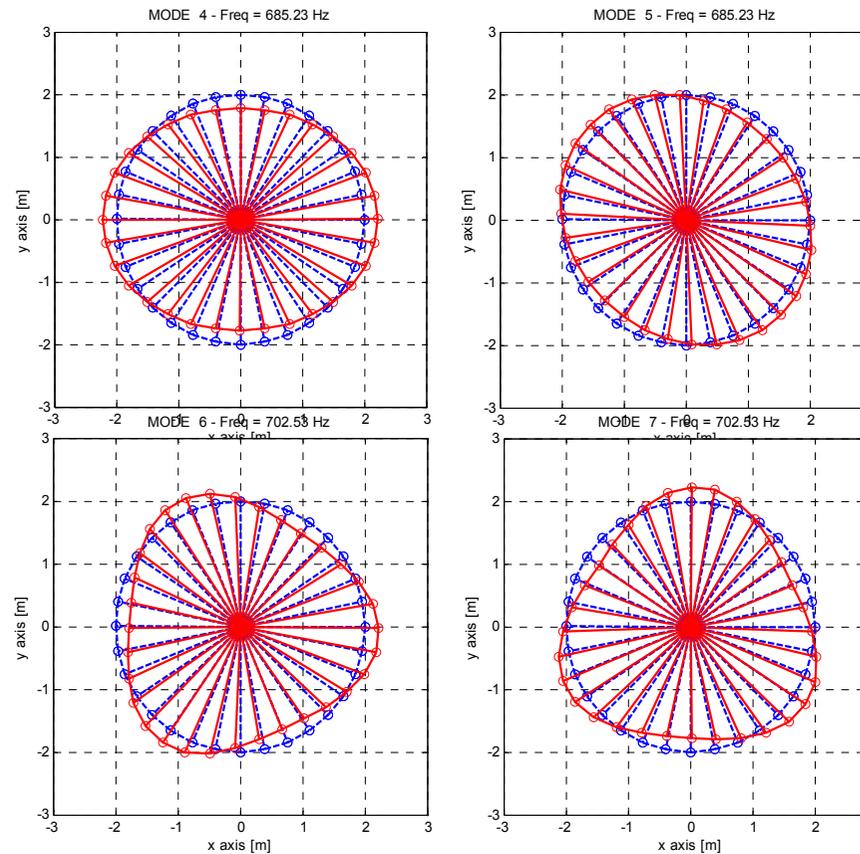
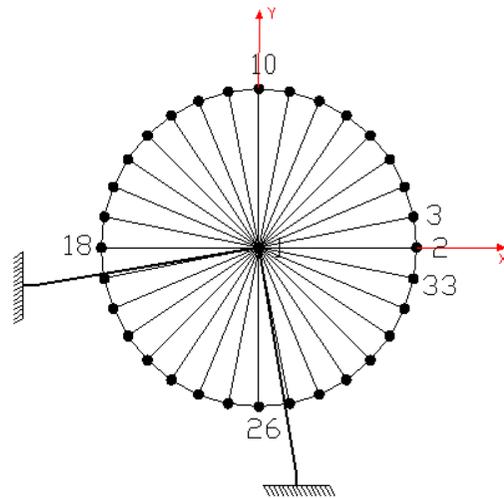
$f_5 = 2860$ Hz



Coincident eigenvalues

Structural modifications:

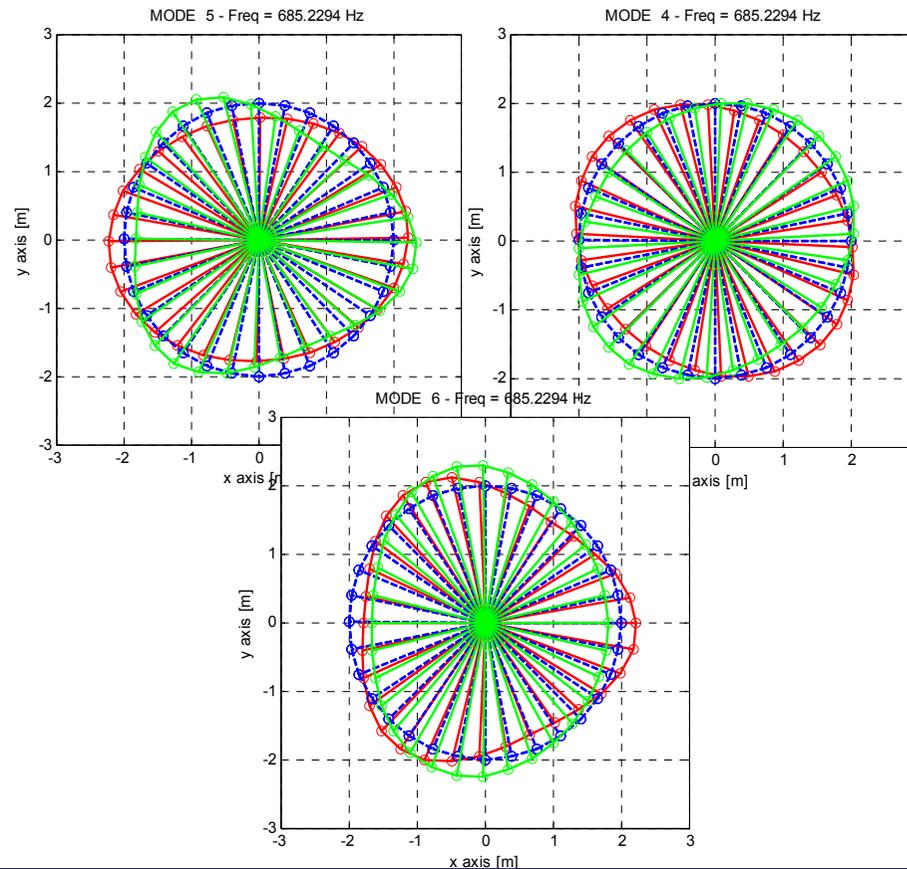
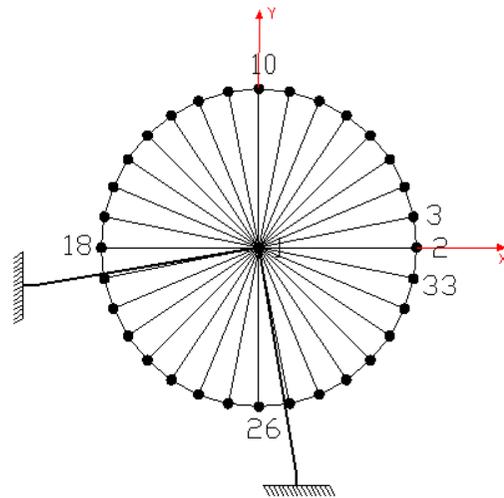
- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes



Coincident eigenvalues

Structural modifications:

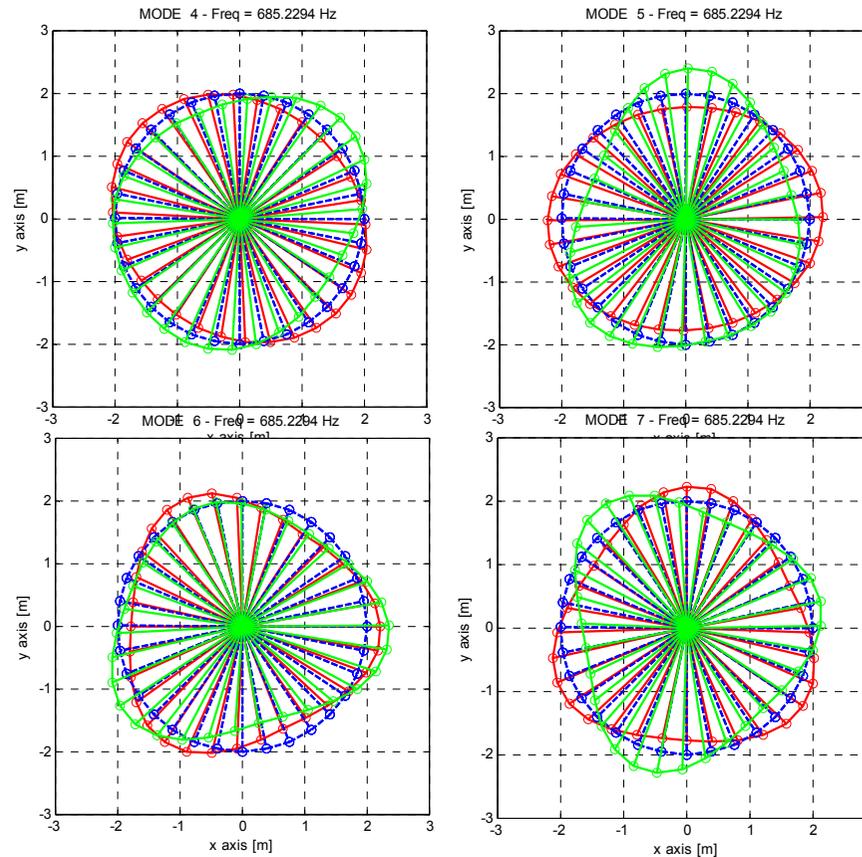
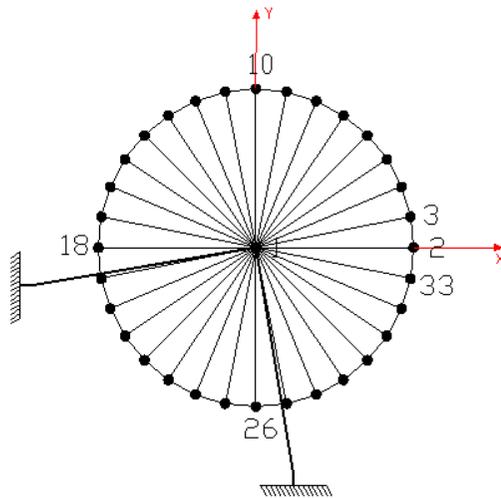
- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes



Coincident eigenvalues

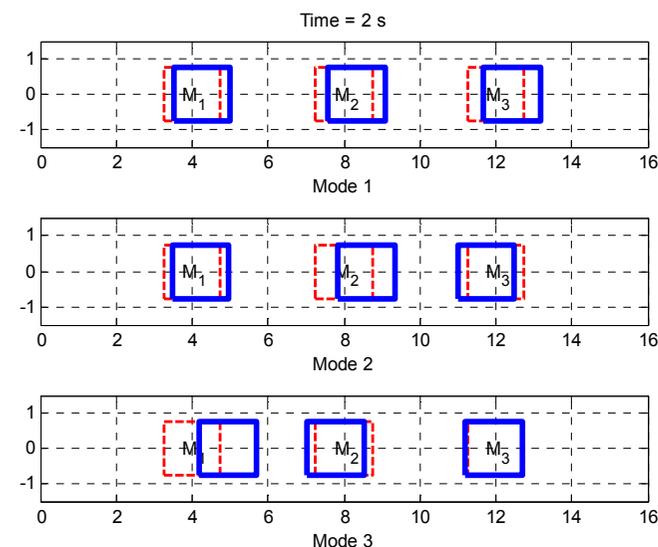
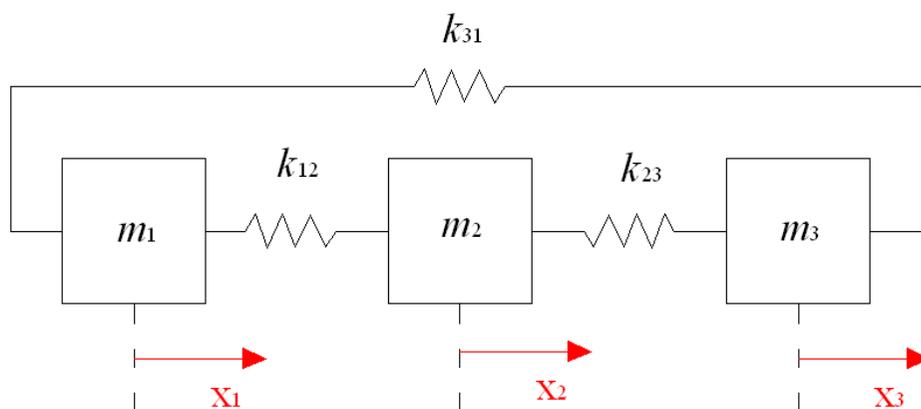
Structural modifications:

- Modify lumped elastic parameters on symmetric structures to enforce coincident mode shapes



Crossing and veering phenomena

Cyclic structures



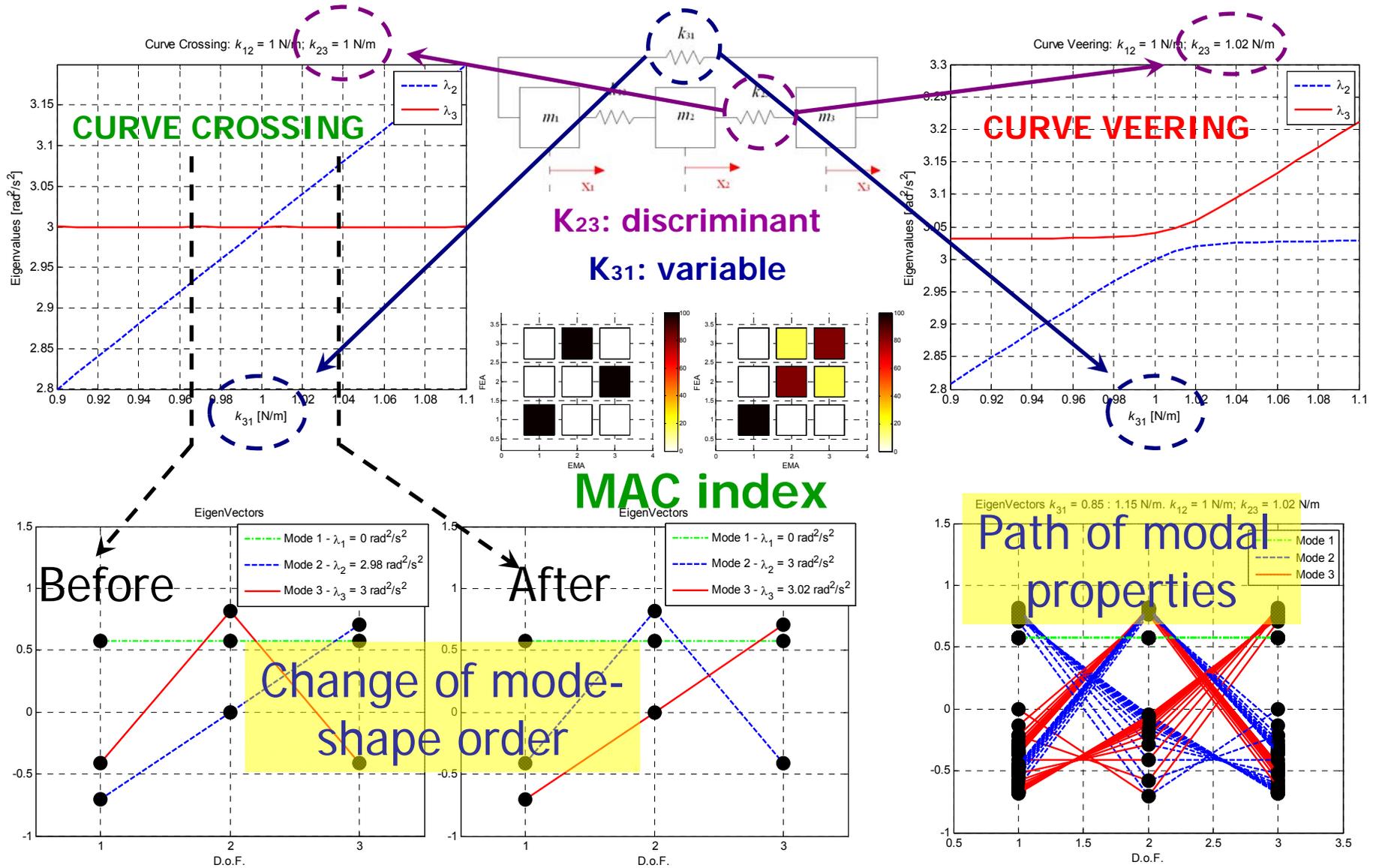
Deterministic analysis of crossing and veering phenomena:

- Crossing $k_{12} = k_{23}$; $k_{31} = 90\% \div 110\% k_{12}$
- Veering $k_{23} = 102\% k_{12}$; $k_{31} = 90\% \div 110\% k_{12}$

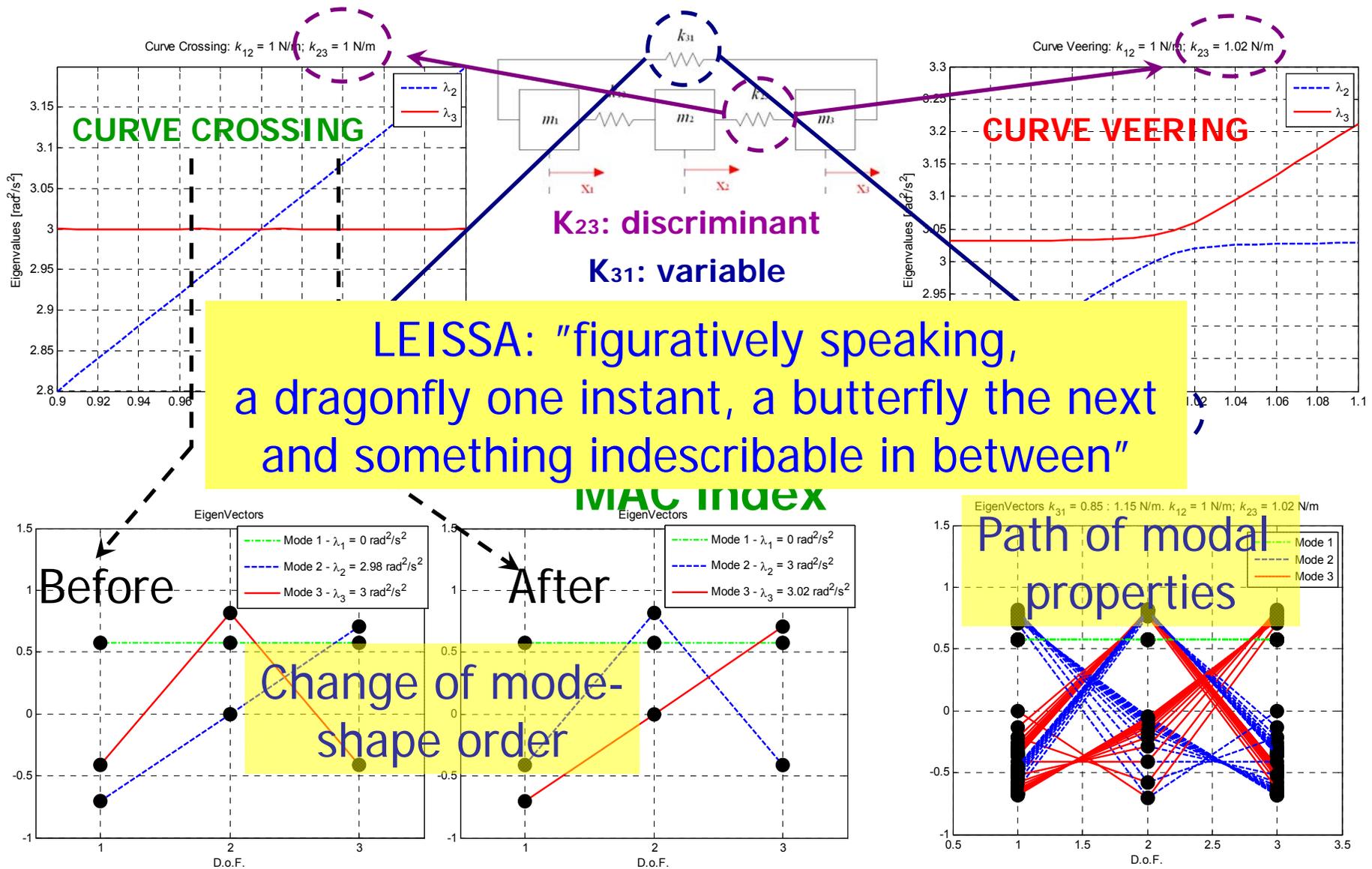
Sensitivity to uncertainty and variability

- Choice of index to discriminate crossing and veering \rightarrow MAC
- Orthogonality properties
- Sensitivity to eigenvalues AND eigenvectors changes

Crossing and veering phenomena



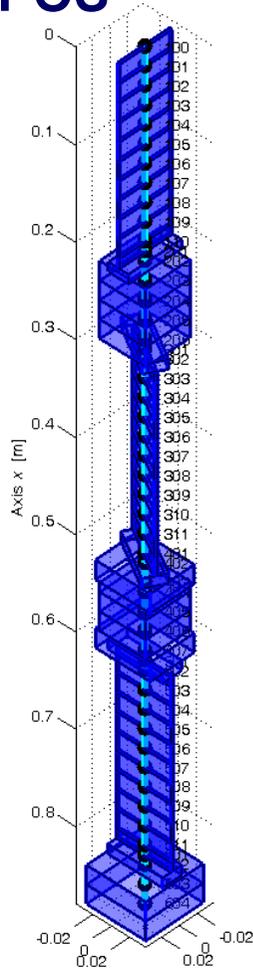
Crossing and veering phenomena



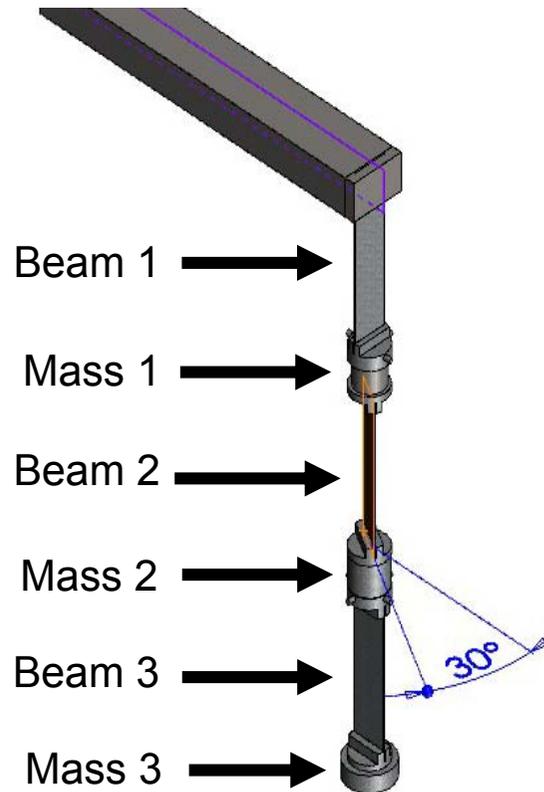
Crossing and veering: “wing” structure

Non-cyclic structure

➤ LUPOS



➤ CAD model



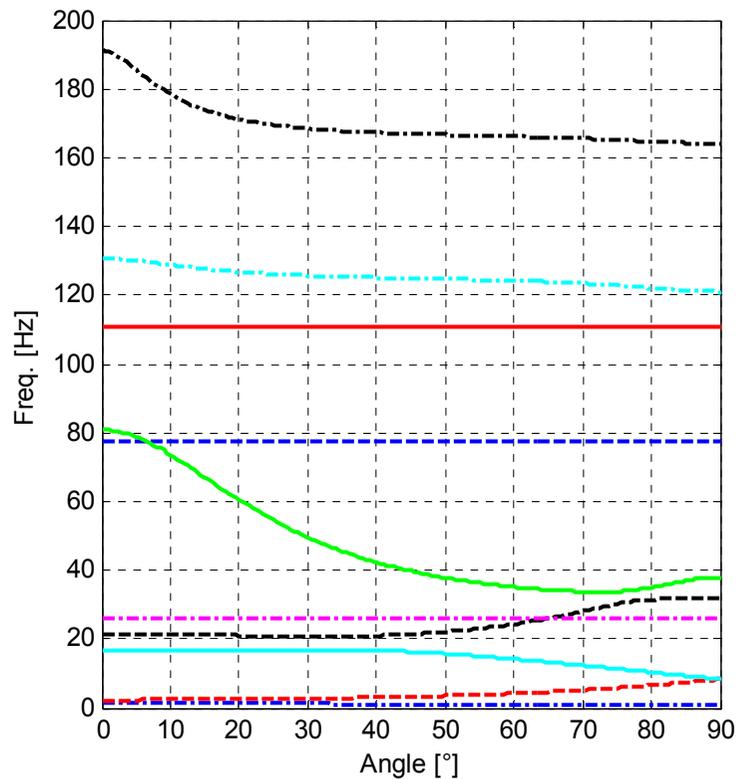
➤ Experimental Test-rig



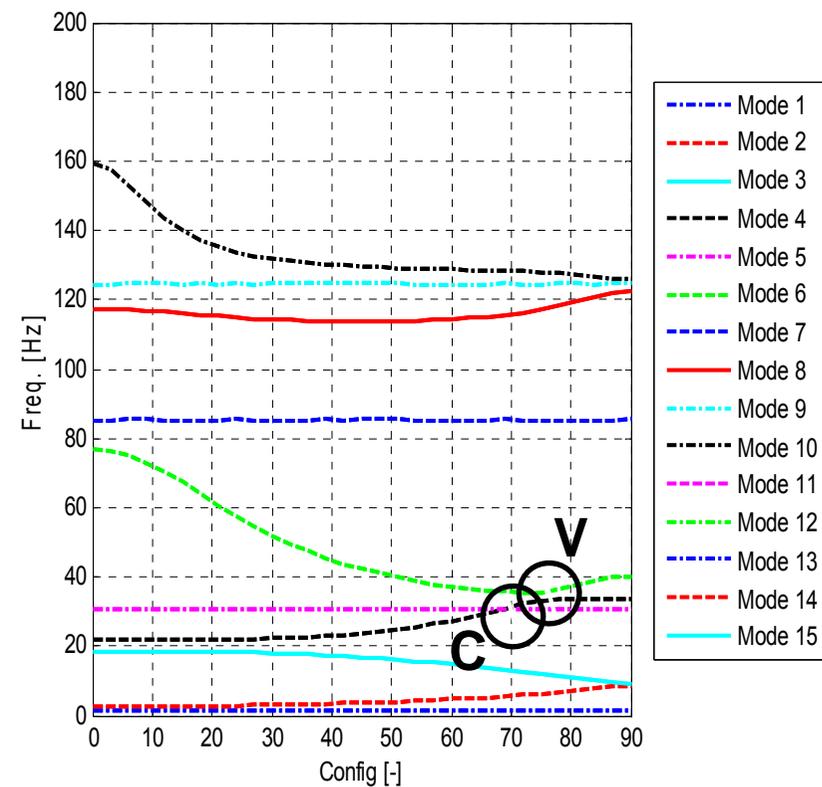
Crossing and veering: comparison LUPOS vs. 3D FEM

Parametric FEM: configuration parameter looped through the configurations to run families of FEA solutions

➤ LUPOS model



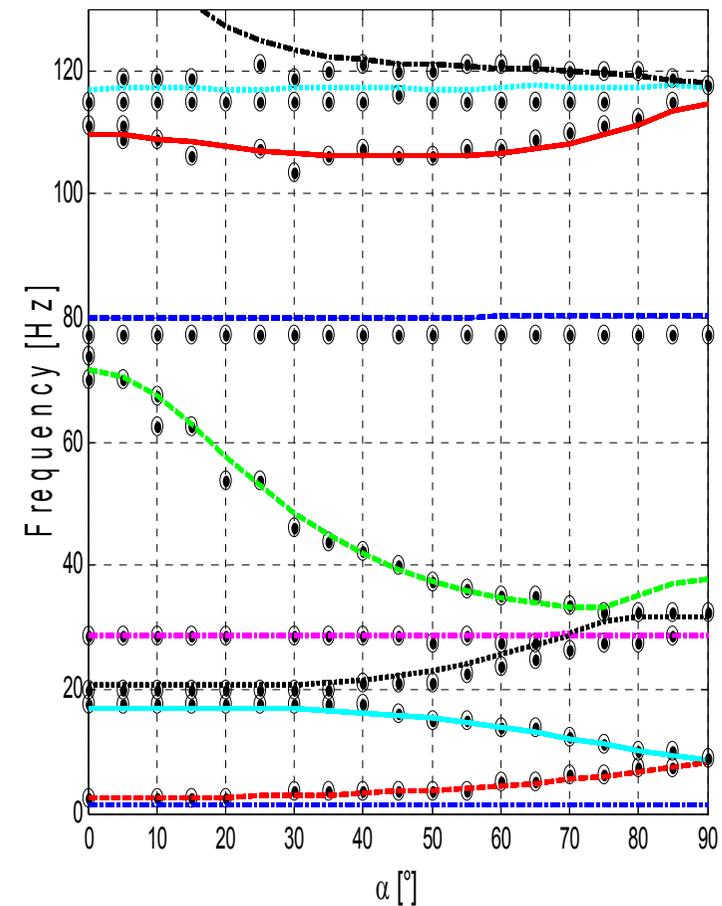
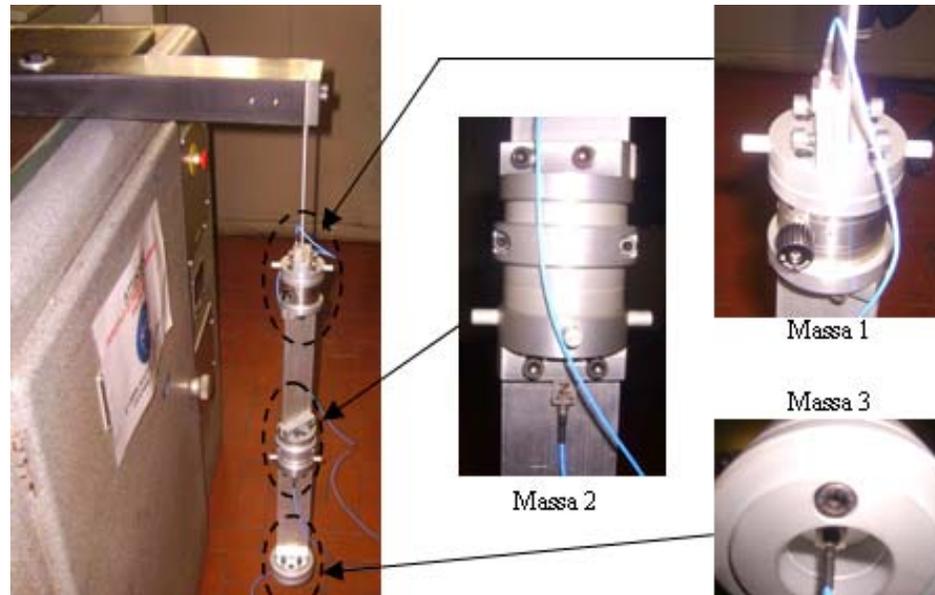
➤ Solidworks CAE model



Crossing and veering: FEM model updating

Experimental Modal Analysis

- 19 configurations (5° steps)

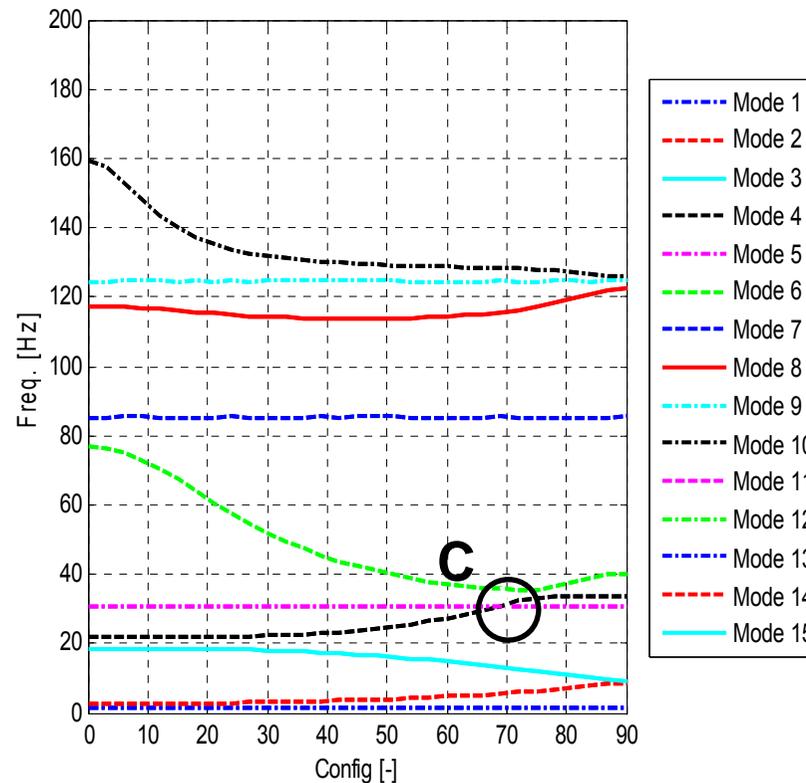


Crossing

Modal Assurance Criterion

$$MAC_{i,j} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \cos^2 \alpha_{i,j}$$

➤ Crossing: modes 4/5 @ 68/69°

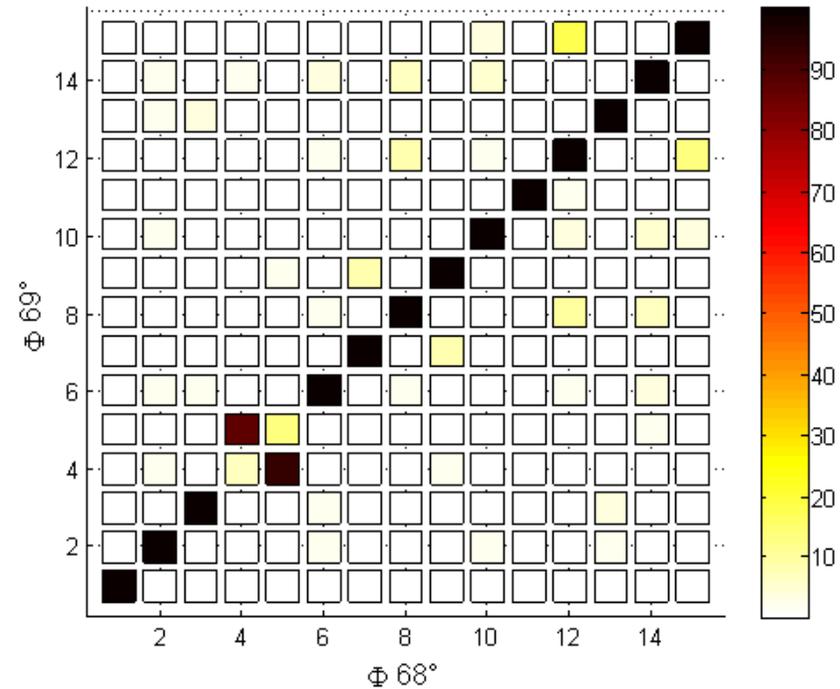
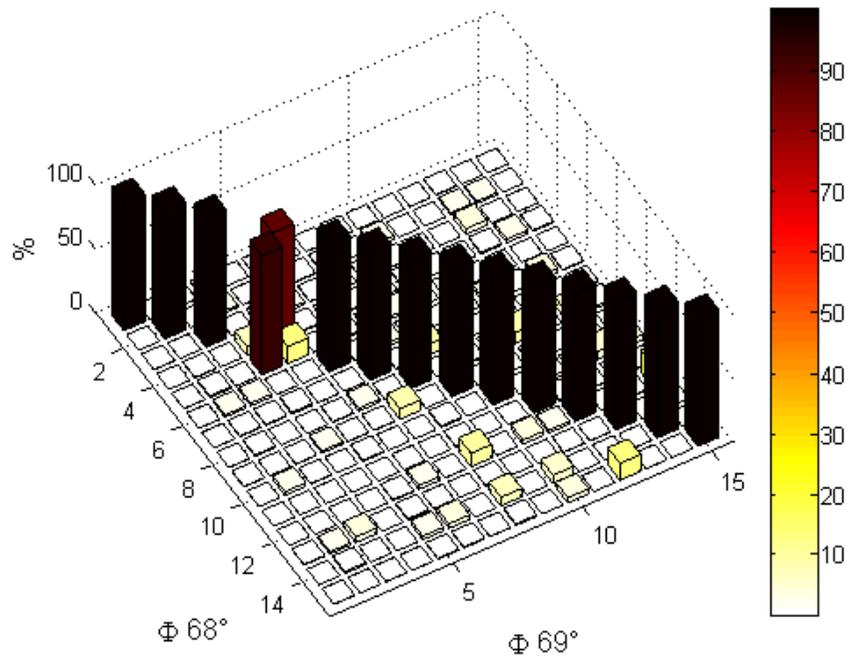


Crossing

Modal Assurance Criterion

$$MAC_{i,j} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \cos^2 \alpha_{i,j}$$

➤ Crossing: modes 4/5 @ 68/69°

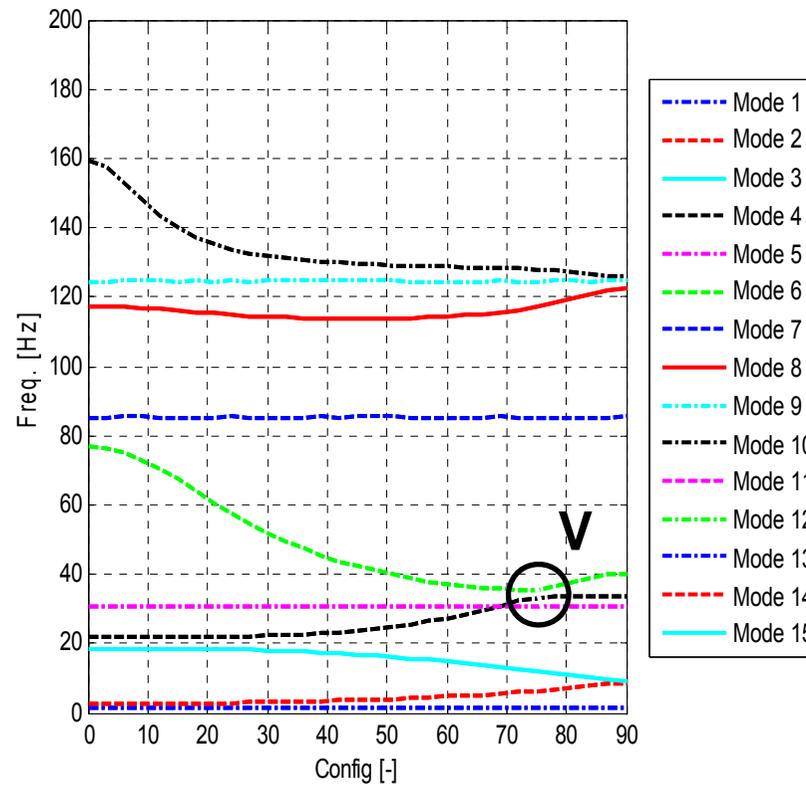


Veering

Modal Assurance Criterion

$$MAC_{i,j} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \cos^2 \alpha_{i,j}$$

➤ **Veering: modes 5/6 @ 72/81°**

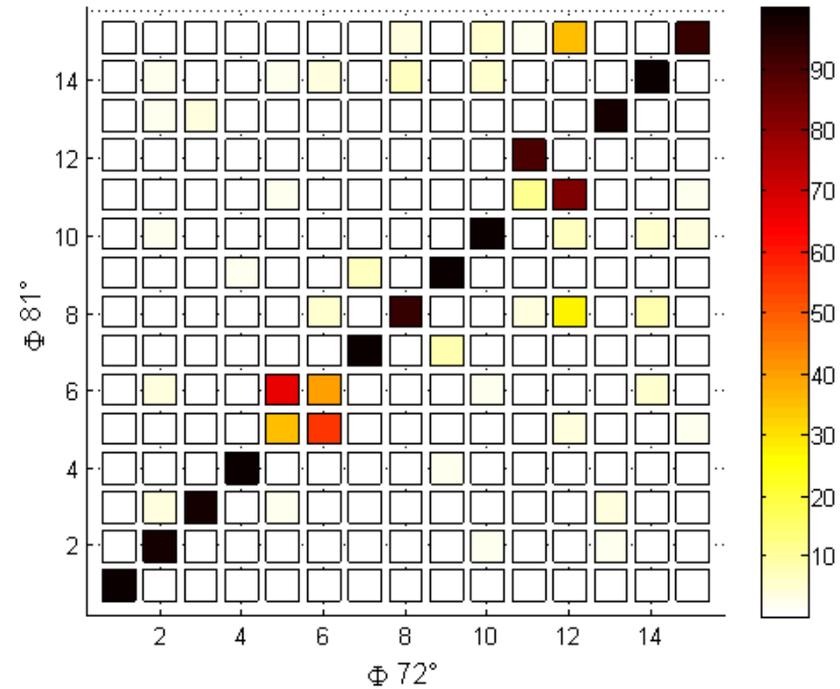
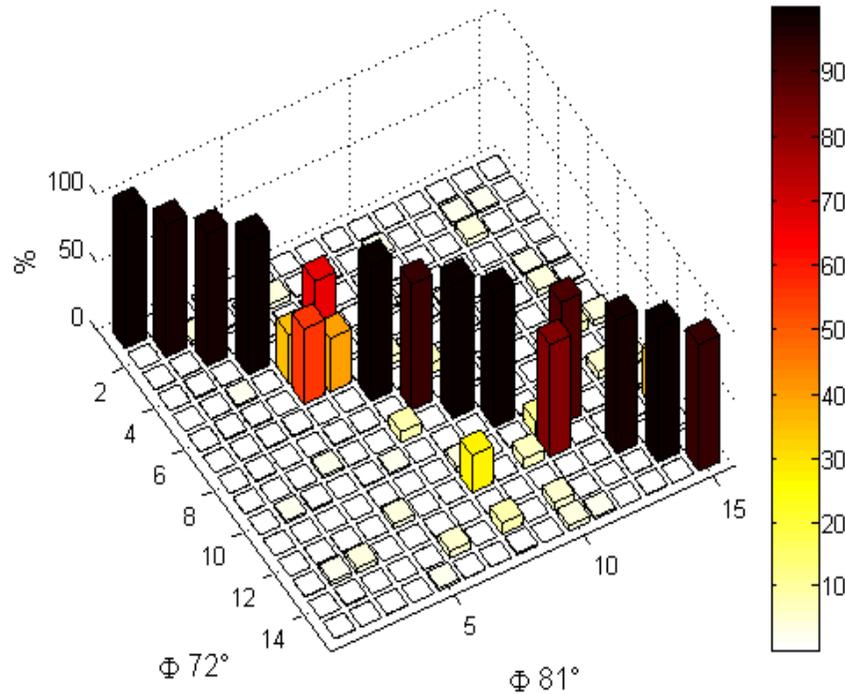


Veering

Modal Assurance Criterion

$$MAC_{i,j} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \cos^2 \alpha_{i,j}$$

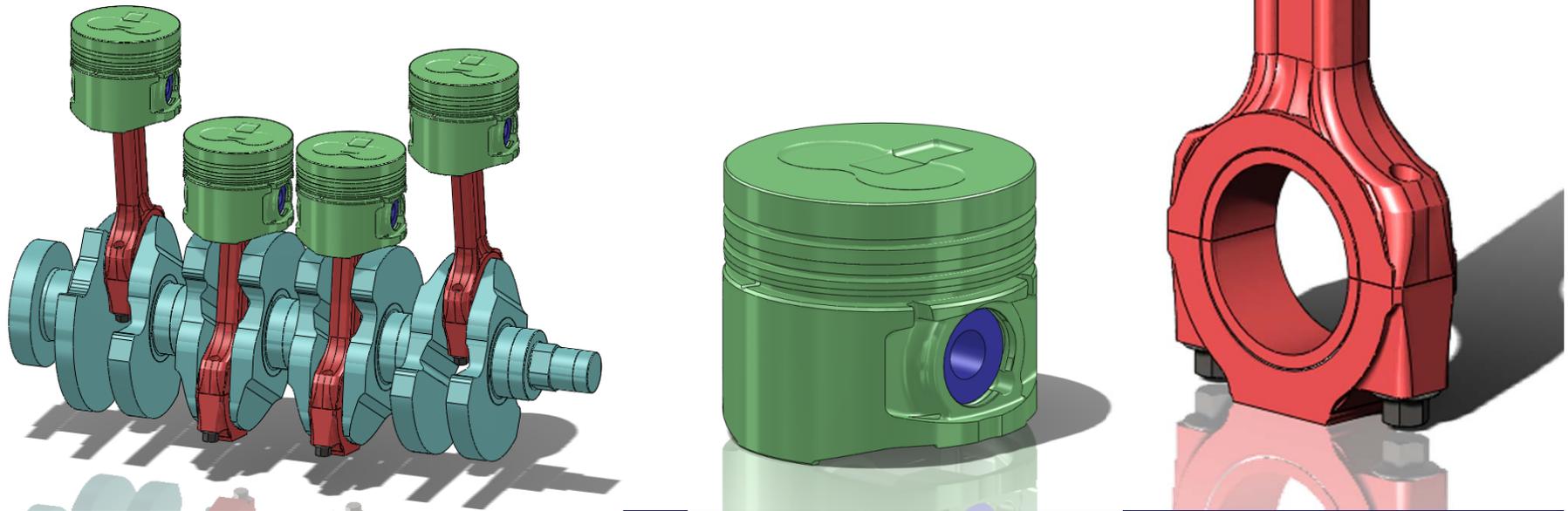
➤ Veering: modes 5/6 @ 72/81°



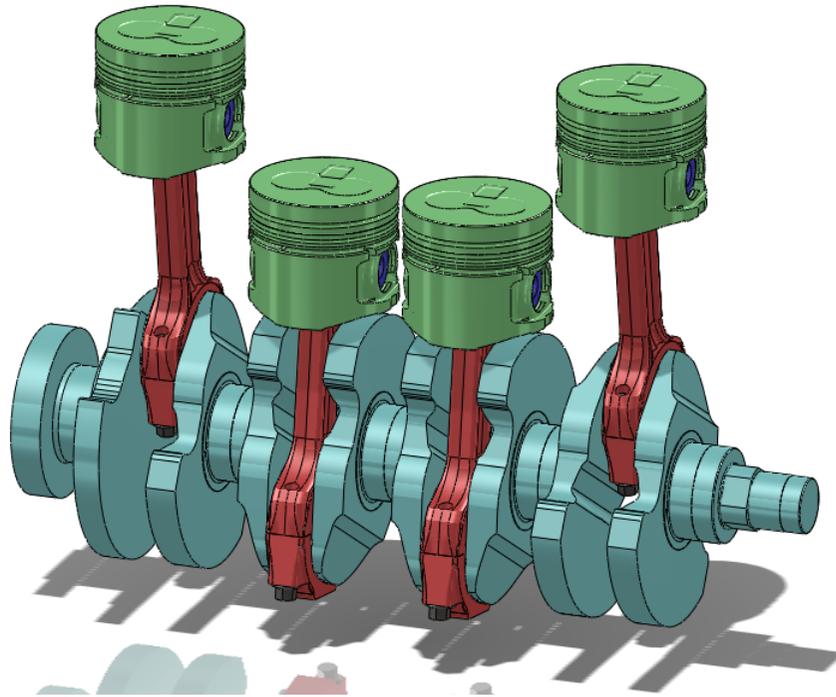
Crossing and veering: the crank-mechanism

Full 4c/4s crank-mechanism:

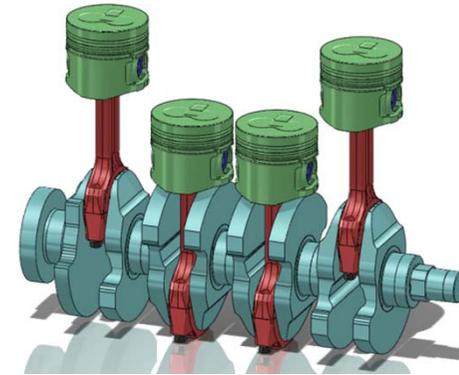
- CAD-CAE parametric modal simulations



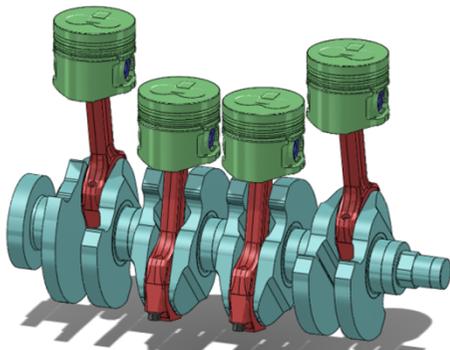
Configuration definition in crank-mechanism



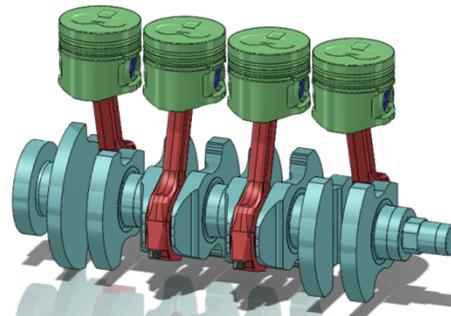
Config. 0°



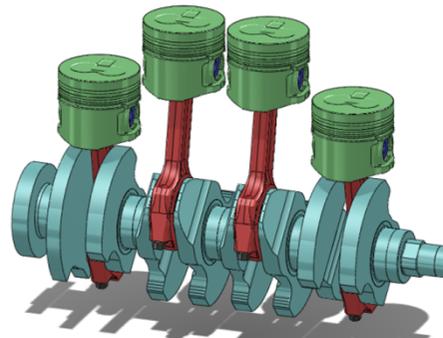
Config. 45°



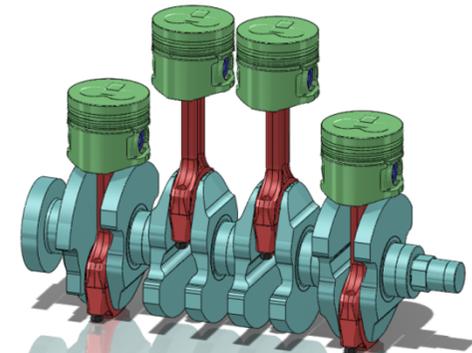
Config. 90°



Config. 135°



Config. 180°

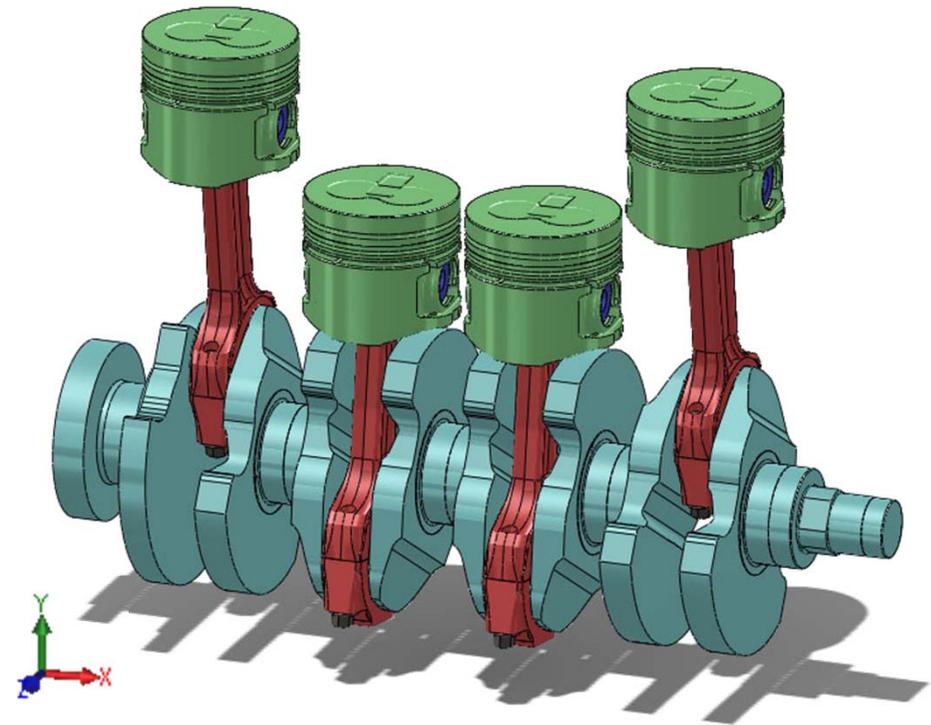


Crank-mechanism CAD-CAE model

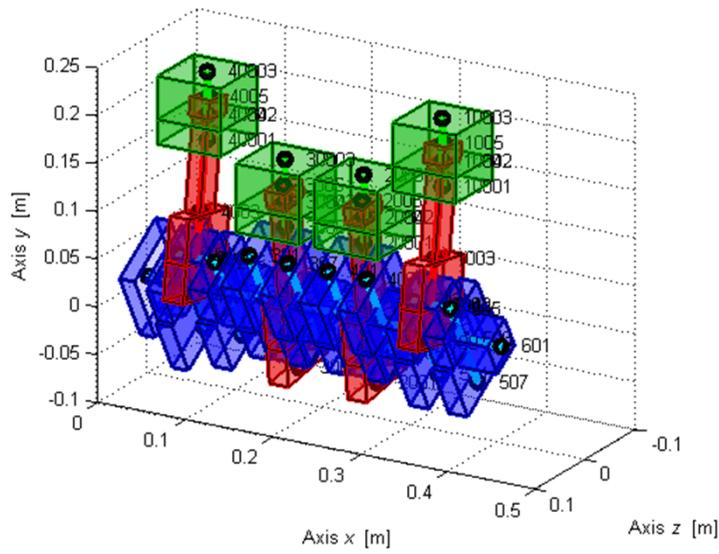
Basic assumptions:

➤ Boundary Conditions

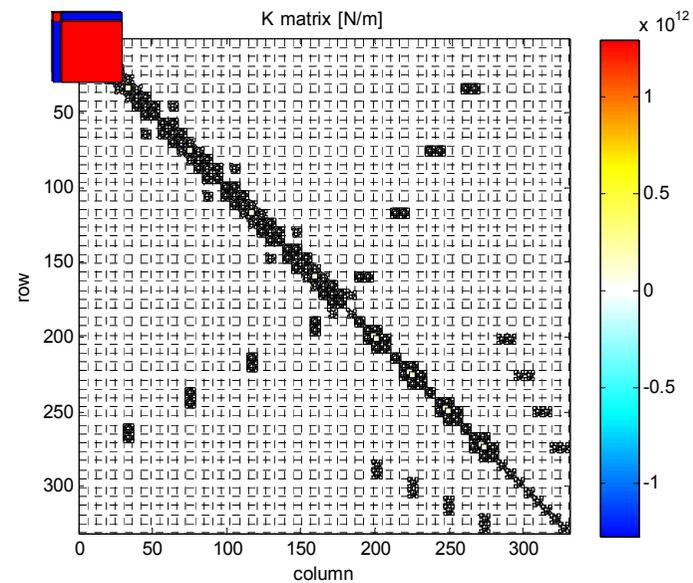
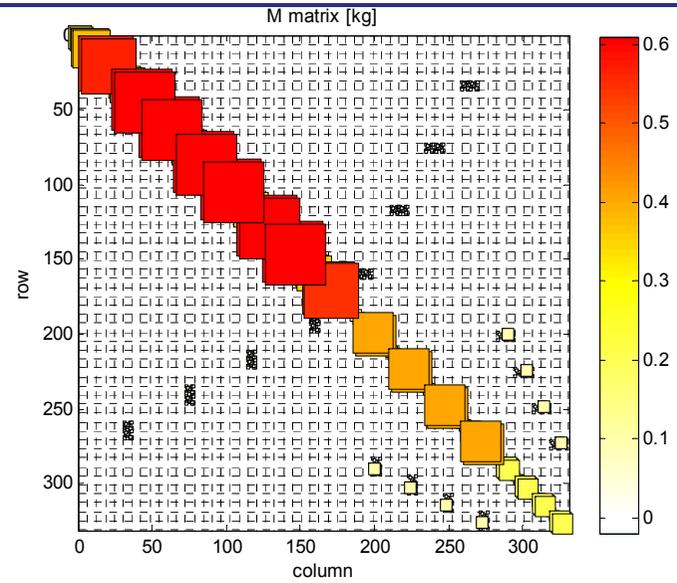
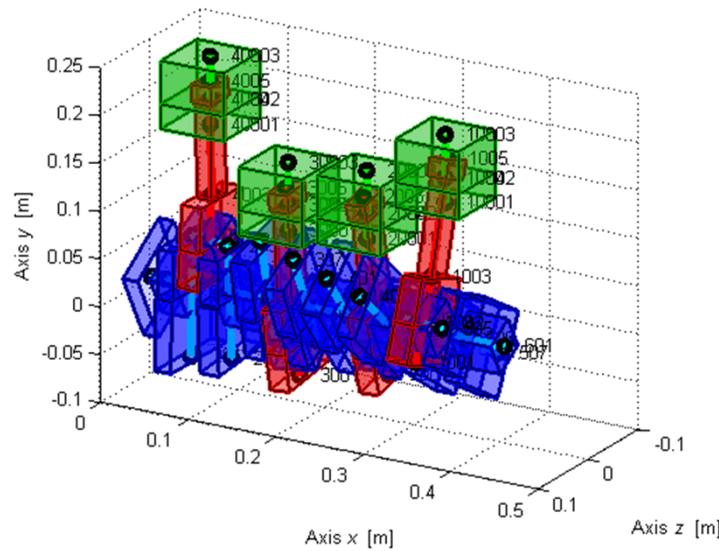
- free-free
- pinned-pinned



Crank-mechanism modelled in LUPOS

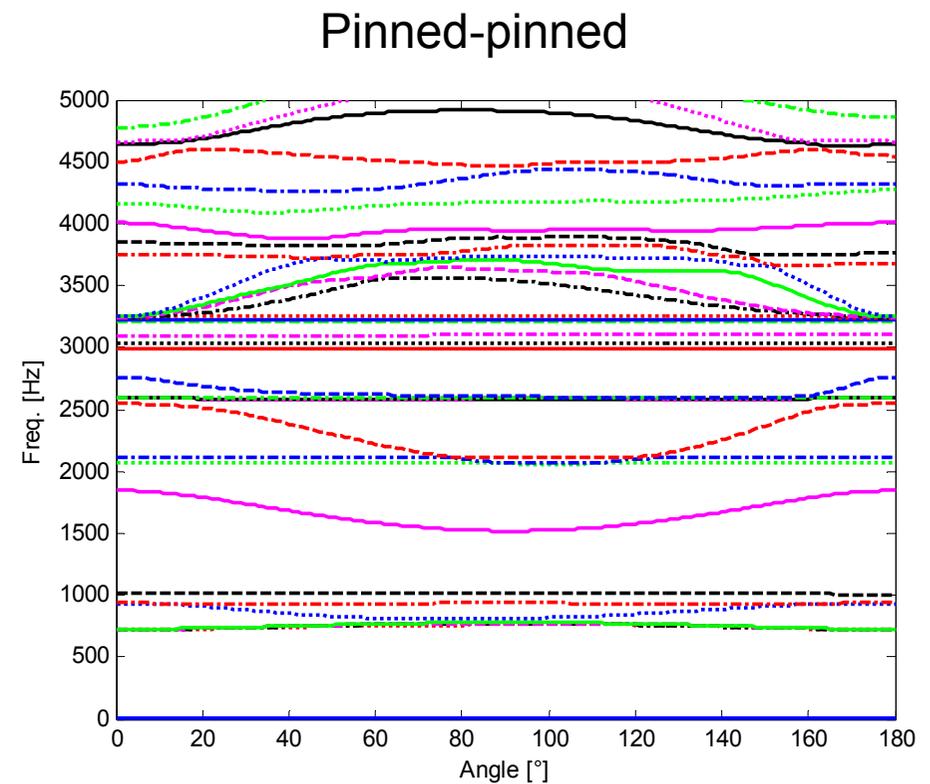
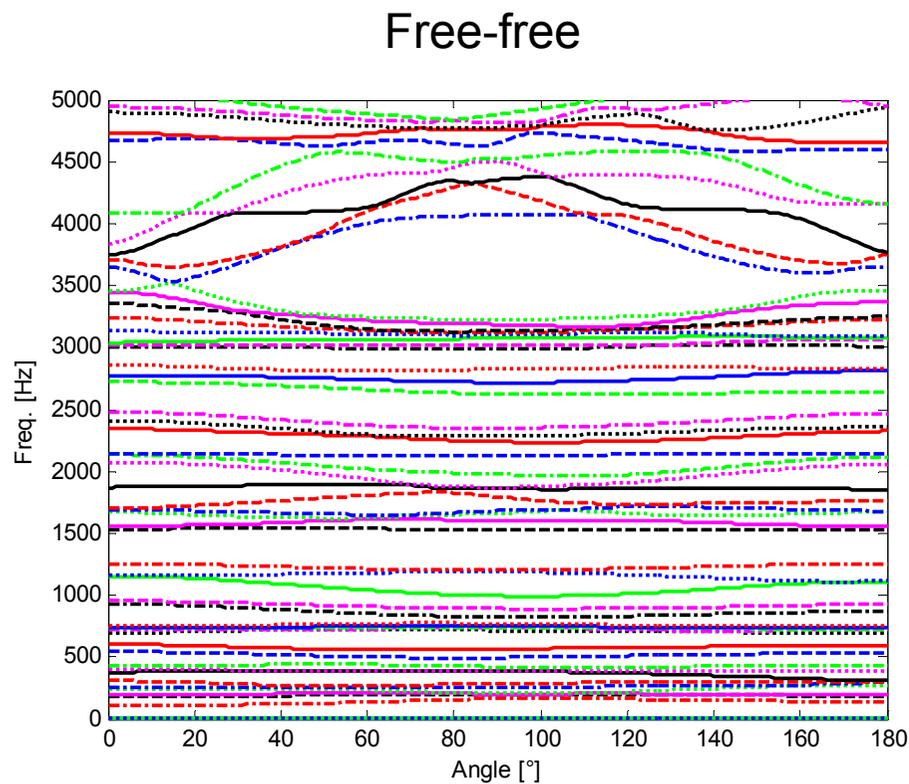


Mode 6: 810.5 Hz



Frequency loci w.r.t. crank angle in both conditions

Different behaviour with different BCs

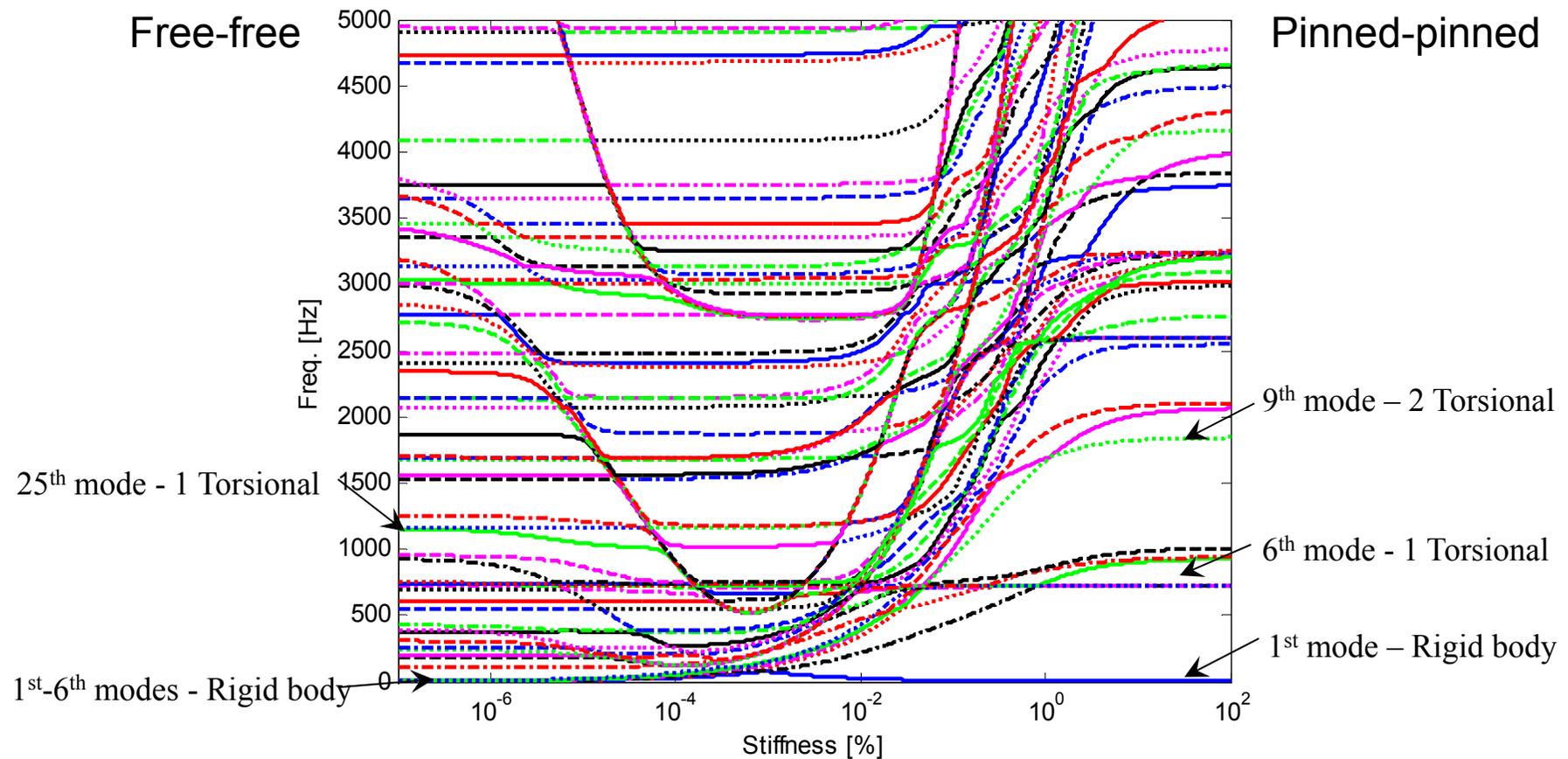
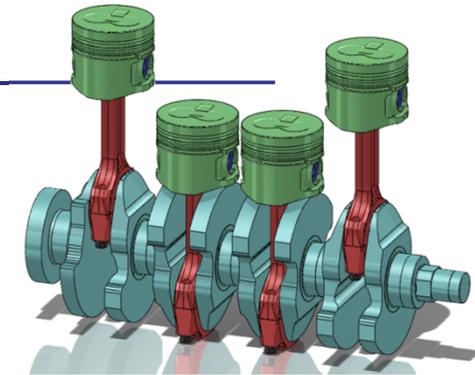


Frequency loci w.r.t. stiffness of BCs

Configuration parameter:

- BC modelled as grounded springs

Config. 0°

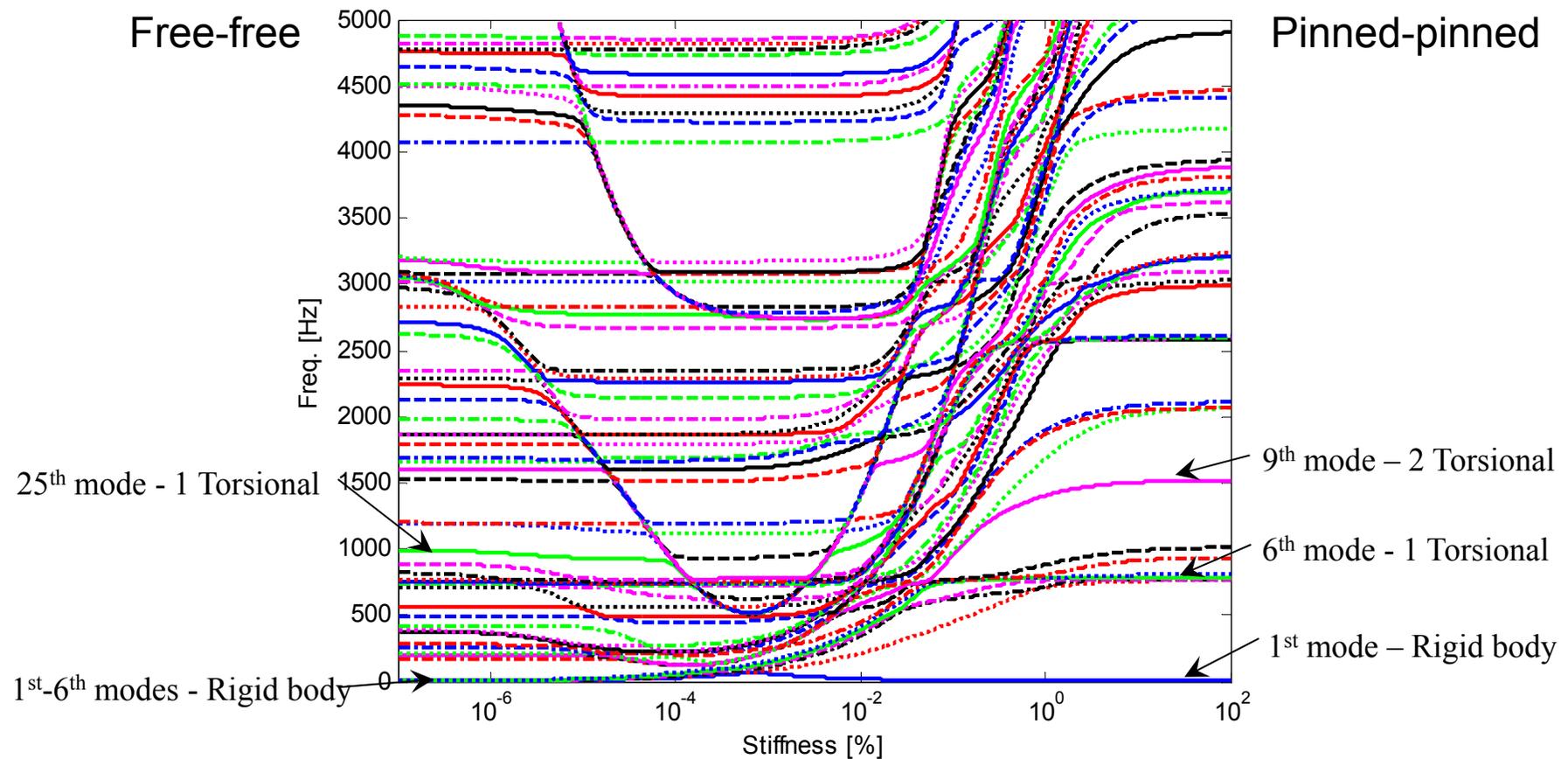
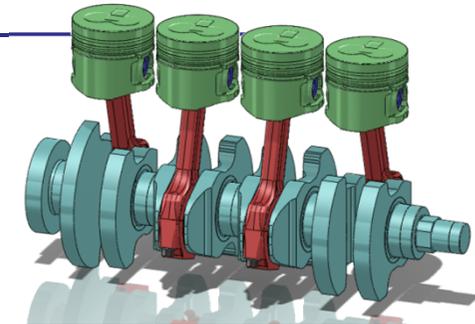


Frequency loci w.r.t. stiffness of BCs

Different configuration:

➤ Crank angle

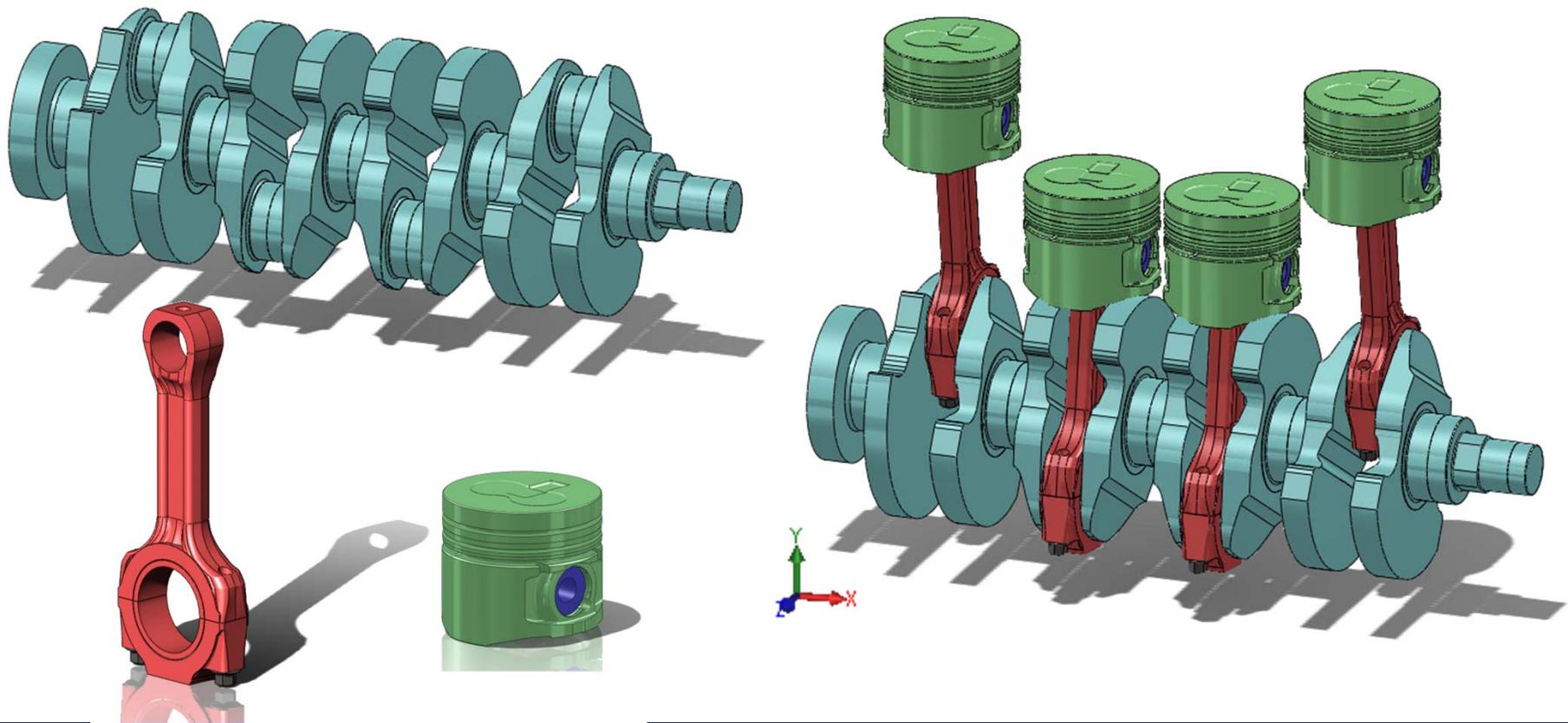
Config. 90°



Another parameter type...

Influence of components coupling:

- From component to system dynamics

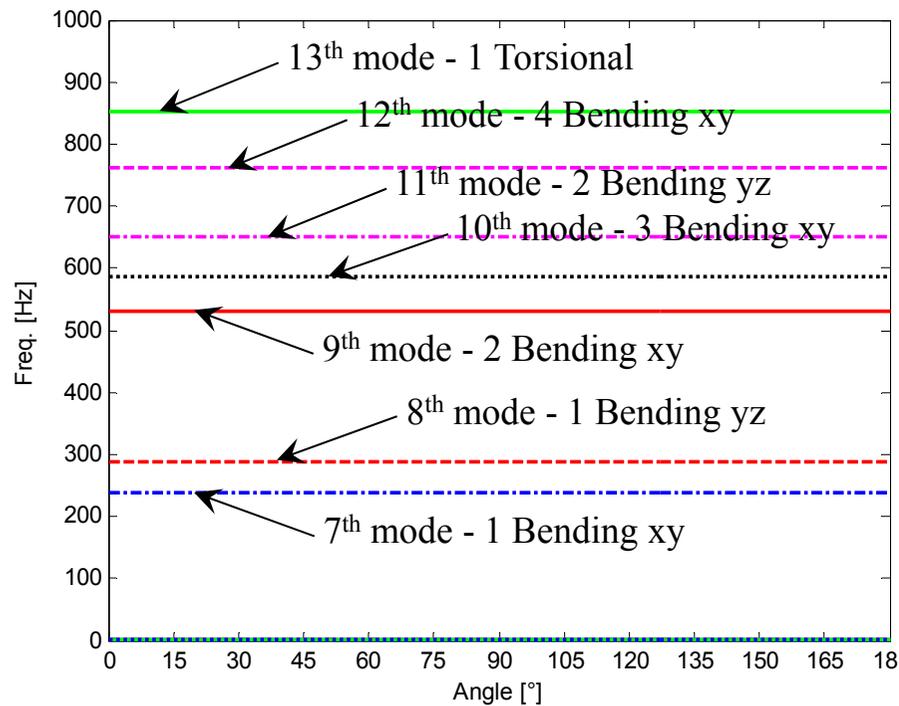


Comparison between component and system dynamics

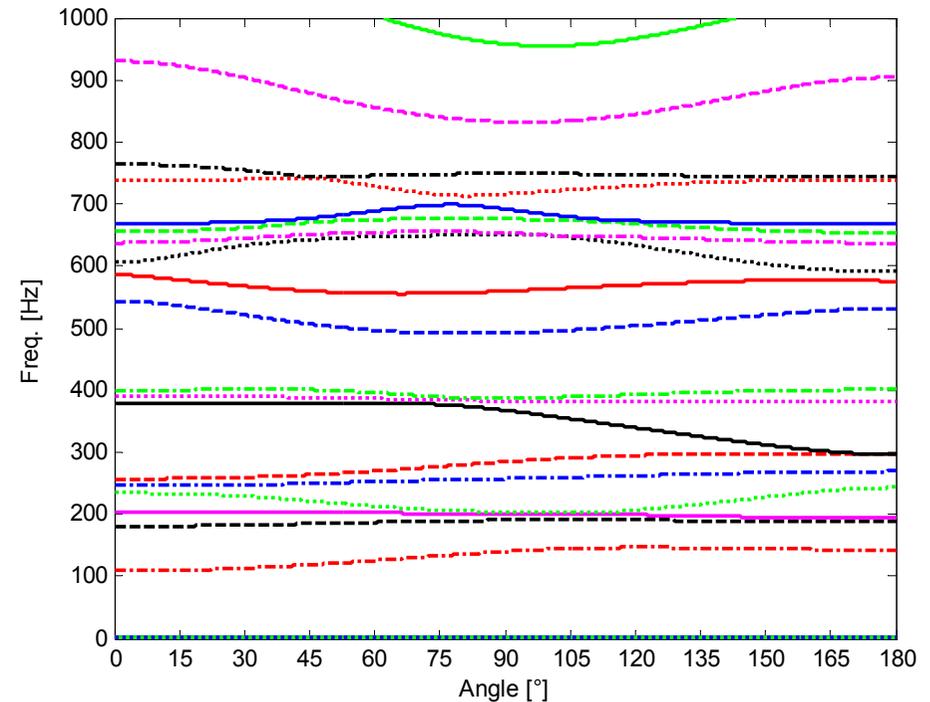
Again different behaviour w.r.t. crank angle

- From component to system dynamics

Crankshaft



Crankmechanism

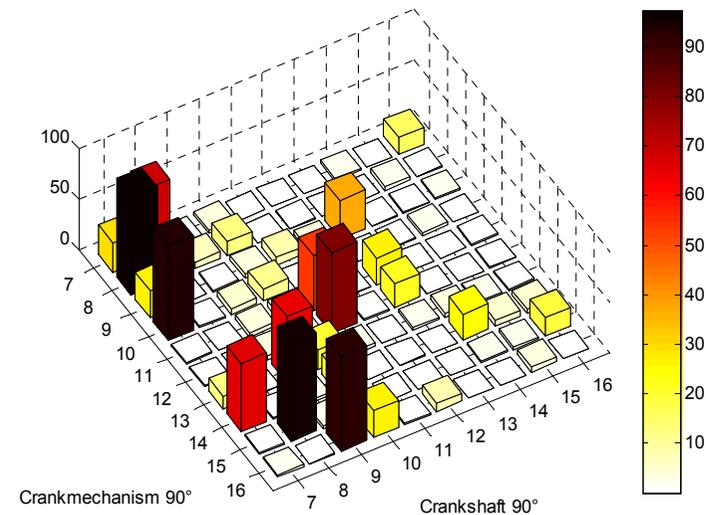
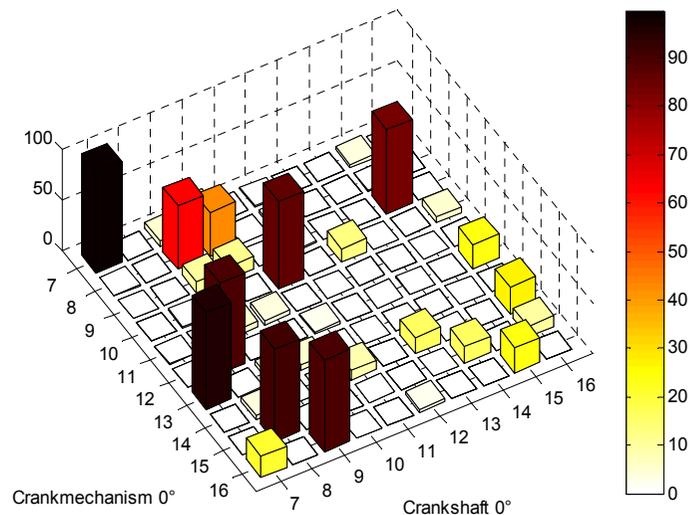


Modal Assurance Criterion

MAC index to compare mode shapes:

- Not a efficient index to compare component and assembly mode shapes

$$MAC_{i,j} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} = \cos^2 \alpha_{i,j}$$

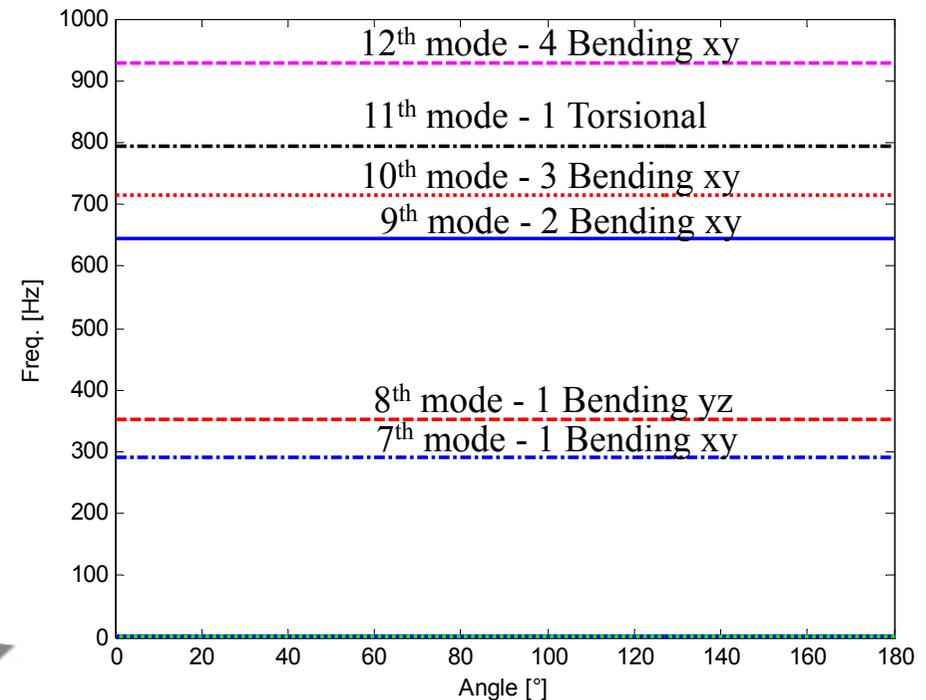
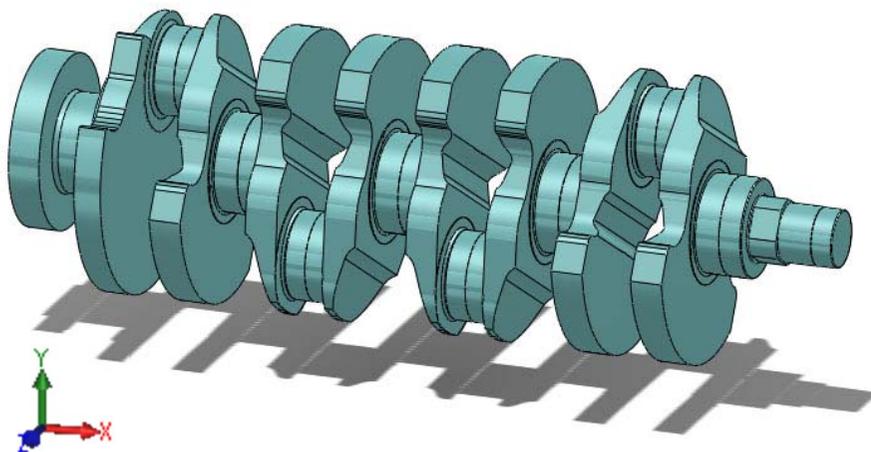


Progressive influence of components interaction

Frequency w.r.t. crank angle

➤ Single component: crankshaft

$$\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 0\%$$

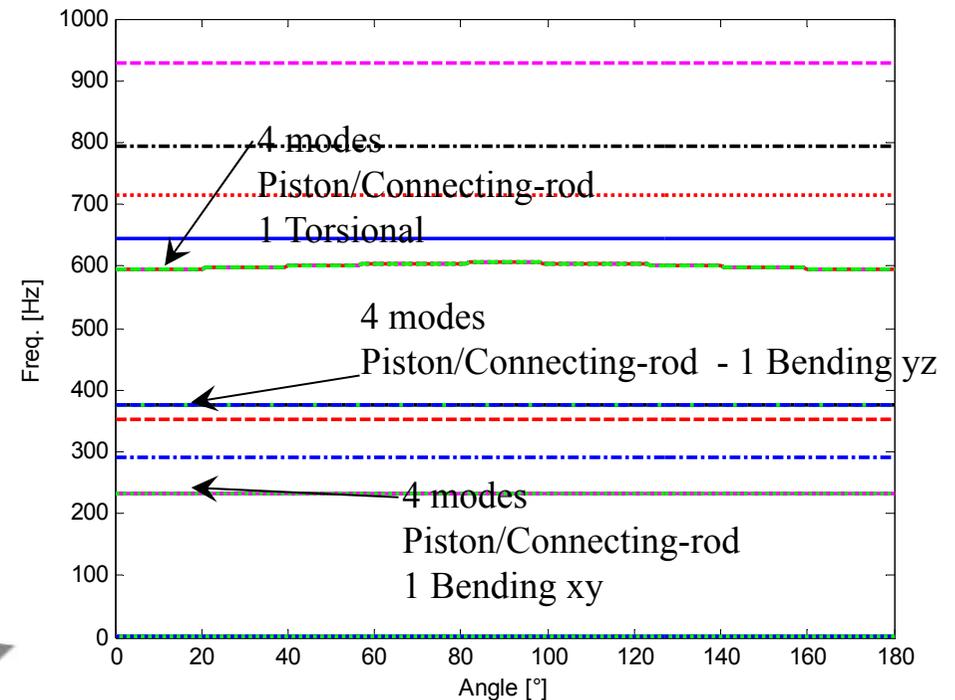
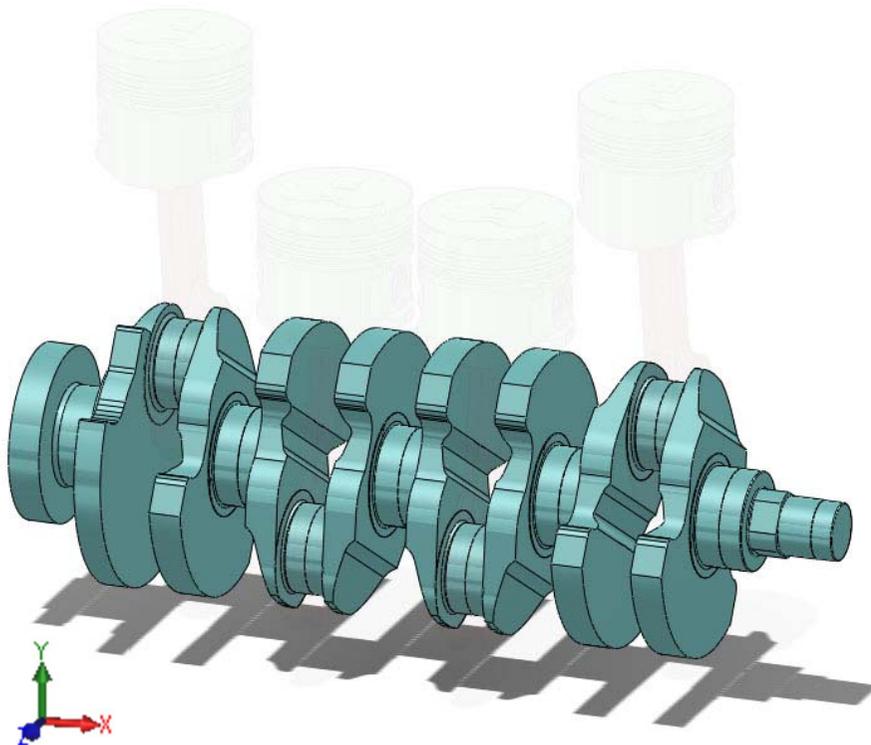


Progressive influence of components interaction

Frequency w.r.t. crank angle

➤ Rods and pistons uncoupled

$$\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 0.01\%$$

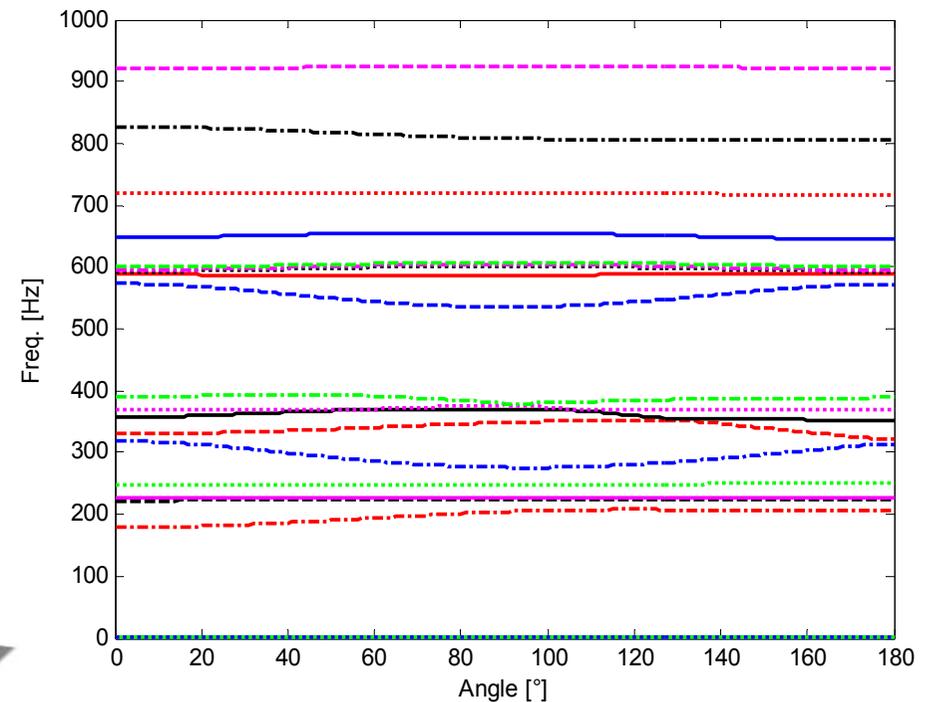
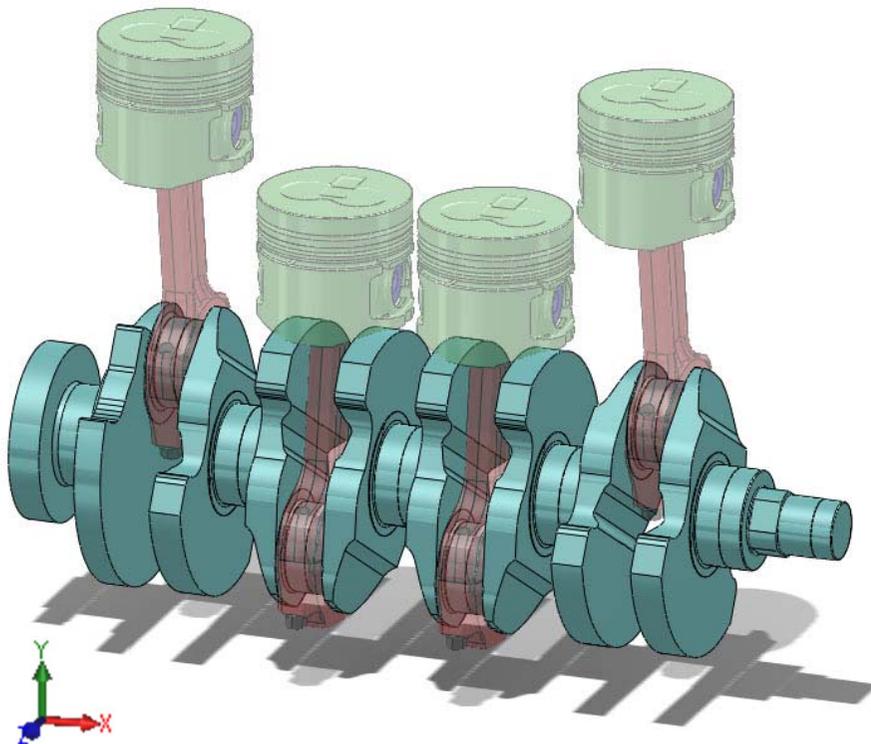


Progressive influence of components interaction

Frequency w.r.t. crank angle

➤ Weak coupling ...

$$\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 10\%$$

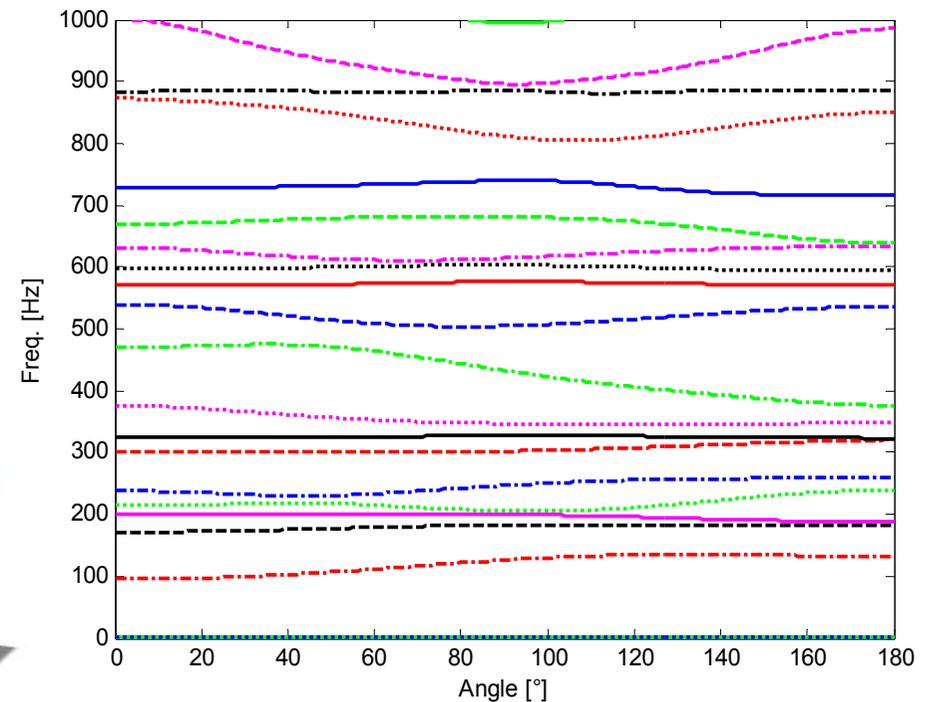
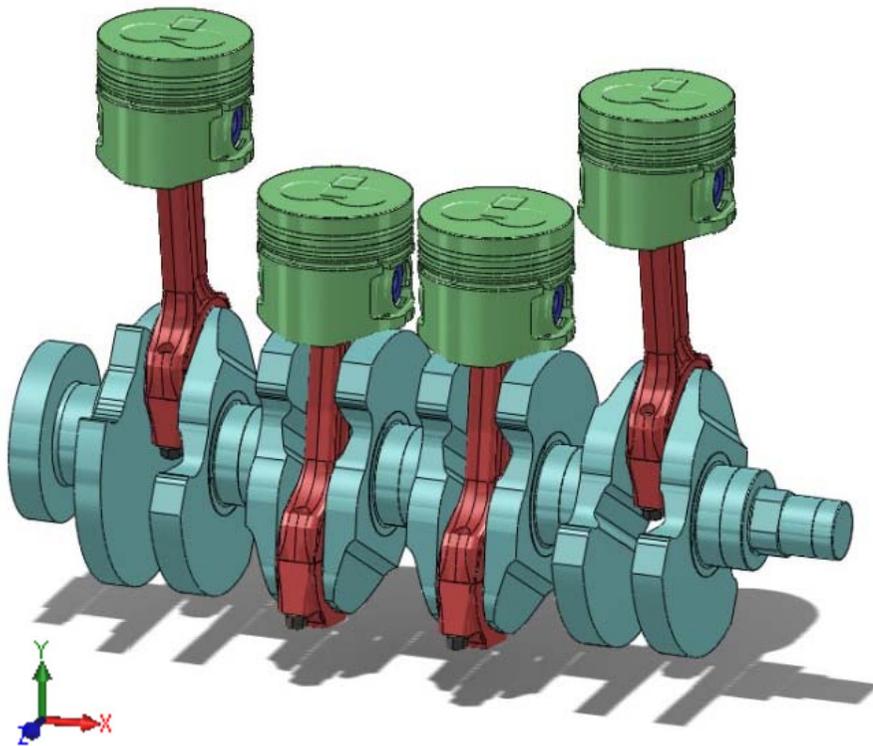


Progressive influence of components interaction

Frequency w.r.t. crank angle

➤ Full coupling

$$\frac{E_{PCR}}{\rho_{PCR}} = \text{const} \quad \rho_{PCR} = 100\%$$

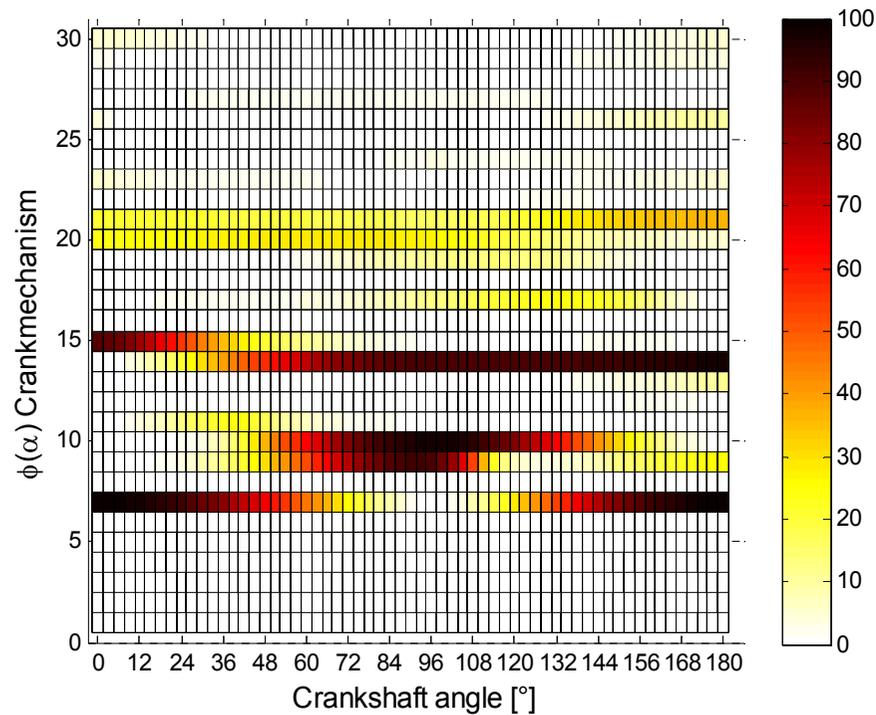


Different use of MAC

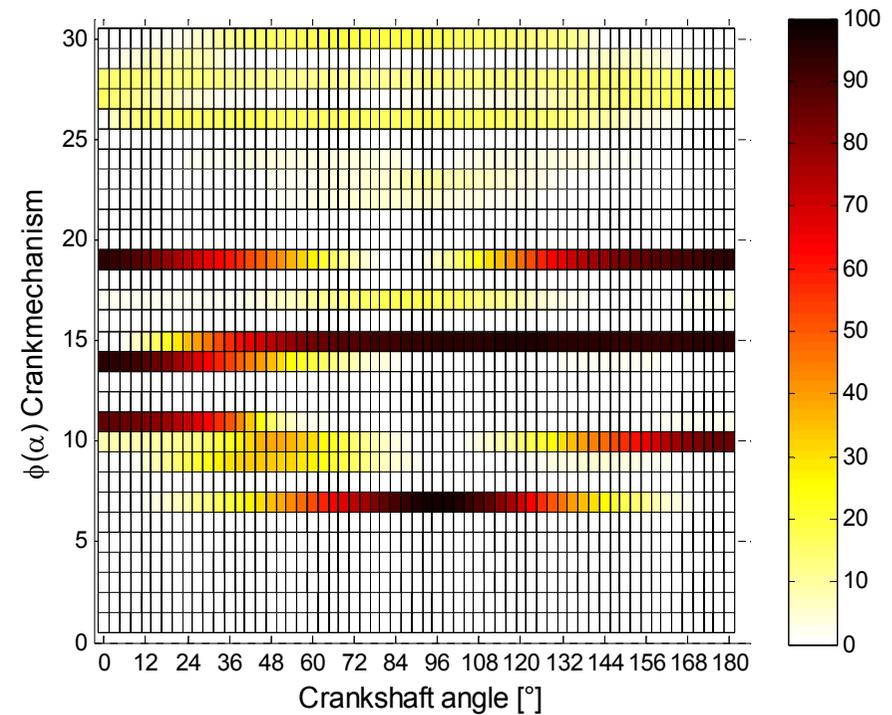
Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 7 – 1 bend xy



Crankshaft mode 8 – 1 bend xz

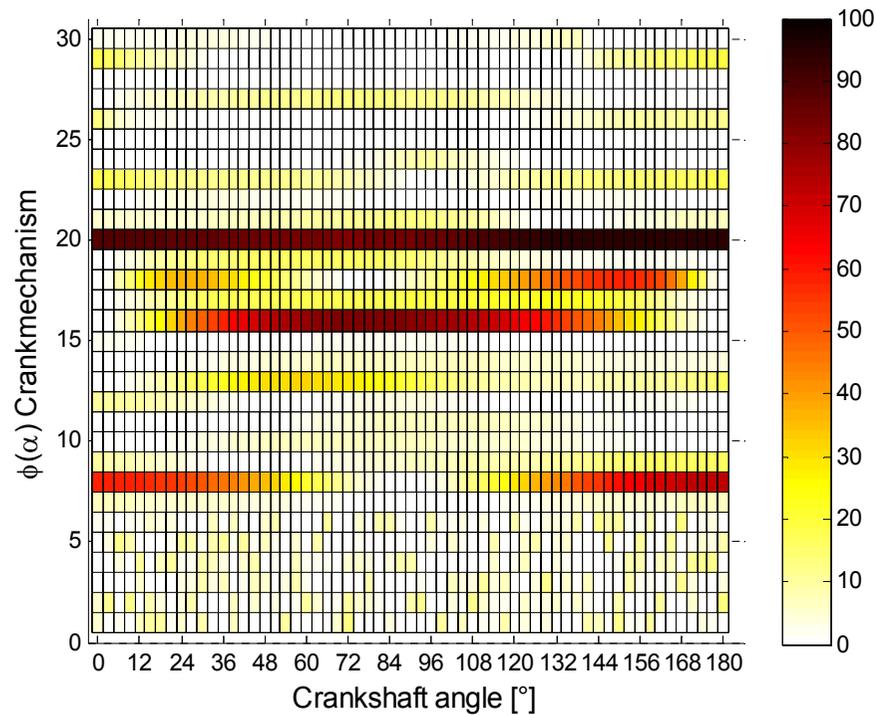


Different use of MAC

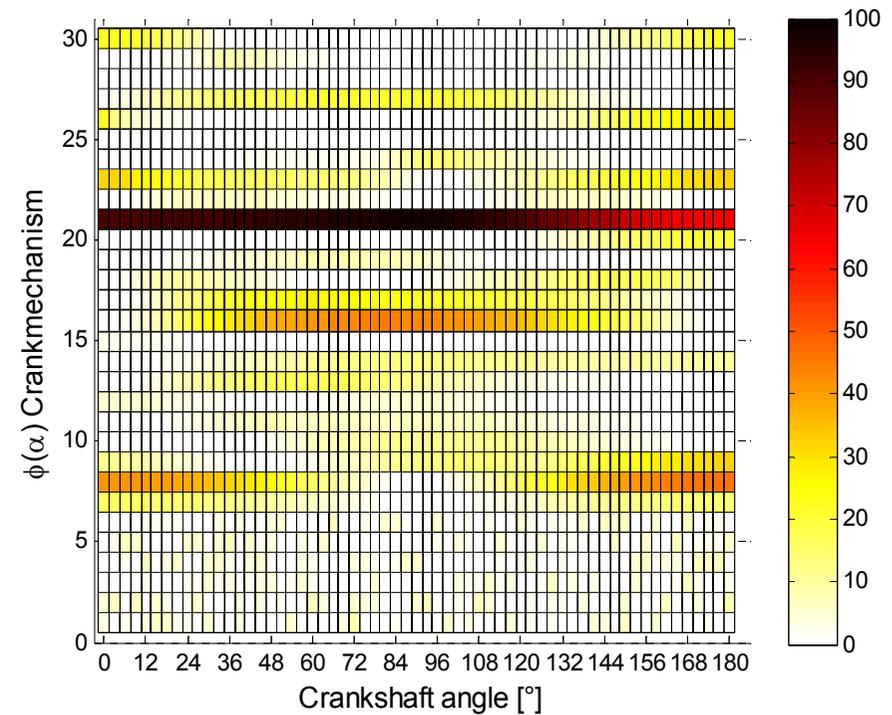
Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 9 – 2 bend xy



Crankshaft mode 10 – 3 bend xy

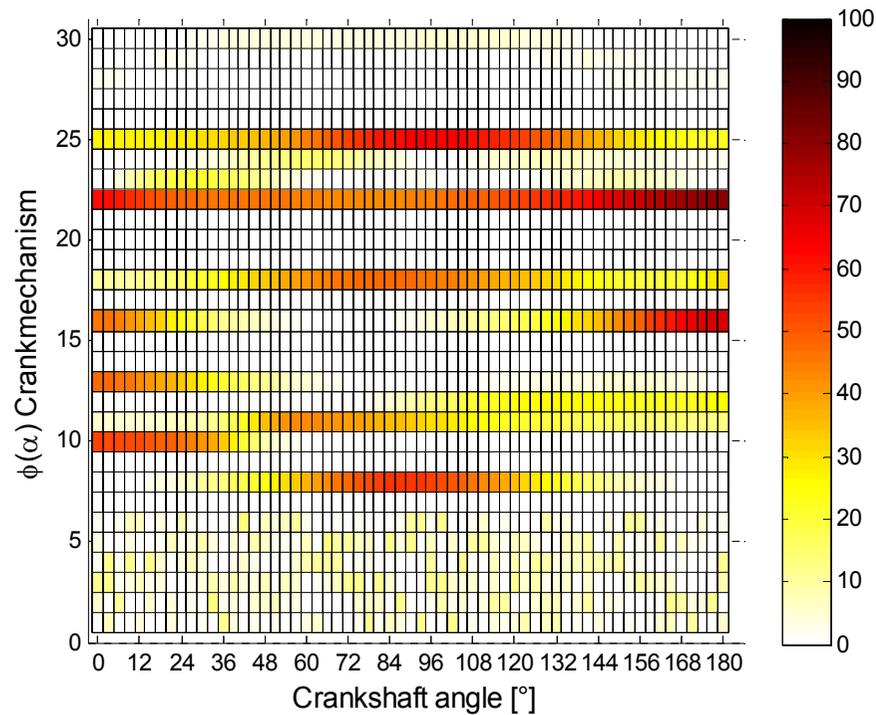


Different use of MAC

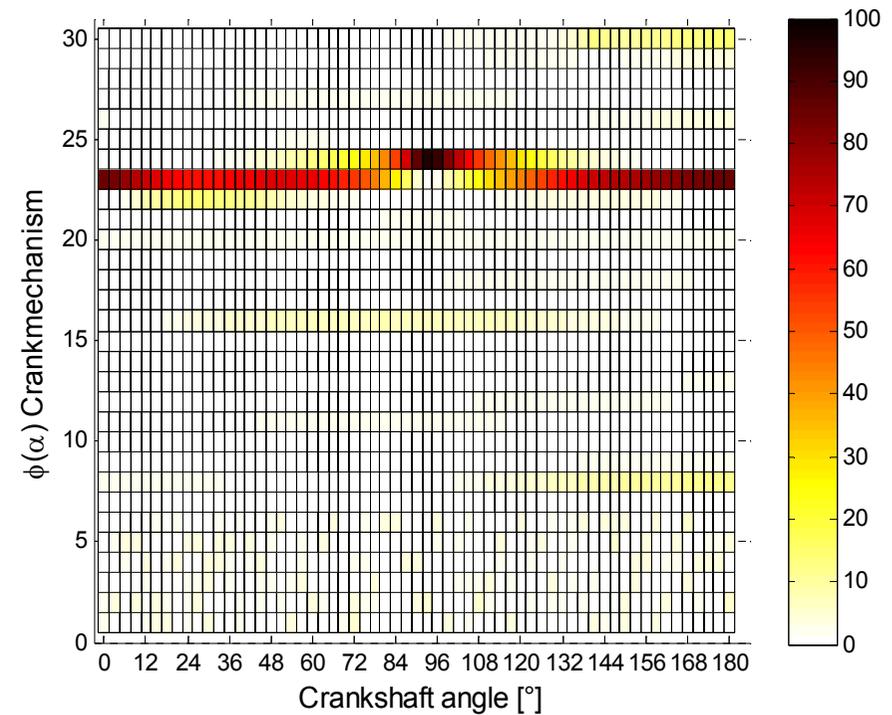
Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 11 – 1 tors



Crankshaft mode 12 – 4 bend xy

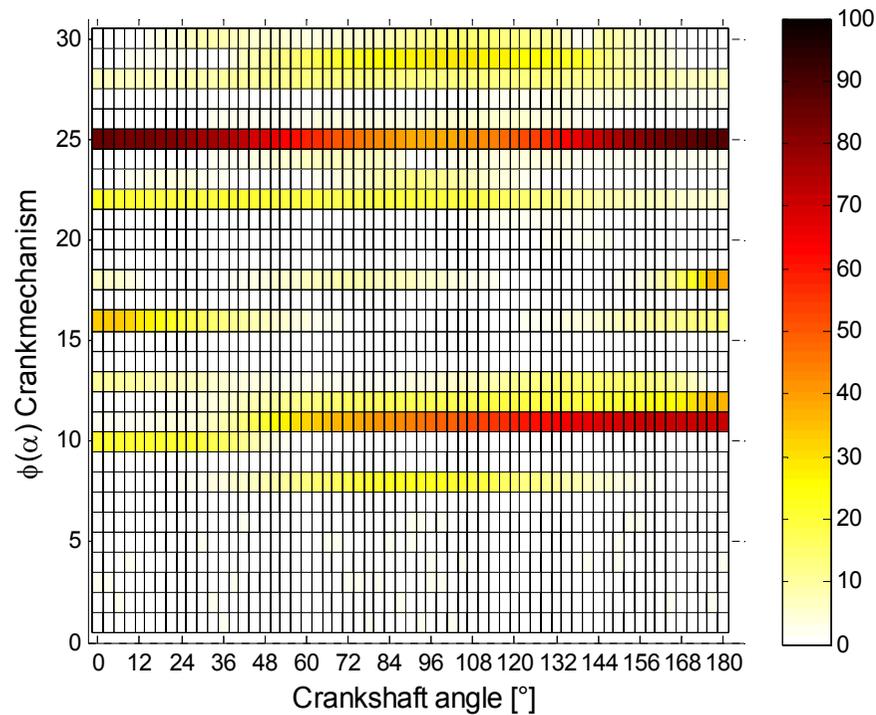


Different use of MAC

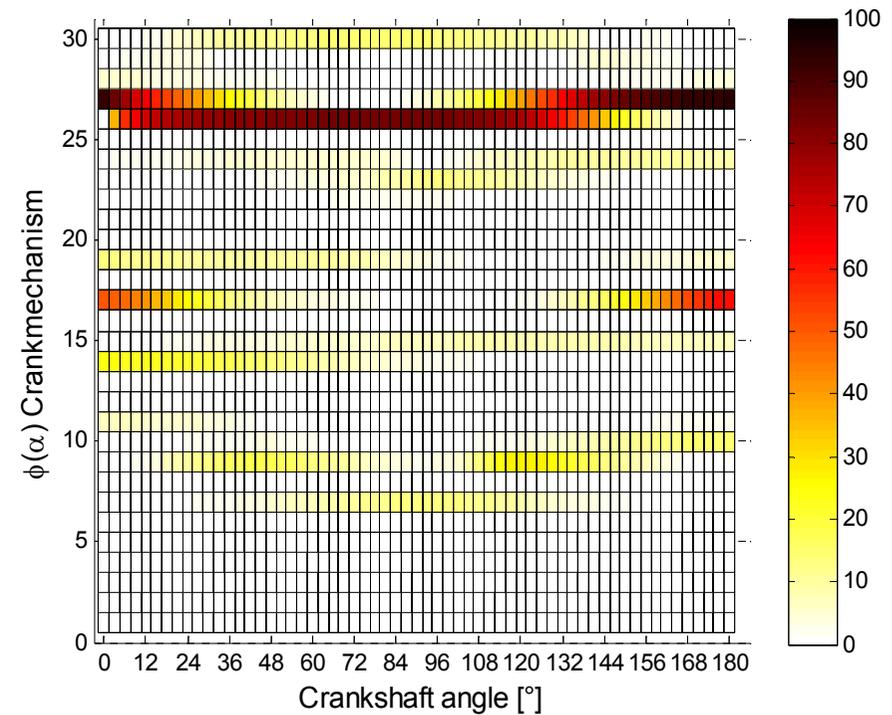
Comparison between crankshaft and crank-mechanism:

- Which system mode shape corresponds to component one?

Crankshaft mode 13



Crankshaft mode 14



Dynamics with close eigenvalues

- Cyclic / symmetric structures
- Non-cyclic / non-symmetric structures
- Effects of boundary conditions
- From component to system dynamics

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