Isogeometric Analysis: An Innovative Paradigm for Computational Mechanics

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Isogeometric Analysis (IGA, see [1,2]) is a recent simulation framework, originally proposed by Tom Hughes, Austin Cottrell, and Yuri Bazilevs in 2005, to bridge the gap between Computational Mechanics and Computer Aided Design (CAD). The basic IGA paradigm consists of adopting the same basis functions used for geometry representation in CAD systems - such as, e.g., Non-Uniform Rational B-Splines (NURBS) - for the approximation of field variables, in an isoparametric fashion. This may lead to a significant cost-saving simplification of the typically expensive mesh generation and refinement processes required by standard finite element analysis. In addition, thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard finite elements in a number of applications ranging from solids and structures to fluids, opening also the door to geometrically flexible discretizations of higher-order partial differential equations in primal form.

I was introduced to IGA at JP's Java (an "historical" coffee shop close to UT Austin's campus which had been the real headquarter of IGA until its closure, last September; see *Figure 1*, for a picture of the place) in the summer of 2004, when for some months I joined the group of Tom Hughes *(see Figure 2)* with the aim of preparing my Master thesis in Earthquake Engineering and carrying out part of my PhD under his guidance. I was supposed to work on some classical topics of computational structural dynamics,

Figure 1:
A picture of the glorious JP's Java coffee shop in Austin, i.e., the real IGA headquarter in the period 2004-2014

Figure 2:
Hanging out in Austin at the dawn of IGA (2004) with two members of the original IGA trio (Y. Bazilevs, middle, J.A. Cottrell, top-right) and the first IGA mathematician (G. Sangalli, bottom-right)

Figure 3:
IGA 2014. The organizers (from left to right): T. Dokken, T.J.R. Hughes, T. Kvamsdal, Y. Bazilevs, A. Reali, D.J. Benson

Figure 4:
IGA 2014. Conference dinner

but Tom easily convinced me of the potential of IGA and I therefore decided to explore the beneficial effect of continuity in spectrum analysis, i.e., in the approximation of structural vibration frequencies (something which could have been of interest also in the Earthquake Engineering community). A few months and several exciting results later, I decided that IGA would have become my main research topic also once back to Pavia and I have been continuously working on that subject ever since, often in collaboration with very talented colleagues.

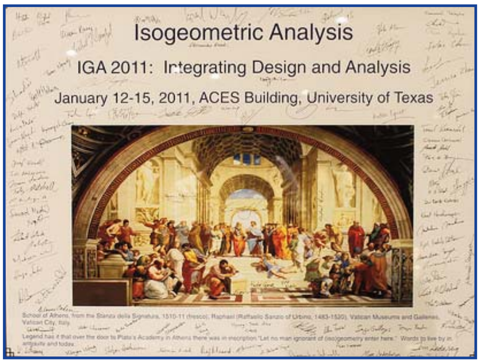


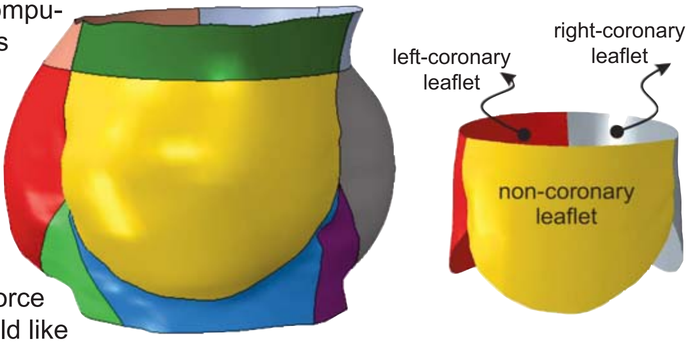
Figure 5:
Signatures of the IGA community. Above: First IGA Conference (Austin, January, 2011). Right: Second IGA Conference (Austin, January, 2014)

Figure 6:
IGA work meeting (wine tasting dinner on September 30, 2014, at the renowned Ricasoli vineyard in the heart of the Chianti area)

I like to mention the important friendship bonds that arose with the people I had the luck to meet along my IGA path with some pictures taken during the IGA conference in Austin last January *(Figures 3-5)*, and with a picture of Tom *(Figure 6)* I took during a nice wine tasting dinner we had together last September 30, at the renowned Ricasoli vineyard in the heart of the Chianti area.

Going back to research and to my beginnings in this field, I have to say that, following the mentioned initial works on spectra [3-5], the superiority of IGA over standard finite elements now appears to be remarkably evident in the approximation of structural vibrations and dynamics, as it is, for instance, clearly shown in a recent paper [6]. Moreover, an impressive example of the potential of IGA is given by the results obtained within a project emanating from one of the frequent Austin-Pavia collaborations and completed with the help of a number of friends, where explicit dynamics simulations of the closure of a patient-specific aortic valve were carried out [7]. The complex geometrical model was built from medical images by means of conforming multiple NURBS patches *(see Figure 7, top)*, and on such a model an explicit nonlinear analysis involving large deformation shells and contact was successfully performed *(see Figure 7, bottom)*. Despite the lack of optimization of the adopted IGA implementation, for a given target level of accuracy, the IGA simulation resulted to be over 440 times faster than that carried out with what is considered to be the fastest shell finite element on the market (i.e., 1h15m versus 23 full 24-hour days!).

Since I mentioned explicit dynamics (where the computational cost is dominated by stress divergence evaluations at quadrature points for the calculation of the residual force vector), I would like to concisely introduce another topic that was initiated during one of my visits in Austin, and which looks indeed promising in all situations where matrix formation costs are dominant: namely, IGA collocation methods. Such novel IGA schemes were first proposed in 2010 [8] with the goal of optimizing computational cost, still taking advantage of IGA geometrical flexibility



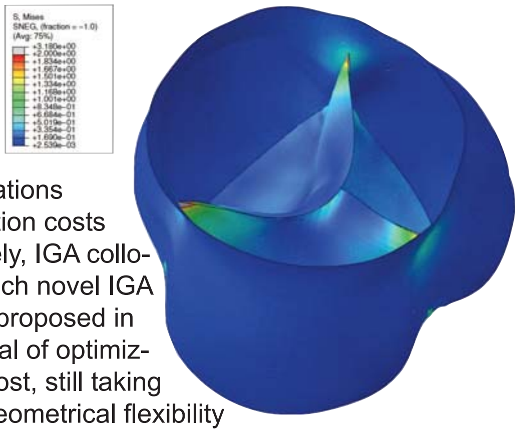


Figure 7:
Multi-patch IGA of a patient-specific aortic valve. Top: Conforming multi-patch NURBS geometry of the aortic valve and of the three leaflets. Bottom: Results of the valve closure simulation in terms of deformed shape and Von Mises stress

and accuracy. The fundamental idea consists of the discretization of the governing partial differential equations in strong form, adopting the isoparametric paradigm and making use of the higher-continuity properties of the IGA shape functions. Detailed comparisons with both IGA and FEA Galerkin-based approaches have shown IGA collocation advantages in terms of accuracy versus computational cost, in particular when higher-order approximation degrees are adopted. Within the IGA collocation context, several promising significant studies have been recently published, including phase-field modeling, contact, and hierarchical local refinement. Moreover, IGA collocation has been very successful in the context of structural elements. In particular, Bernoulli-Euler beam and Kirchhoff plate elements have been proposed and shear-deformable structural elements have been considered as well. In fact, locking-free mixed formulations both for initially-straight planar Timoshenko beams and for curved spatial rods have been advanced and analyzed, and IGA collocation has been also successfully applied to the solution of Reissner-Mindlin plate problems. All these results have been published in several papers, and interested readers are referred to the review reported in [9] and to references therein.

Going back to the Galerkin framework, I remarked above that IGA opened the door to geometrically flexible discretizations of higher-order partial differential equations in primal form. In particular, I want to mention that it gave new life to Kirchhoff-Love shell models (see, among others, [10] and references therein), which possess several key advantages over Reissner-Mindlin models typically adopted in finite element simulations (such as no rotation degrees of freedom, no shear locking, etc.). In addition, IGA higher-regularity gives the possibility of efficiently implementing locking-free Reissner-Mindlin models, or novel structural models able to retain the advantages of Kirchhoff-Love formulations but including also shear deformation, as it happens with Reissner-Mindlin approaches (see, e.g., [11]).

As a final representative case study, I want to briefly mention the results of some nonlinear bending simulations of complex structures like shape memory alloy stents, just obtained within the group of Computational Mechanics and Advanced Materials at the University of Pavia [12]. These nonlinear analyses do not involve contact

Figure 8:
SMA stent bending: IGA versus FEA.
Top: Reaction force convergence plots.
Middle: Force-displacement plots for finest FEA and coarsest IGA.
Bottom: Comparison of deformed shapes for finest FEA and coarsest IGA

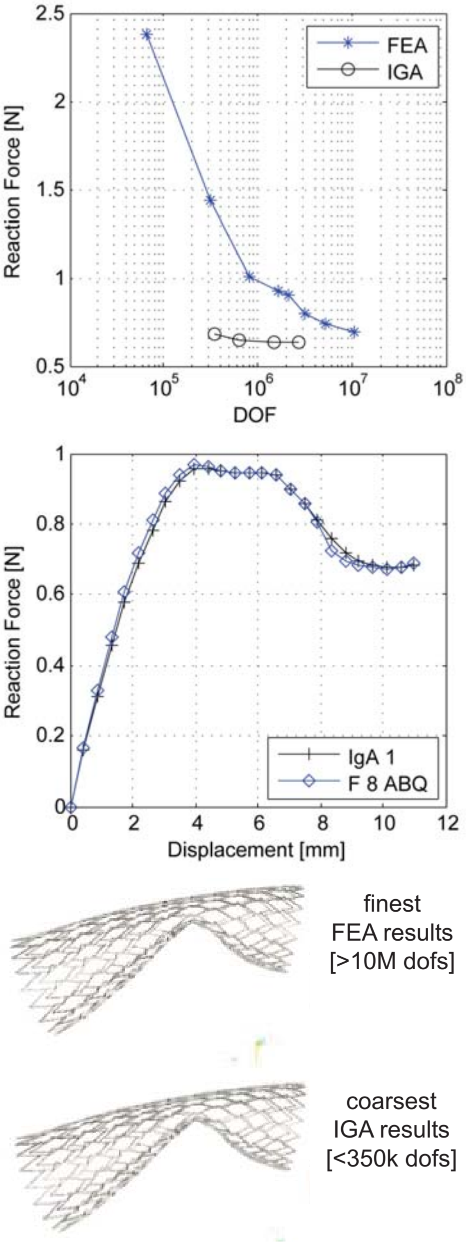
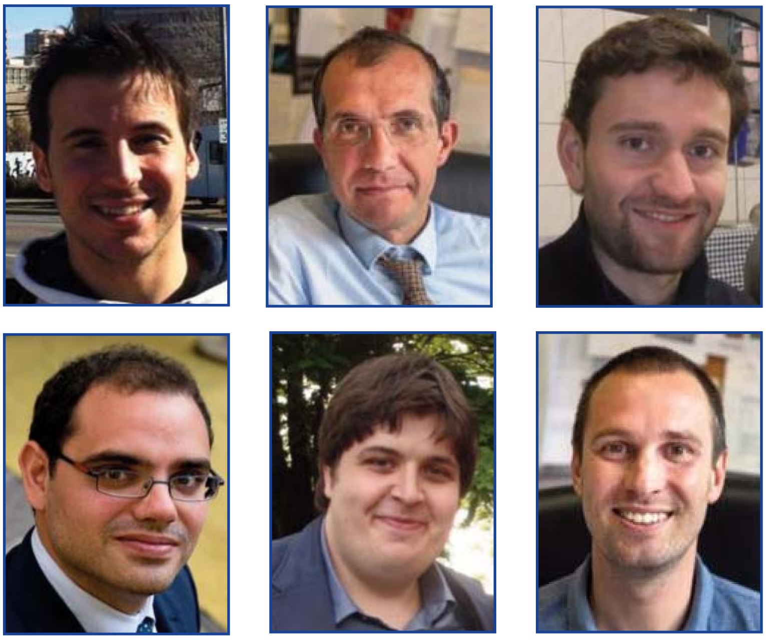


Figure 9:
Some members of the group of Computational Mechanics & Advanced Materials at the University of Pavia currently involved in IGA activities: S. Morganti, F. Auricchio, J. Kiendl, M. Conti, M. Ferraro, A. Reali



nor explicit dynamics (as in the valve case described above), but the geometrical complexity, the use of non-trivial inelastic constitutive models, and the presence of significant buckling phenomena imply that extremely fine meshes - comprising a number of degrees of freedom well beyond what is typically adopted in these kinds of computational biomechanics problems - are required to correctly describe the physical phenomenon by means of standard finite elements. Instead, IGA simulations are proven to correctly reproduce the actual physics of the problem already with coarse meshes. A remarkable note is that, within the adopted finite element framework (refer to [12] for details), 10,622,016 d.o.f.'s are required to reproduce the same results obtained within IGA with only 346,413 d.o.f.'s *(see Figure 8)*. In this case, the overall computational cost for IGA is four times lower, despite the use of a single processor versus the eight processors employed in the case of finite elements (a serial implementation within FEAP was used for IGA, while ABAQUS parallel capabilities were employed for finite elements).

Many other impressive examples could be cited here, but for the sake of brevity I would just refer to the fast-growing IGA literature. I instead would like to conclude with a picture *(Figure 9)* of the members of the group of Computational Mechanics and Advanced Materials at the University of Pavia, which are currently contributing to some of the research activities described above. I also want to acknowledge the fundamental interactions and contributions of the "Numerical Analysis crew" in Pavia, as well as of all the people I had the privilege to meet over the last decade at the Institute of Computational Engineering and Sciences, UT Austin (cf. the acknowledgement section below for a, probably still incomplete, list of collaborators and friends).

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