Un esempio di materiale avanzato: le leghe a memoria di forma

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Presentation outline

- **Introduction to shape-memory alloys**
  - ✓ SE: superelastic effect
  - ✓ SME: shape memory effect
- **Traditional and innovative application**
  - ✓ SE: eyeglasses / stents
  - ✓ SME: actuators / intervertebral spacers
- **1D constitutive models**
  - SE + SME time-continuous constitutive models
  - Algorithmical considerations
  - Algorithm numerical validation
  - A home-made round-robin test
- **3D constitutive models**
  - ✓ SE + SME time-continuous constitutive models
  - ✓ Algorithmical / mathematical considerations
  - ✓ Algorithm numerical validation (uni-axial & multi-axial)
- **3D numerical simulations**
  - ✓ SE: stents [free-expansion / crush test]
  - ✓ SME: actuators / intervertebral spacers
  - ✓ Hybrid composites

Shape memory alloy: an introduction

Shape memory alloys (SMA) : materials with an intrinsic ability to recover initial "shape" also after severe deformations

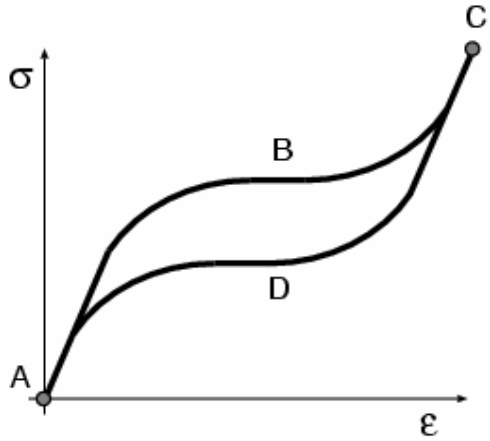
Macroscopic point of view : two unusual effects, in general non available in traditional materials

Superelastic effect

Shape memory effect

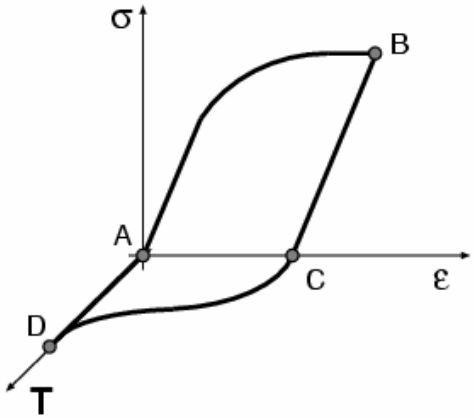
Shape memory alloy: an introduction

Superelastic effect



Mechanical recovery

Shape memory effect



Thermal recovery

Application fields

Macroscopic effects non available in traditional materials

⇩

Innovative and commercially valuable applications

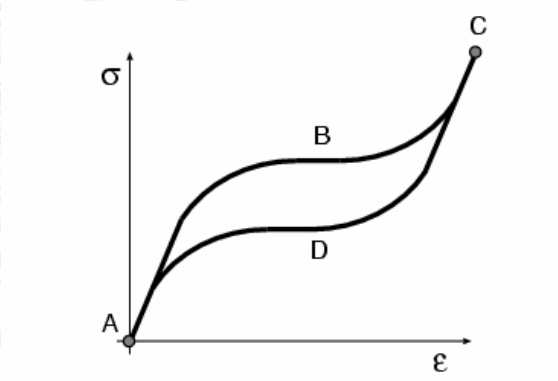
Biomedical area : orthodontics, orthopedics, guidewires, stents, eyeglasses, micro-actuators

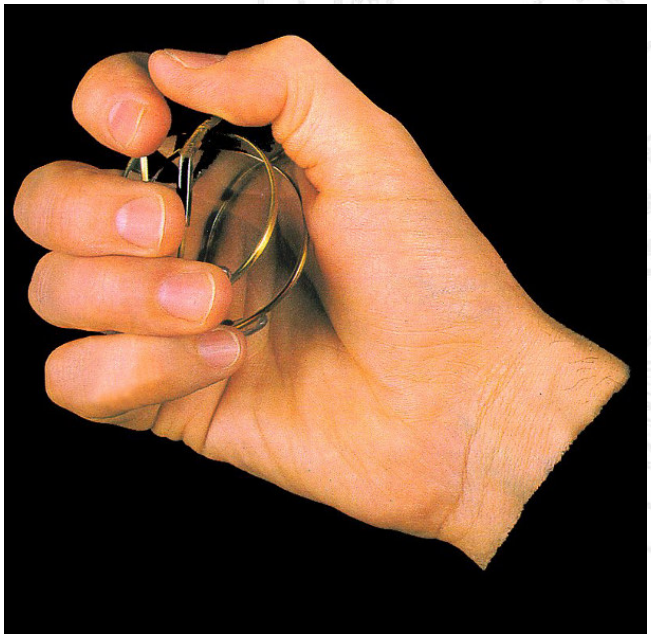
Mechanical area : actuators, thermal valves, connectors, closing/opening systems

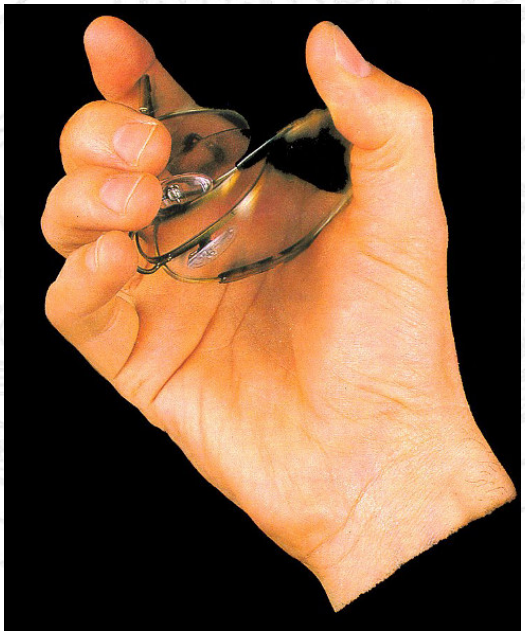
Structural area : shape & vibration control, energy dissipation (civil engineering, aeronautics, car industry)

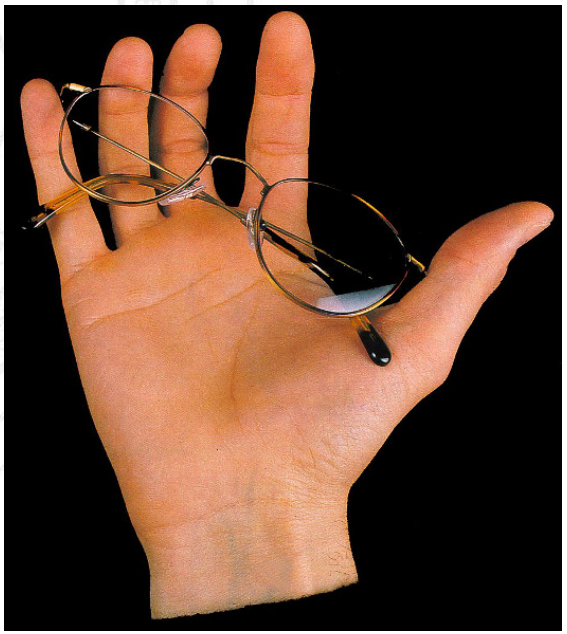
Superelasticity: applications

Superelastic effect (SE) :



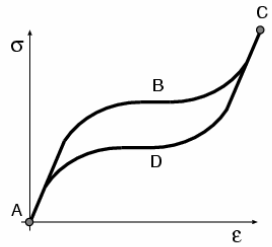


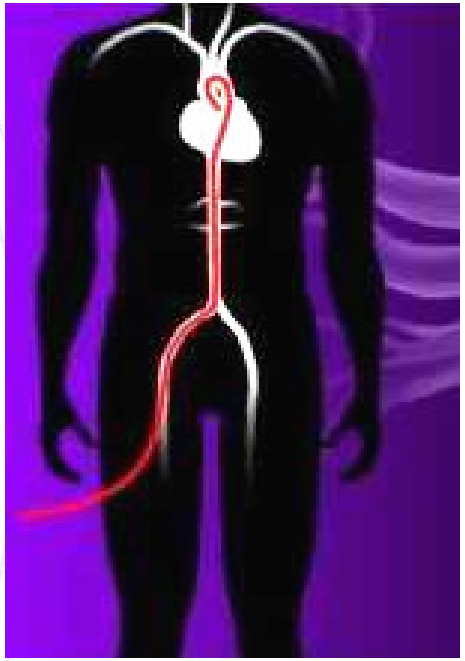




Superelasticity: cardio-surgery

Superelastic effect (SE) :

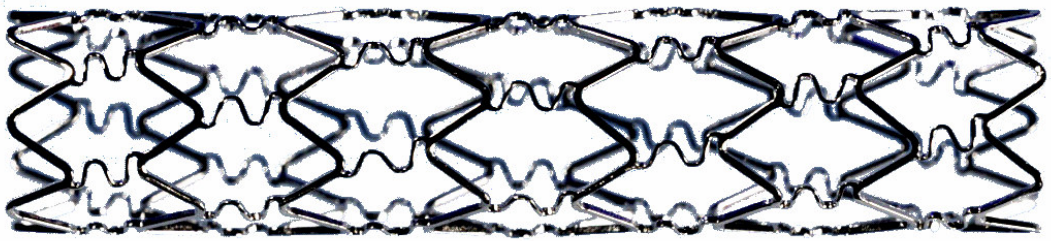




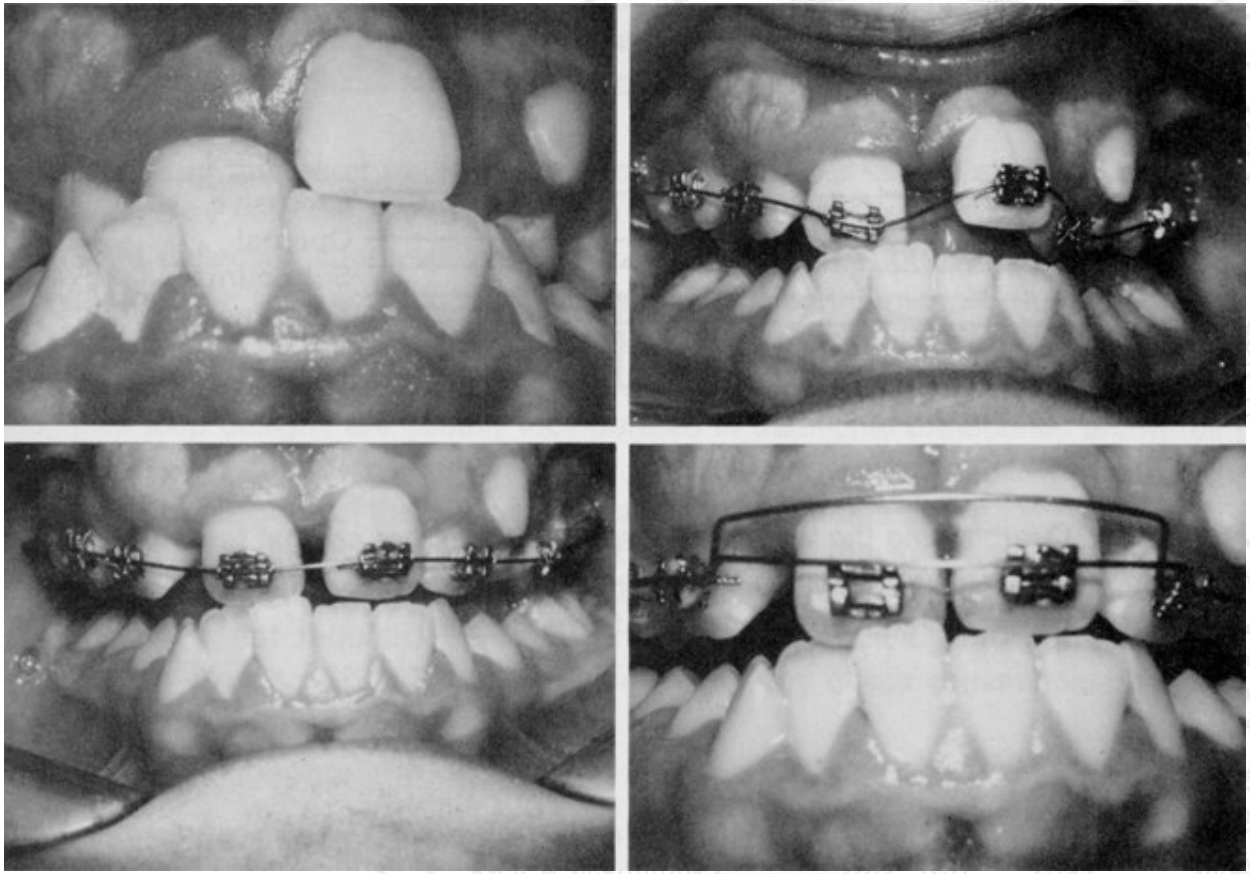
Stenosis : occlusion of a physiological lumen (ex.: coronaries)

Non invasive treatments:
peripherical access

SMA intravascular stents : expandable tube-like structures permanently placed into stenotic arteries to restore blood perfusion / serve as scaffold to increase artery blood flow to downstream tissues



Superelasticity: orthodontics



Orthodontic archwires for the correction of tooth mal-position

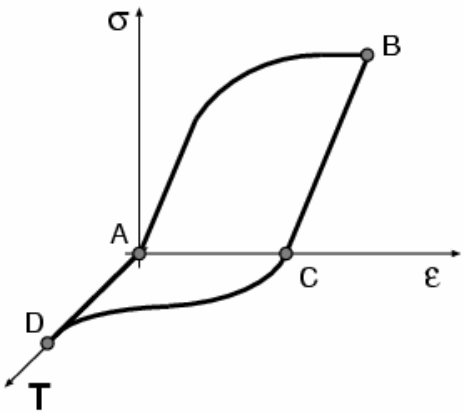
[results in only 3 weeks of therapy !!]

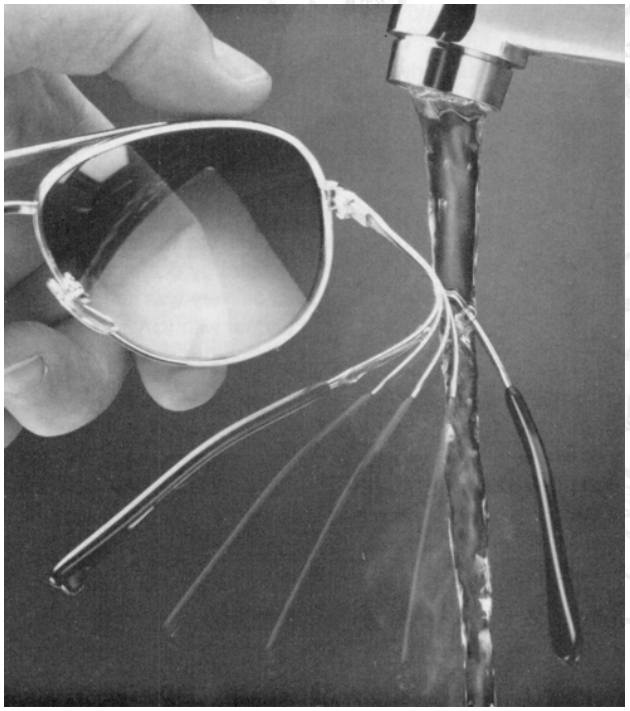
Advantages : application of a constant and low intensity force

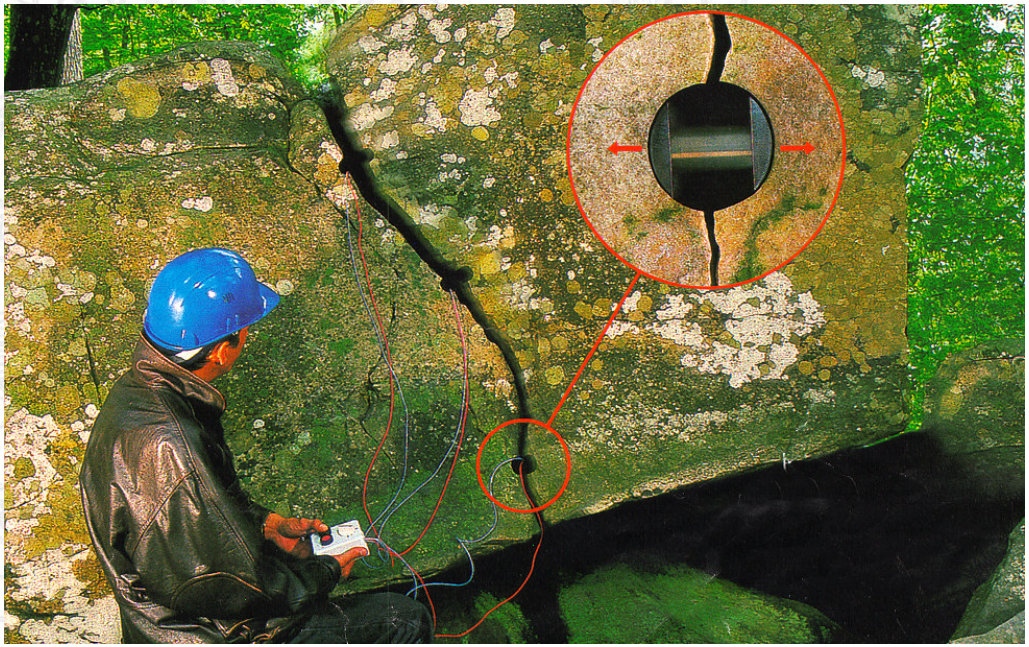
reduced tissue damage and increased remodeling speed

Shape-memory effect: applications

Shape-memory effect (SME) :



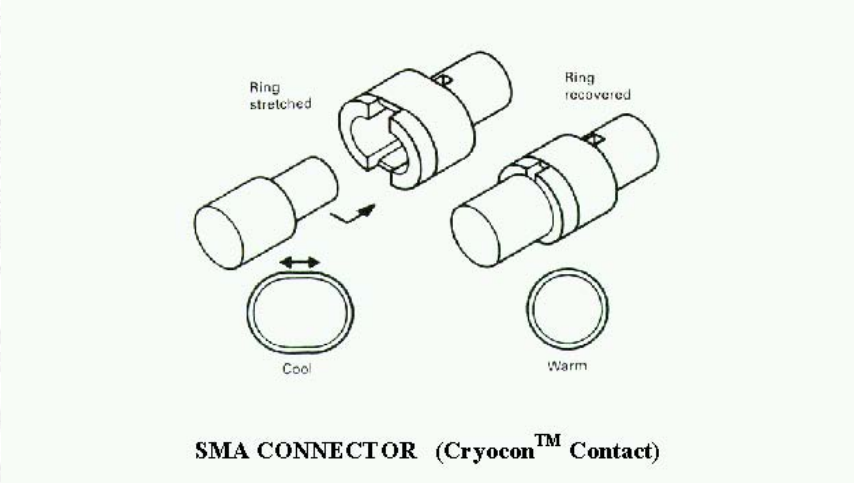




Shape-memory effect: connectors

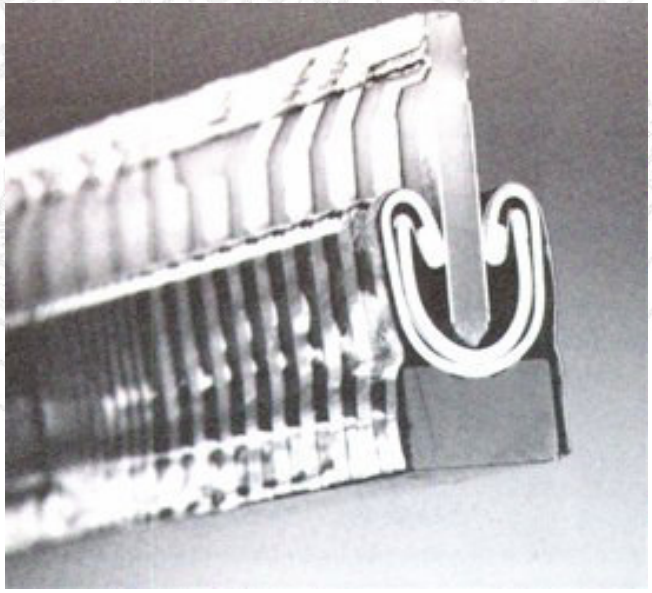
Most known application:
constrained recovery

Recovery constrained by a rigid element

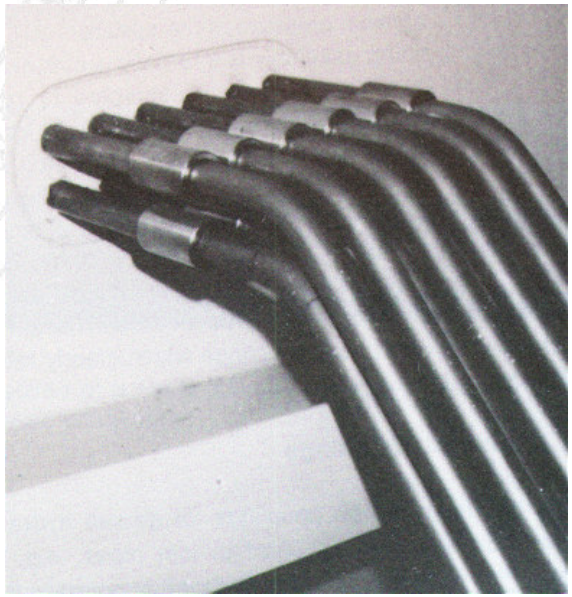


Different materials

High precision

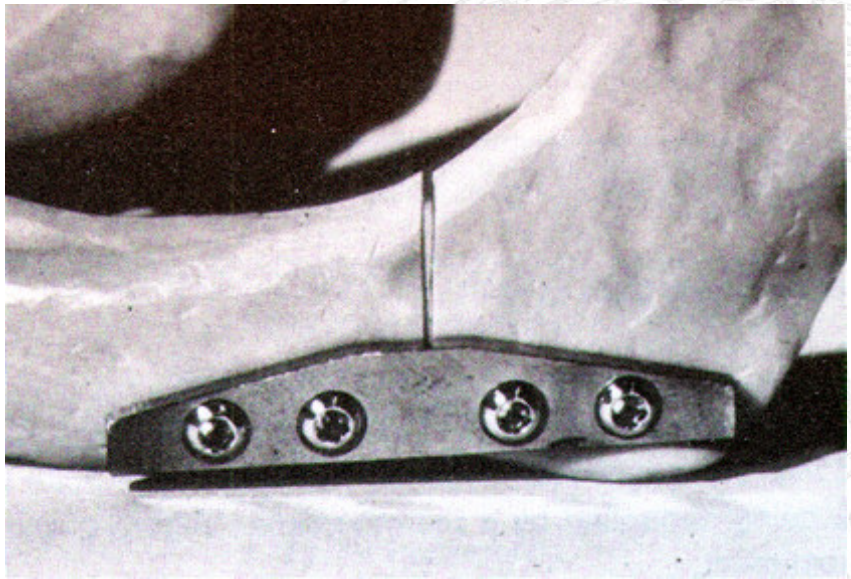


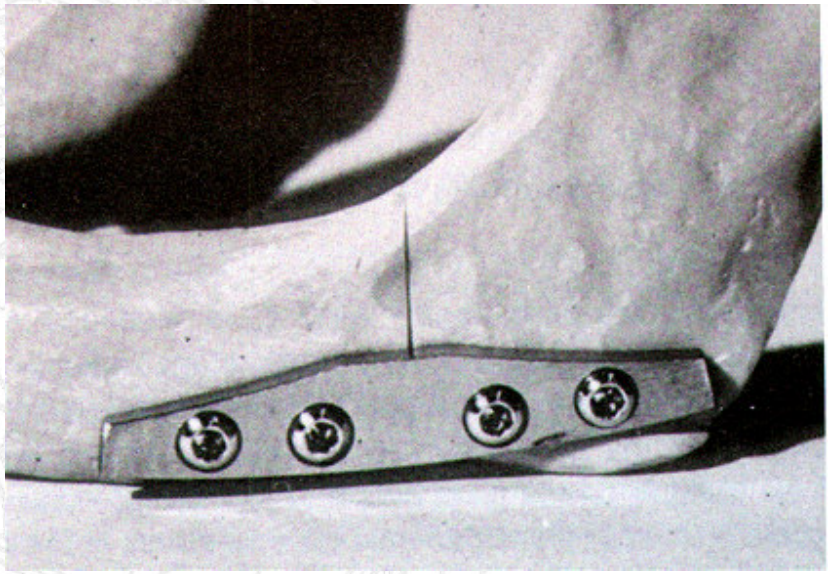
Easy use



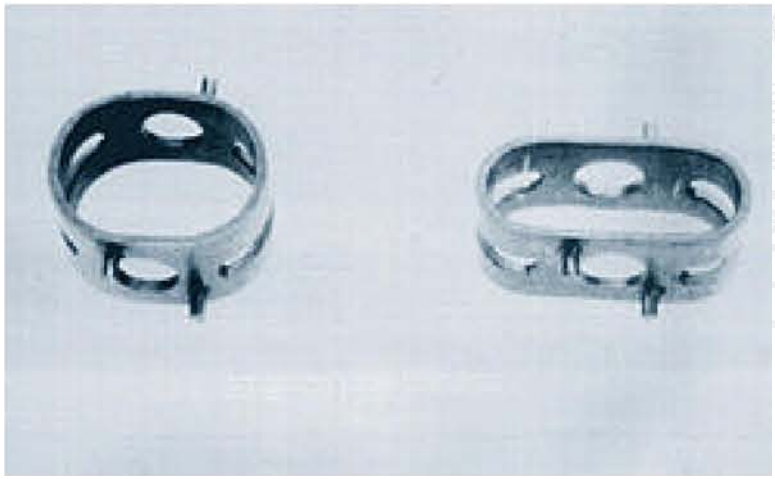
Shape-memory effect: orthopedics

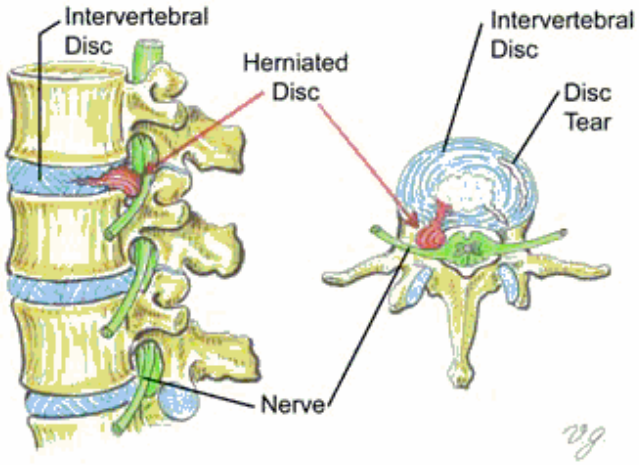
Cambers : *elements to help bone healing*





Spinal fixtures: *devices for the treatment of herniated disks*



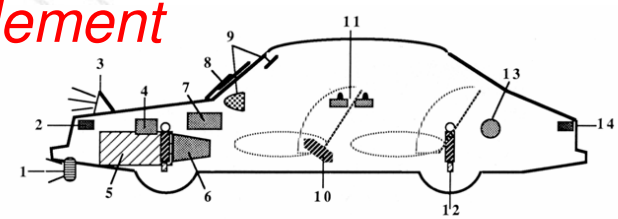


Shape-memory effect: automotive

Coupling between : a wire with shape-memory effect

a spring-like deformable element

⇨ *Two-way SME (repeatable)*

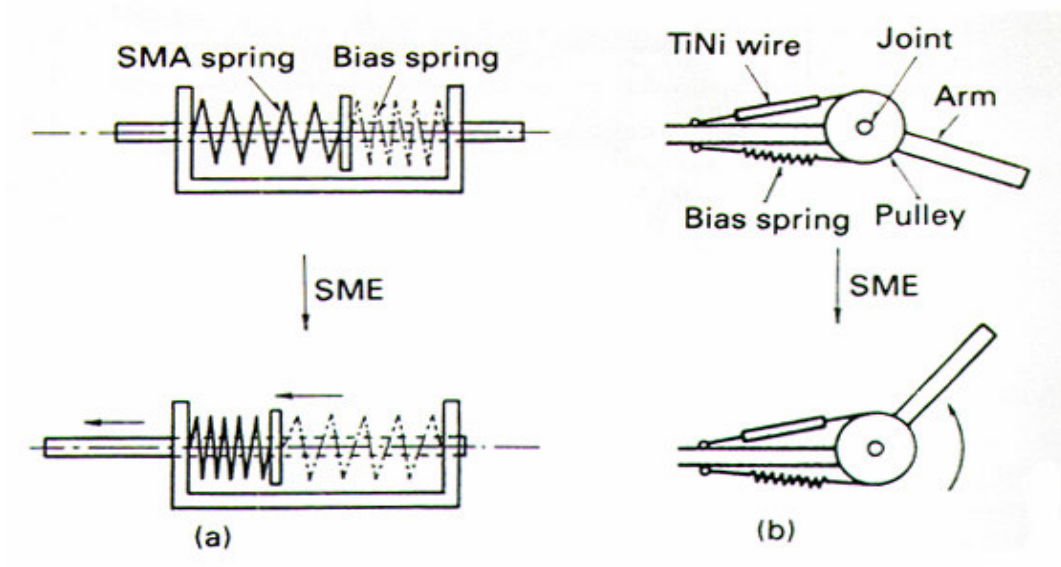


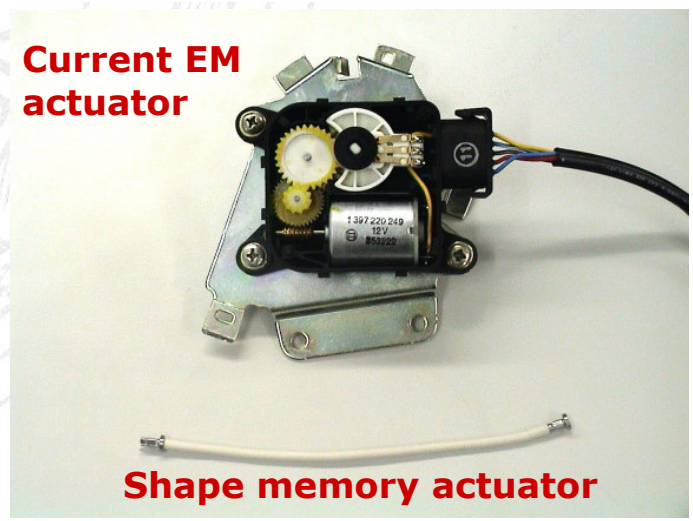
1 Foglamp Louver
2 Engine Hood Lock
3 Retractable Head-Light
4 Fuel Management
5 Engine Control
6 Transmission Control
7 Climate Control
8 Wiper Pressure Control
9 Rear-View Mirror Adjustement
10 Seat-Belt Adjustement
11 Central Locking System
12 Shock Absorber Adjustment
13 Filler Inlet Lock
14 Trunk Lock

Figure 12: Potential applications for electrical shape memory actuators in automobiles.

Linear actuators

Angular actuators

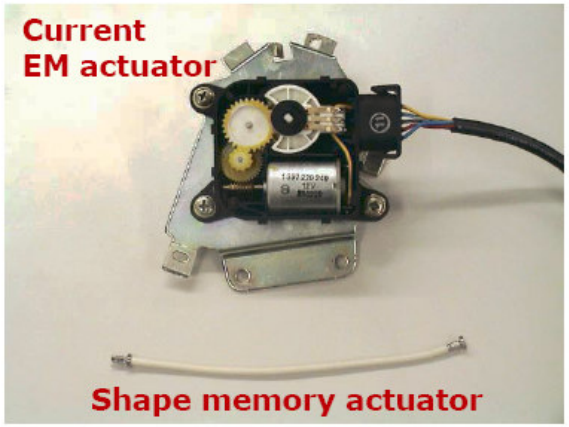




A collaboration with an Italian company: SAES getters

Shape Memory Alloys for innovative micro-actuators

Few centimeter of shape memory wire
can replace bulky electromagnetic actuators



Main advantages:

- Compactness and flexibility
- Reduced costs
- Direct linear or angular movement
- High reliability
- Noiseless operation
- Work in "harsh environment"
- No EMI

	Attuatore a memoria di forma	Attuatore elettromagnetico
Corsa di attuazione	4 - 6 mm	6 - 8 mm
Forza max di attuazione	60 N	35 N
Tempo di risposta	< 0,2 s	< 0,5 s
Temperatura di esercizio	-30 °C + +80 °C	-30 °C + +80 °C
Cicli	> 100000	> 50000
Volume	10 cm^3	60 cm^3
Peso	15 g	75 g
Alimentazione	12 V DC	12 V DC

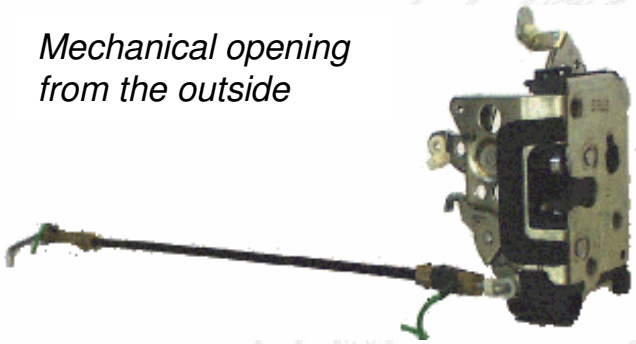
SMA linear actuator vs electromagnetic
Performances comparison

we support your innovation

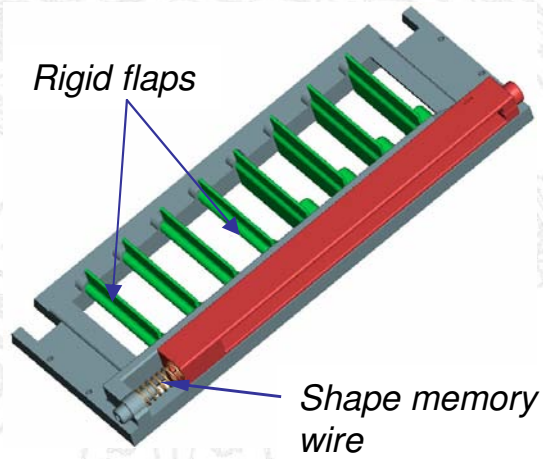
saes getters

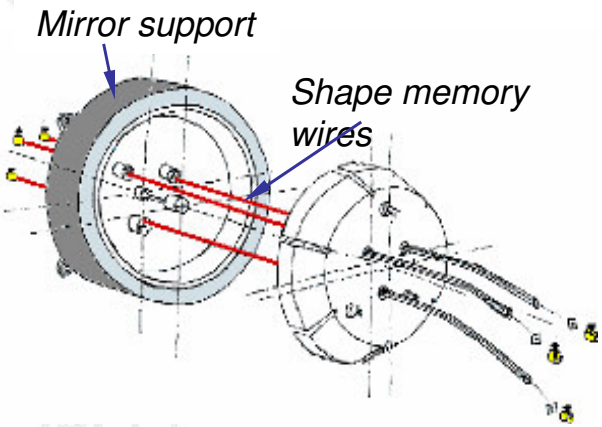
SAES getters: some ideas in automotive industry

From concept design…

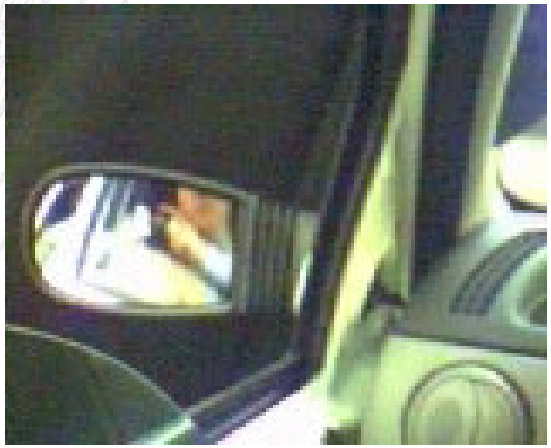


Internal electrical and mechanical opening based on SMA bowden

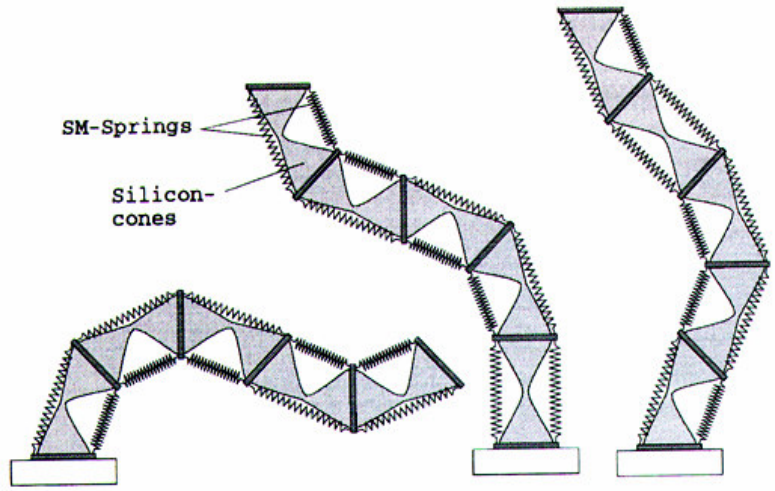


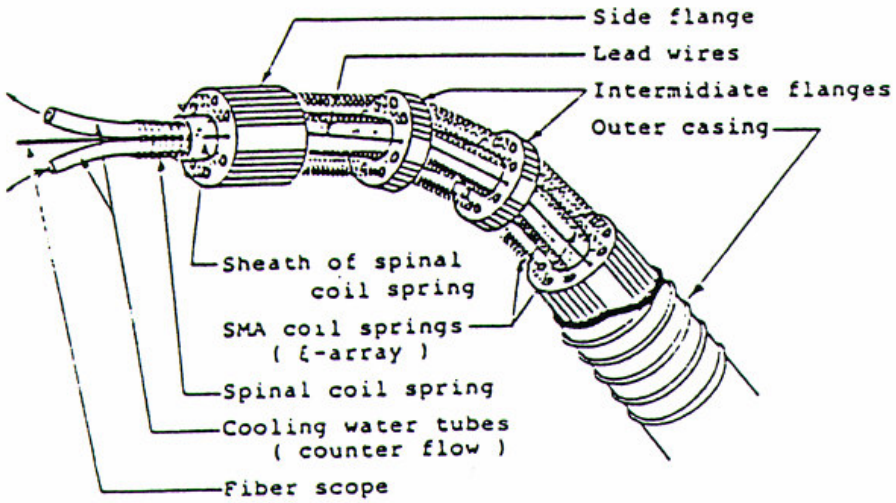


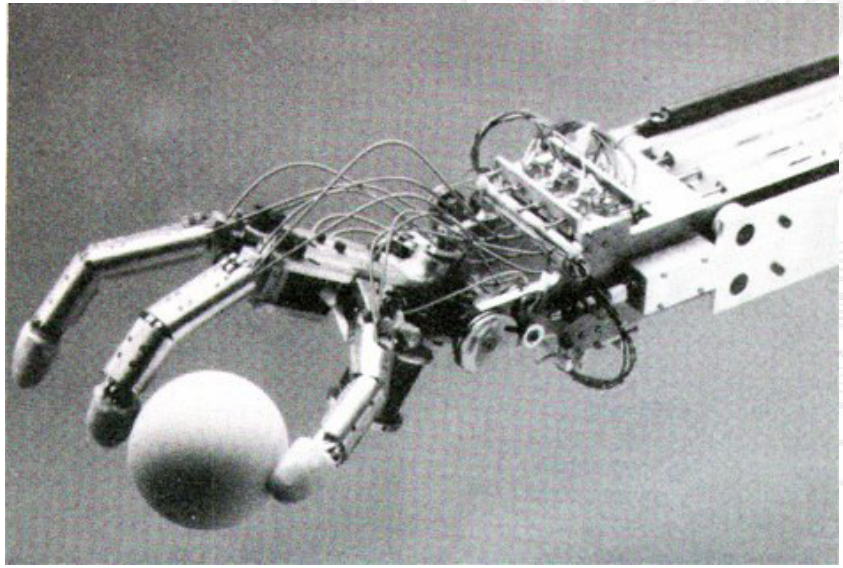
…to prototypes:

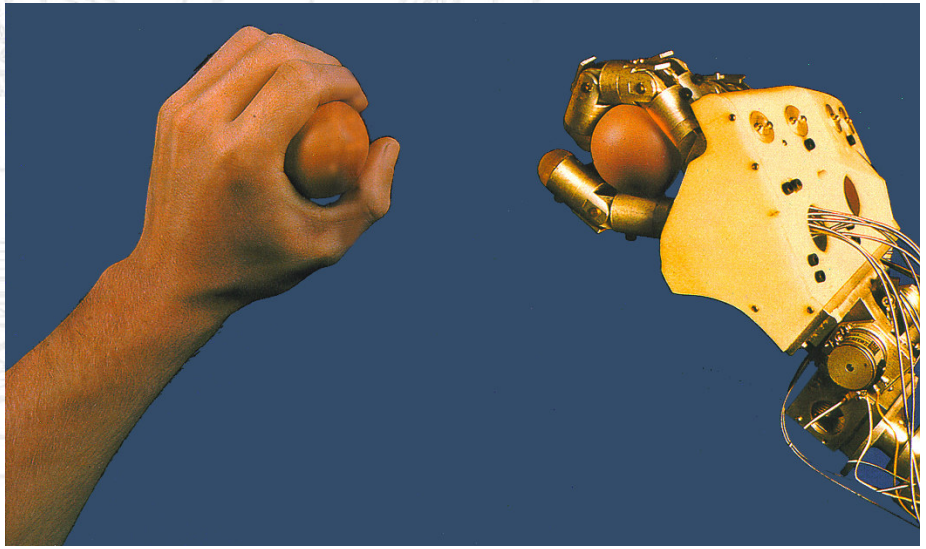


Shape-memory effect: actuators









Shape-memory alloys: micro-mechanics

Key aspect : reversible crystallographic transformation

solid-solid phase transformation

martensitic phase transformation [no diffusion]

Martensite : stable at lower temperatures

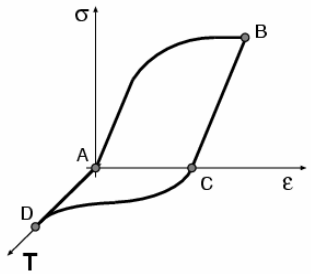
lower crystallographic symmetry

Austenite : stable at higher temperatures

higher crystallographic symmetry

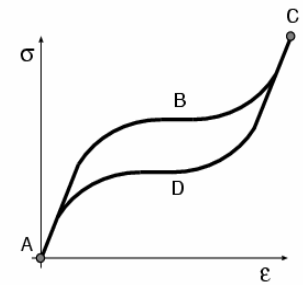
Shape-memory alloys: micro-mechanics

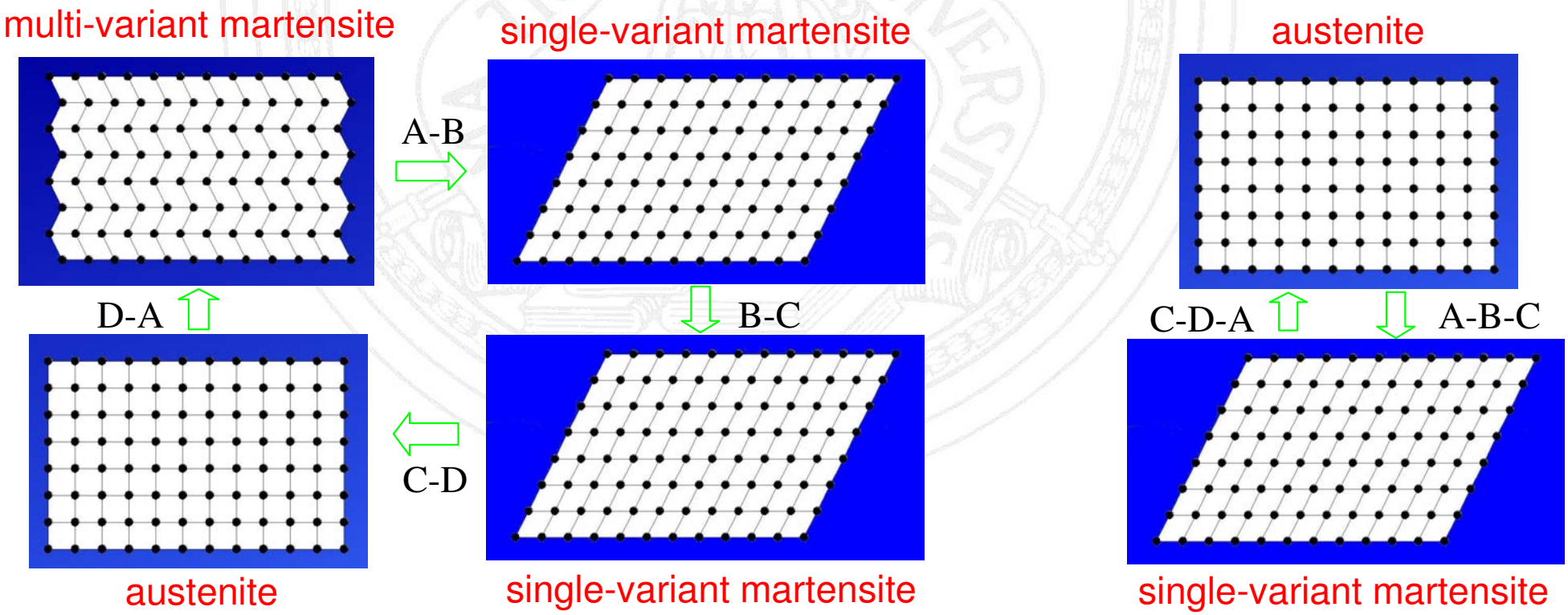
Thermo-elastic martensitic transformation crystallographically reversible
(diffusionless, first type, athermal, two phases, hysteresis)



Austenite
stable at higher temperatures $T>A_f$

Martensite
stable at lower temperatures $T<M_f$
single/multi variant





Shape-memory alloys: micro-mechanics

Single crystal transformation: change of crystallographic group

Crystallographic theories : *compute the deformation gradient to obtain the change of group symmetry* *(Wayman 1964)*

PROBLEM : *interaction between product and parent phase*

Single-variant martensite *i.e.* *with a preferred direction (stress)*

Multiple-variant martensite *i.e* *with no preferred direction*

PROBLEM : *interaction also between different grains and other micro-structure*

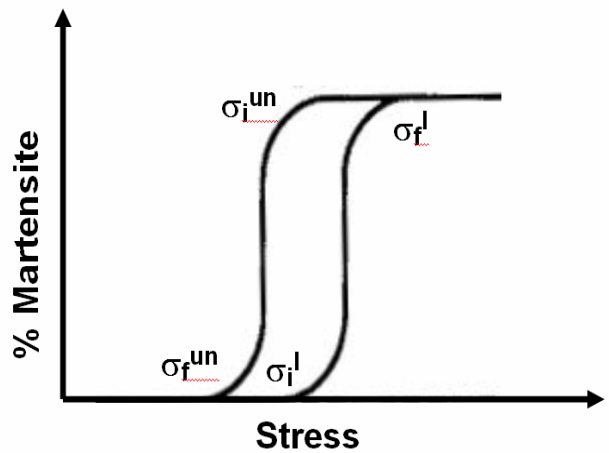
Approach the problem from the micro-mechanics level

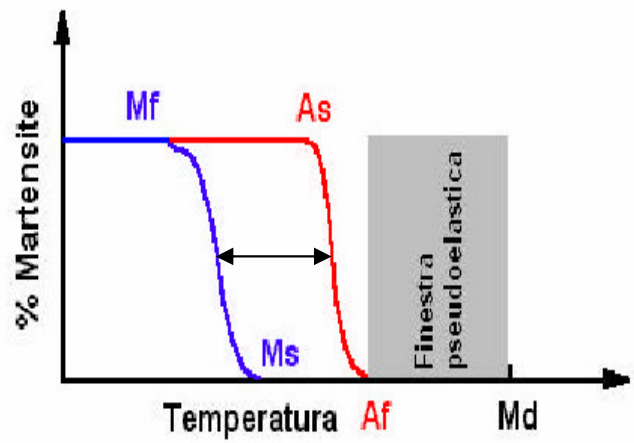
to build a macroscopic model is quite complex

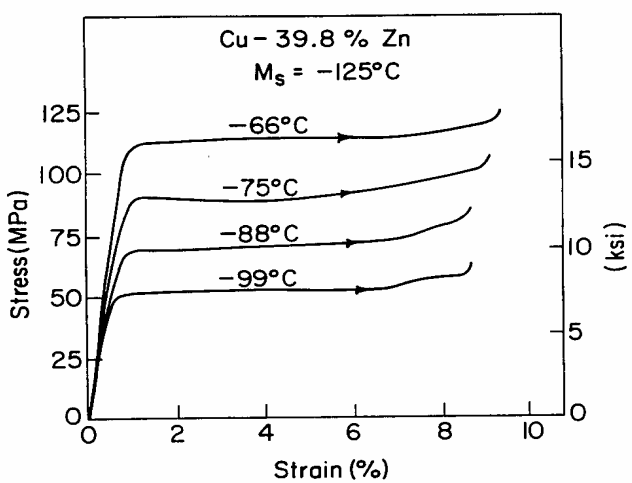
Shape-memory alloys: thermodynamical aspects

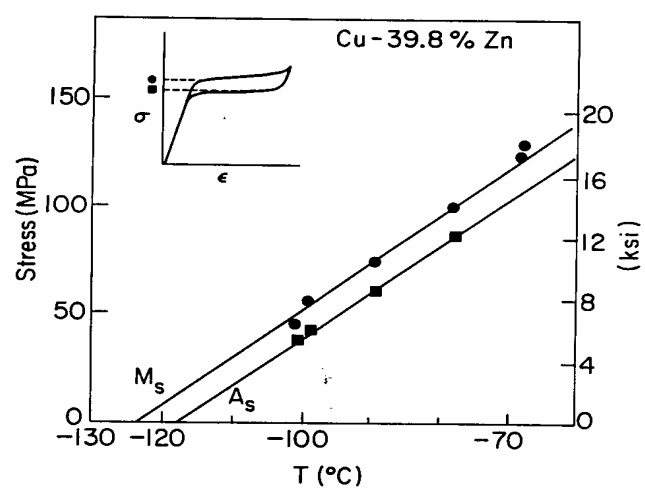
Strong thermo-mechanical coupling :

- balance between chemical (thermal component) and mechanical (elastic component) thermodynamical forces characterize PTs
- temperature and stress are thermodynamically equivalent driving forces



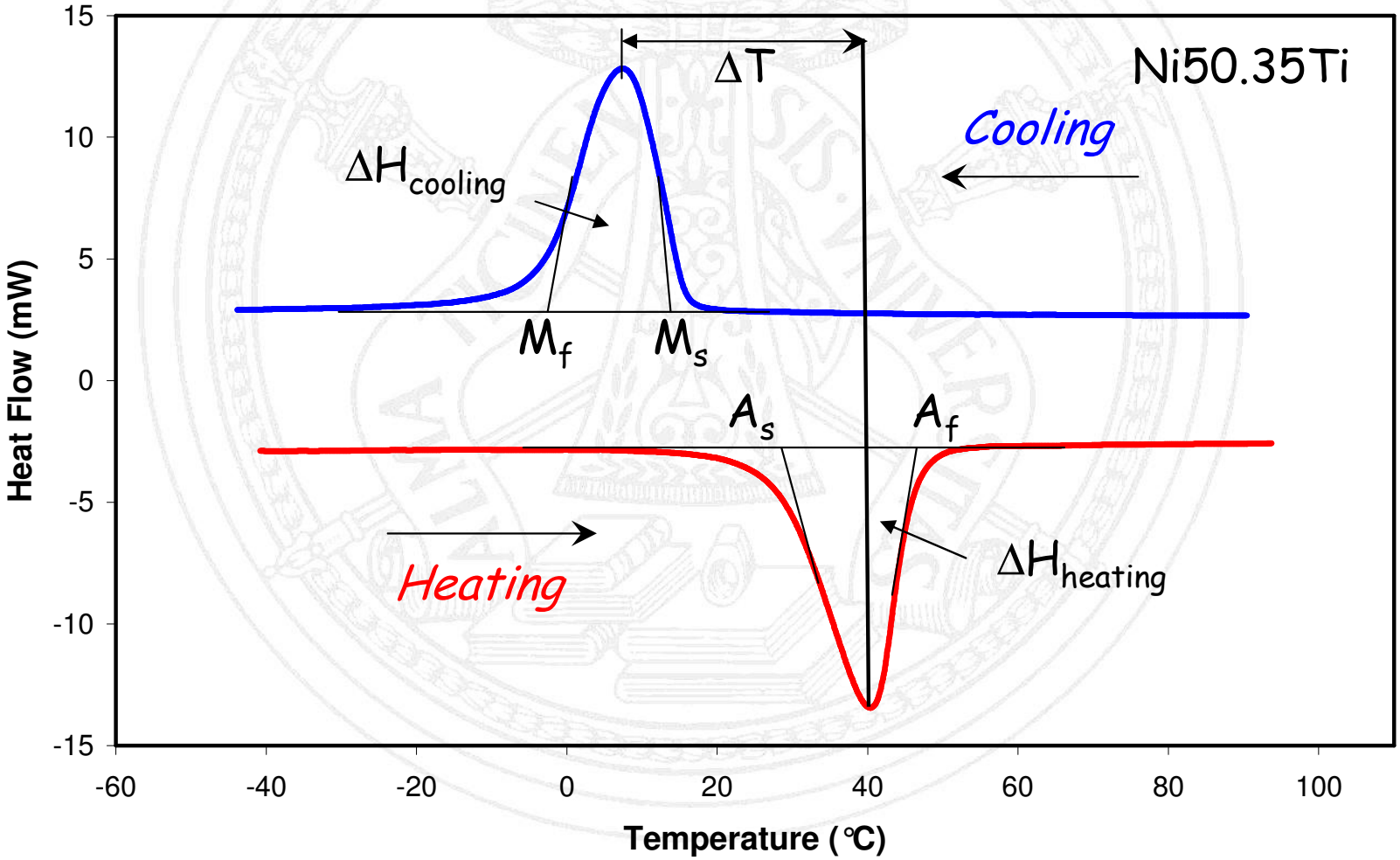






Shape-memory alloys: thermomechanical coupling

Latent heat release/absorption as well as body thermal exchange with surroundings may strongly influence body temperature



Versus SMA constitutive models

- Important to have a 3D computational tool
  - ✓ to perform structural analysis of existing devices
  - ✓ to perform structural analysis of new possible devices

- Complex constitutive behavior
  - ✓ Mechanical response
  - ✓ Thermo-mechanical coupling

We prefer to approach the problem directly

from

a macroscopic phenomenological point of view

⇨ Models non always available in commercial codes

⇨ Engineers play a fundamental role ➔ correct design

Constitutive model

Souza et al. (EJM 1998) Auricchio & Petrini (IJNME 2002 / 2004 …)

3D phenomenological model: generalized standard materials

✓ **Control variables** → strain ε, temperature T

✓ **Internal variables** → transformation strain $\mathbf{e}^{tr}$

- second order tensor
- traceless, following experimental evidences
- dealing only with a single internal variable second-order tensor, <u>at most</u> the model may distinguish between
  - <u>generic parent phase</u> (not associated to macroscopic strain)
  - <u>generic product phase</u> (associated to a macroscopic strain)
- need to satisfy a constraint (complete phase transformation)

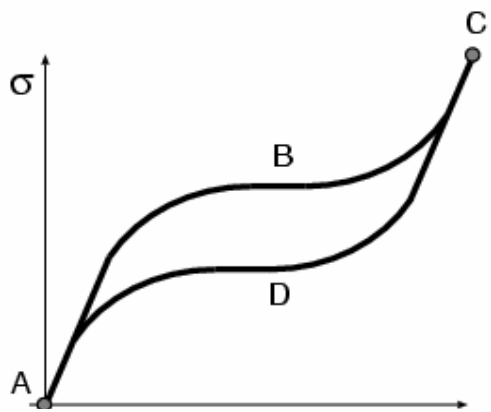
$$0 \leq \| \mathbf{e}^{tr} \| \leq \varepsilon_L$$

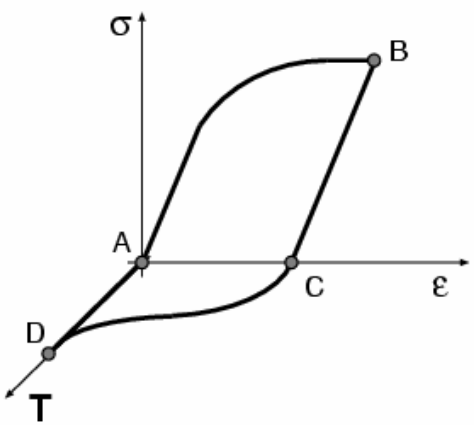
✓ **Convex potentials** → constitutive and evolutive laws

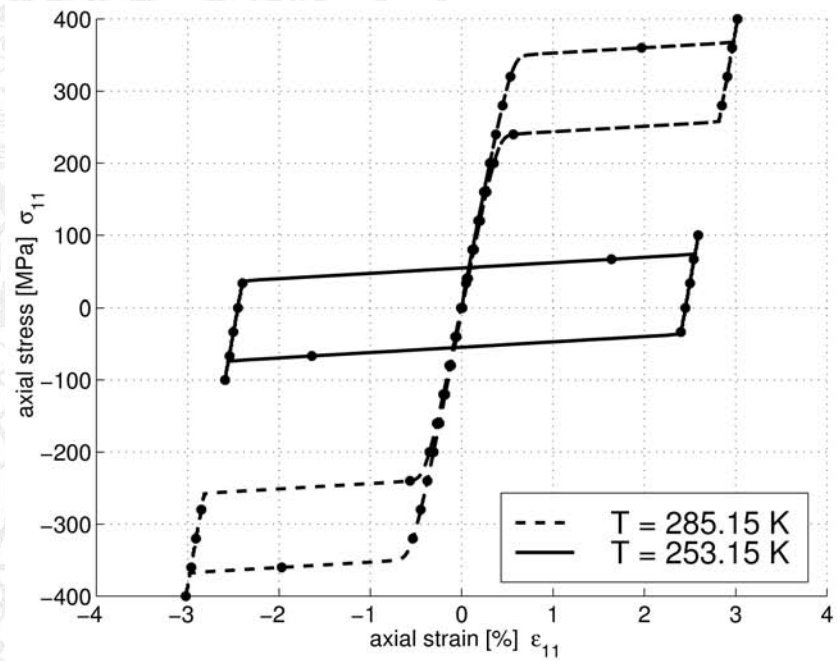
- Helmholtz free-energy + dissipation pseudo-potential

Constitutive model: toward a plasticity-like mode

Tension-compression







Stress increment per step :

40 Mpa (dots); 2 Mpa (lines)

Le leghe a memoria di forma: un modello 1D

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1D constitutive modeling

Helmholtz free-energy: $\psi = \psi_{el} + \psi_{ch} + \psi_{tr} + I_{\varepsilon_L}$

$$\begin{cases} \psi_{el} = \dfrac{1}{2} E \left(\varepsilon - \varepsilon^{tr} \right)^2 \\ \psi_{ch} = \beta \left\langle T - T^* \right\rangle \left| \varepsilon^{tr} \right| \end{cases} \qquad \begin{cases} \psi_{tr} = \dfrac{h}{2} \left| \varepsilon^{tr} \right|^2 \\ I_{\varepsilon_L} = \begin{cases} 0 & \text{if } \left| \varepsilon^{tr} \right| \le \varepsilon_L \\ +\infty & \text{otherwise} \end{cases} \end{cases}$$

- **Elastic energy**: due to thermo-elastic material deformation
  [E: elastic modulus]
- **Chemical energy**: due to thermally-induced martensitic transformation
  [β: dependence of critical stress on temp.; T*: reference temp.; < • >: positive part]
- **Transformation energy**: due to transformation-induced hardening
  [*h:* slope of linear stress-transformation strain relation in uni-axial case]
- **Indicator function**: introduced to satisfy constraint on transformation strain norm

1D constitutive modeling

$$\psi = \frac{1}{2}E\left(\varepsilon - \varepsilon^{tr}\right)^2 + \beta\left\langle T - T^*\right\rangle\left|\varepsilon^{tr}\right| + \frac{h}{2}\left|\varepsilon^{tr}\right|^2 + I_{\varepsilon_L}\left(\varepsilon^{tr}\right)$$

- ✓ Following standard arguments, we derive constitutive equations

$$\sigma = \frac{\partial\psi}{\partial\varepsilon} = E\left(\varepsilon - \varepsilon^{tr}\right)$$

- ✓ Derive also <u>thermodynamic force</u> X associated to internal variable ε^{tr}

$$X = -\frac{\partial\psi}{\partial\varepsilon^{tr}} = \sigma - \left[\beta\left\langle T - T^*\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$

- ✓ **X** : interpretable as “relative stress” (classical plasticity)

$$X = \sigma - \alpha$$

with α back stress $\alpha = \left[\beta\left\langle T - T^*\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$

1D constitutive modeling

✓ Compute differential of indicator function

$$I_{\varepsilon_L} = \begin{cases} 0 & \text{if } \left|\varepsilon^{tr}\right| \le \varepsilon_L \\ +\infty & \text{otherwise} \end{cases}$$

hence

$$\gamma = \frac{\partial I_{\varepsilon_L}}{\partial \left|\varepsilon^{tr}\right|} = \begin{cases} 0 & \text{if } \left|\varepsilon^{tr}\right| < \varepsilon_L \\ +\mathcal{R} & \text{if } \left|\varepsilon^{tr}\right| = \varepsilon_L \\ \varnothing & \text{if } \left|\varepsilon^{tr}\right| > \varepsilon_L \end{cases} \Leftrightarrow \begin{cases} \gamma = 0 & \text{if } 0 < \left|\varepsilon^{tr}\right| < \varepsilon_L \\ \gamma \ge 0 & \text{if } \left|\varepsilon^{tr}\right| = \varepsilon_L \end{cases}$$

Model completed through the following ingredients

- ✓ Limit function $$F(X) = |X| - R$$
- ✓ Associative flow rule $$\dot{\varepsilon}^{tr} = \dot{\zeta}\,\frac{\partial F}{\partial X}$$
- ✓ Kuhn-Tucker condition $$\dot{\zeta} \geq 0 \;,\; \mathrm{F} \leq 0 \;,\; \dot{\zeta} F = 0$$

- *R:* elastic domain radius

Complete 1D time-continuous model

- ✓ Stress definition $$\sigma = E\left(\varepsilon - \varepsilon^{tr}\right)$$
- ✓ Thermodyn. force $$X = \sigma - \left[\beta\left\langle T - T^{*}\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$
- ✓ Limit function $$F(X) = |X| - R$$
- ✓ Associative flow rule $$\dot{\varepsilon}^{tr} = \dot{\zeta}\,\frac{\partial F}{\partial X}$$
- ✓ Kuhn-Tucker condition $$\dot{\zeta} \geq 0 \;,\; F \leq 0 \;,\; \dot{\zeta} F = 0$$
- ✓ Constraint on PT $$\gamma \geq 0$$

Time-integration

$$\sigma = E\left(\varepsilon - \varepsilon^{tr}\right)$$

$$X = \sigma - \left[\beta\left\langle T - T^{*}\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$

$$F(X) = |X| - R$$

$$\gamma \geq 0$$

$$\dot{\varepsilon}^{tr} = \dot{\zeta}\frac{\partial F}{\partial X}$$

$$\dot{\zeta} \geq 0 \;,\; F \leq 0 \;,\; \dot{\zeta} F = 0$$

$$\rightarrow$$

$$\varepsilon^{tr} = \varepsilon_{n}^{tr} + \lambda\frac{X}{|X|}$$

$$\lambda \geq 0 \;,\; F \leq 0 \;,\; \lambda F = 0$$

Complete 1D time-discrete model

$$\sigma = E\left(\varepsilon - \varepsilon^{tr}\right)$$

$$X = \sigma - \left[\beta\left\langle T - T^{*}\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$

$$F(X) = |X| - R$$

$$\varepsilon^{tr} = \varepsilon_{n}^{tr} + \lambda\frac{X}{|X|}$$

$$\lambda \geq 0 \quad , \quad F \leq 0 \quad , \quad \lambda F = 0$$

$$\gamma \geq 0$$

Non trivial aspect: obtained non-linear algebraic system is highly non-linear due to the presence of **<u>2 constraints</u>**

Constraint due to limit function
Constraint due to phase transformation

1D constitutive modeling

Return map:
apply predictor-corrector
2 times

```
assume elastic step

if (elastic predictor non admissable)
      assume non-saturated PT

If non-saturated PT non admissable
      assume saturated PT
```

```
compute elastic predictor
if (elastic predictor admissible) then
   solution found !!
else
   compute non-saturated PT predictor
   if (non-saturated PT admissible) then
      solution found !!
   else
      compute saturated PT predictor
      check solution
   end
end
```

1D constitutive modeling

Elastic step

$$\lambda = 0 \qquad \gamma = 0$$

$$\varepsilon^{tr,TR} = \varepsilon_n^{tr}$$

$$\sigma^{TR} = E\left(\varepsilon - \varepsilon^{tr,TR}\right)$$

$$X^{TR} = \sigma^{TR} - \left[\beta\left\langle T - T^*\right\rangle + h\left|\varepsilon^{tr,TR}\right|\right]\frac{\varepsilon^{tr,TR}}{\left|\varepsilon^{tr,TR}\right|}$$

$$F^{TR} = F\left(X^{TR}\right) = \left|X^{TR}\right| - R$$

Check: $F^{TR} < 0$

1D constitutive modeling

Non saturated PT

$$\lambda > 0 \quad \gamma = 0$$

$$\sigma = E\left(\varepsilon - \varepsilon^{tr}\right)$$

$$X = \sigma - \left[\beta\left\langle T - T^{*}\right\rangle + h\left|\varepsilon^{tr}\right|\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$

$$F(X) = |X| - R$$

$$\varepsilon^{tr} = \varepsilon_{n}^{tr} + \lambda\frac{X}{|X|}$$

Check: $|\varepsilon^{TR}| < \varepsilon_L$

1D constitutive modeling

Saturated PT

$$\lambda > 0 \quad \gamma > 0$$

$$\sigma = E\left(\varepsilon - \varepsilon^{tr}\right)$$

$$X = \sigma - \left[\beta\left\langle T - T^{*}\right\rangle + h\left|\varepsilon^{tr}\right| + \gamma\right]\frac{\varepsilon^{tr}}{\left|\varepsilon^{tr}\right|}$$

$$F(X) = |X| - R$$

$$\varepsilon^{tr} = \varepsilon_n^{tr} + \lambda\frac{X}{|X|}$$

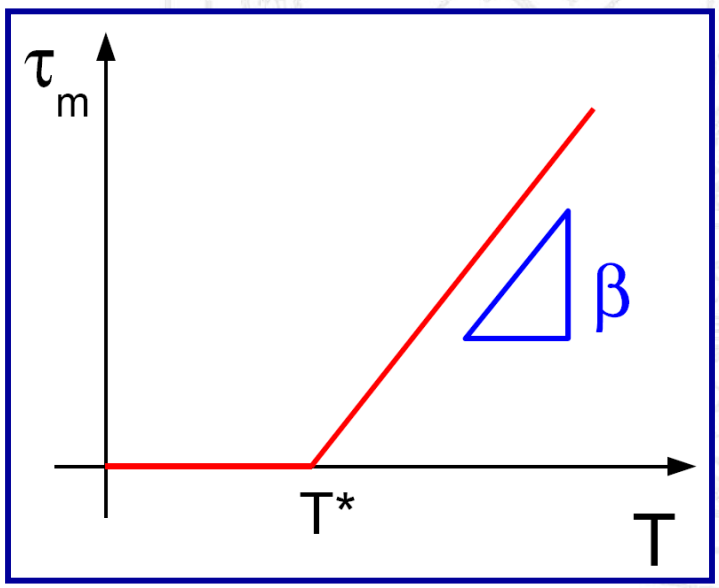
$$\left|\varepsilon^{tr}\right| = \varepsilon_L$$

A collaboration with an Italian company: SAES getters

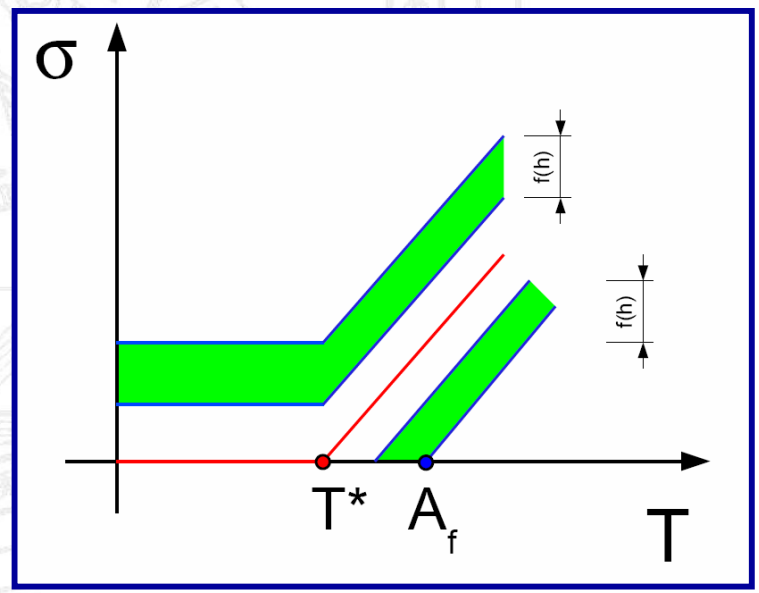
GOAL: 1. interpretation of model parameter in a 1D setting
2. calibration with SAES material

E:	elastic modulus
h:	slope of linear stress-transformation strain relation
T*:	reference temperature
β :	dependence of critical stress on temperature
R :	hysteresis width

$\tau_m = \beta < T - T^* >$ with $< \bullet >$: positive part

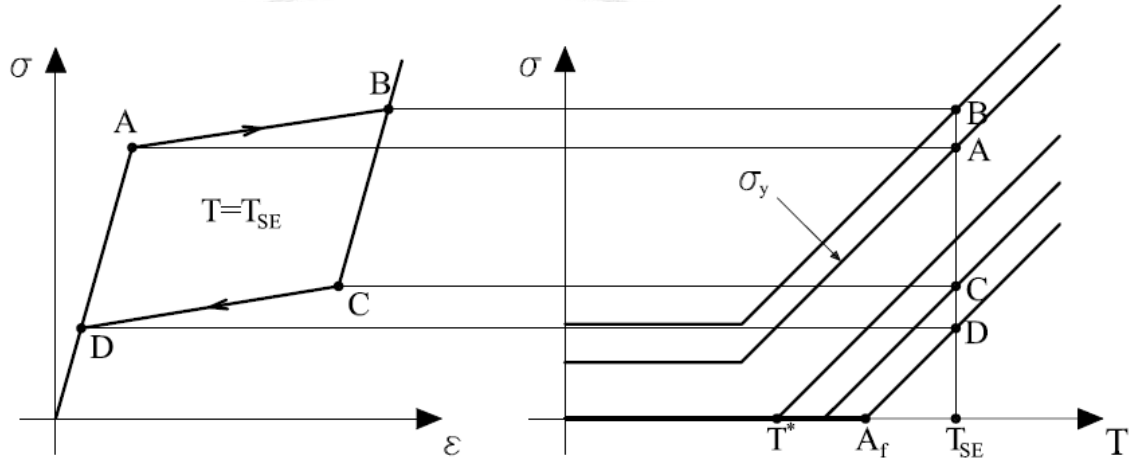


Temperature dependence for τ_m

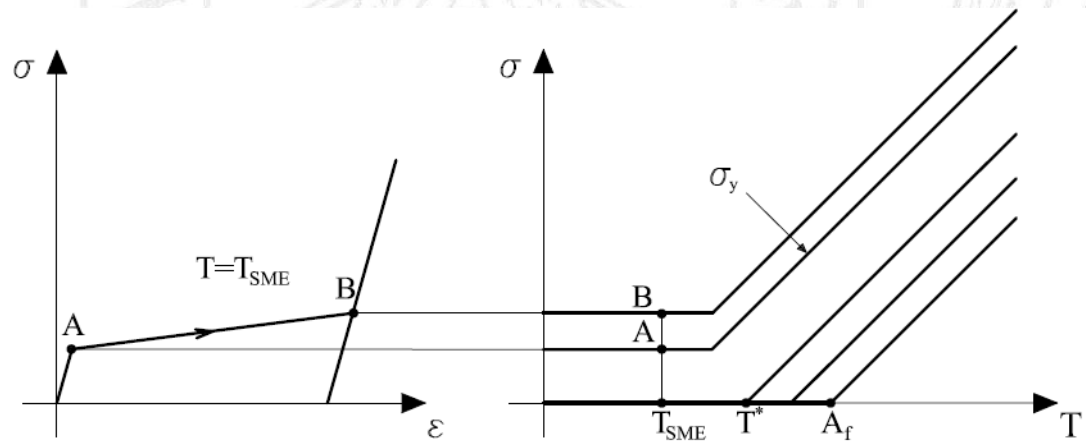


Temperature dependence for σ_y
phase transformation stress activation

SAES getters: reproduction of basic effects



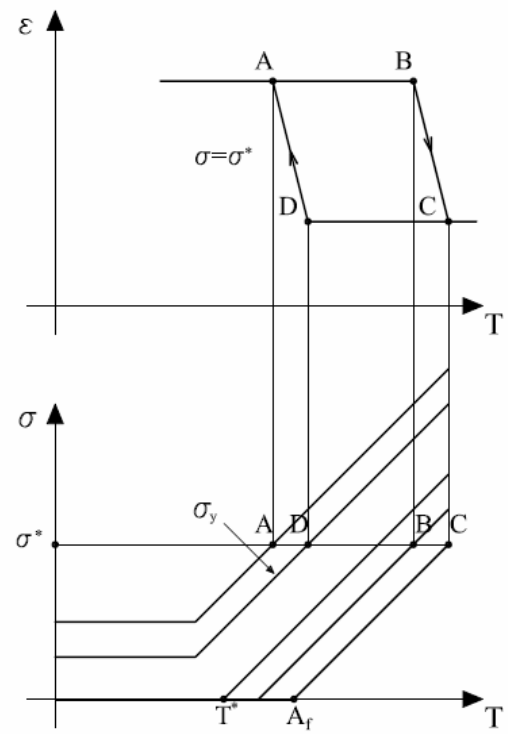
Superelastic test (at constant temperature $T = T_{SE}$): stress-strain diagram (left) and stress-temperature diagram (right)



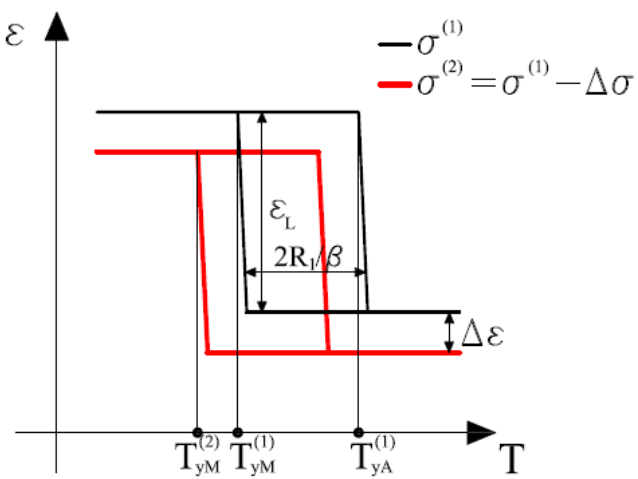
Shape-memory test (at constant temperature $T = T_{SME}$): stress-strain diagram (left) and stress-temperature diagram (right)

SAES getters: reproduction of basic effects

ε – T test (at constant stress σ *): strain-temperature diagram (top) and stress-temperature diagram (bottom)



Model parameter calibration: given 2 strain-temperature curves at 2 constant stresses $\sigma^1 > \sigma^2$, identify in a simple way all needed material parameters

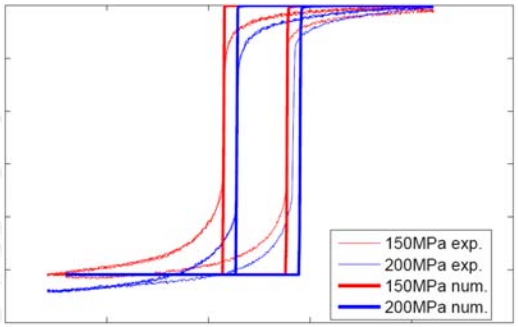


SAES getters: “local” round-robin test

Experimental vs numerical curves relative to wires @ 150 and 200 MPa;

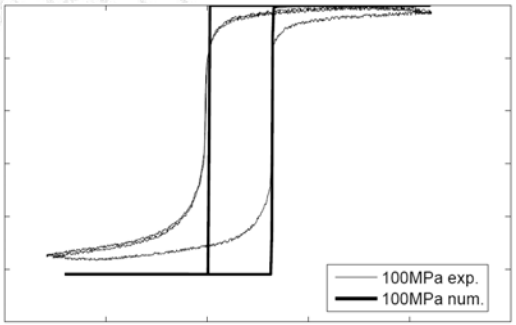
numerical curve @ 150 MPa: fully fitted

numerical curve @ 200 MPa: partially fitted

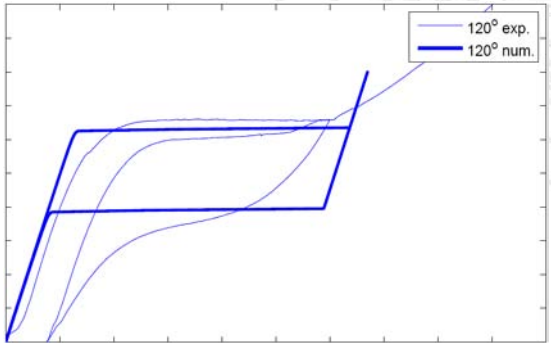


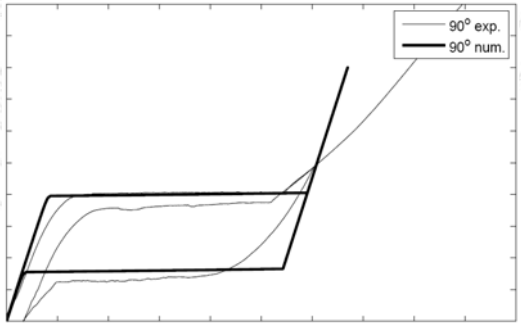
Experimental vs numerical curves relative to 0.2mm wires at 100 MPa

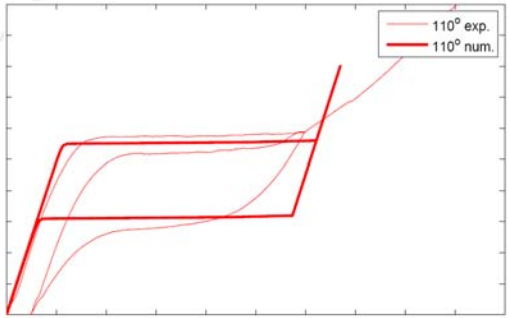
numerical curve fully predicted



Stress-strain tests @ different temperatures: numerical curve fully predicted







Le leghe a memoria di forma: un modello 3D

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Constitutive model

Souza et al. (EJM 1998) Auricchio & Petrini (IJNME 2002 / 2004 …)

3D phenomenological model: generalized standard materials

✓ **Control variables** → strain ε, temperature T

✓ **Internal variables** → transformation strain $\mathbf{e}^{tr}$

- second order tensor
- traceless, following experimental evidences
- dealing only with a single internal variable second-order tensor, <u>at most</u> the model may distinguish between
  - <u>generic parent phase</u> (not associated to macroscopic strain)
  - <u>generic product phase</u> (associated to a macroscopic strain)
- need to satisfy a constraint (complete phase transformation)

$$0 \leq \| \mathbf{e}^{tr} \| \leq \varepsilon_L$$

✓ **Convex potentials** → constitutive and evolutive laws

- Helmholtz free-energy + dissipation pseudo-potential

3D constitutive modeling

Helmholtz free-energy: $\psi = \psi_{el} + \psi_{ch} + \psi_{tr} + \psi_{id} + I_{\varepsilon_L}$

$$\begin{cases} \psi_{el} = \dfrac{1}{2} K\theta^2 + G\left\|\mathbf{e} - \mathbf{e}^{tr}\right\|^2 \\ \qquad - 3\,\alpha K\theta\,(T - T_0) \\ \psi_{ch} = \tau_m(T)\left\|\mathbf{e}^{tr}\right\| \end{cases}$$

$$\begin{cases} \psi_{tr} = \dfrac{h}{2}\left\|\mathbf{e}^{tr}\right\|^2 \\ \psi_{id} = (u_0 - T\eta_0) + c\left[(T - T_0) - T\log(T/T_0)\right] \\ I_{\varepsilon_L} = \begin{cases} 0 & \text{if } \left\|\mathbf{e}^{tr}\right\| \le \varepsilon_L \\ +\infty & \text{otherwise} \end{cases} \end{cases}$$

- **Elastic energy**: due to thermo-elastic material deformation $\boldsymbol{\varepsilon} = \mathbf{e} + (\theta/3)\,\mathbf{1}$
  [*K, G:* bulk & shear modulus; α: thermal expansion coefficient; T_0: reference temp.]
- **Chemical energy**: due to thermally-induced martensitic transformation [$\tau_m = \beta < T - T^* >$ with β: dependence of critical stress on temp.; T^*: reference temp.; $< \bullet >$: positive part]
- **Transformation energy**: due to transformation-induced hardening
  [*h:* slope of linear stress-transformation strain relation in uni-axial case]
- **Ideal free energy**: due to changes in temp. wrt reference temp. in incompressible ideal solid
  [*c:* heat capacity, u_0 and η_0: internal energy and entropy at the reference state]
- **Indicator function**: introduced to satisfy constraint on transformation strain norm

3D constitutive modeling

$$\psi = \frac{1}{2}K\theta^2 + G\left\|\mathbf{e}-\mathbf{e}^{tr}\right\|^2 - 3\,\alpha K\theta\,\left(T - T^*\right) + \beta\left\langle T - M_f\right\rangle\left\|\mathbf{e}^{tr}\right\| +$$

$$+\frac{h}{2}\left\|\mathbf{e}^{tr}\right\|^2 + \left(u_0 - T\eta_0\right) + c\left[\left(T - T_0\right) - \log(T/T_0)\right] + I_{\varepsilon_L}$$

✓ Following standard arguments, we derive constitutive equations

$$\begin{cases} p = \dfrac{\partial\psi}{\partial\theta} = K\left[\theta - 3\,\alpha\left(T - T_0\right)\right] \\ \mathbf{s} = \dfrac{\partial\psi}{\partial e} = 2G\left(\mathbf{e}-\mathbf{e}^{tr}\right) \\ \eta = -\dfrac{\partial\psi}{\partial T} = \eta_0 + 3\alpha K\theta - \beta\left\|\mathbf{e}^{tr}\right\|\dfrac{\left\langle T - T^*\right\rangle}{\left|T - T^*\right|} + c\left(T - T_0\right) \end{cases}$$

p and **s:** volumetric and deviatoric part of the stress η: entropy

3D constitutive modeling

- ✓ Derive also thermodynamic force $\mathbf{X}$ associated to internal variable $\mathbf{e}^{tr}$

$$\mathbf{X} = -\frac{\partial \psi}{\partial \mathbf{e}^{tr}} = \mathbf{s} - \left[\beta\left\langle T - T^{*}\right\rangle + h\|\mathbf{e}^{tr}\| + \gamma\right]\frac{\mathbf{e}^{tr}}{\|\mathbf{e}^{tr}\|}$$

where

$$\gamma = \frac{\partial I_{\varepsilon_L}}{\partial \|\mathbf{e}^{tr}\|} = \begin{cases} 0 & \text{if } \|\mathbf{e}^{tr}\| < \varepsilon_L \\ +\mathcal{R} & \text{if } \|\mathbf{e}^{tr}\| < \varepsilon_L \\ \varnothing & \text{if } \|\mathbf{e}^{tr}\| > \varepsilon_L \end{cases} \Leftrightarrow \begin{cases} \gamma = 0 & \text{if } 0 < \|\mathbf{e}^{tr}\| < \varepsilon_L \\ \gamma \geq 0 & \text{if } \|\mathbf{e}^{tr}\| = \varepsilon_L \end{cases}$$

- ✓ $\mathbf{X}$: interpretable as “relative stress” (classical plasticity)

$$\mathbf{X} = \mathbf{s} - \boldsymbol{\alpha}$$

with $\boldsymbol{\alpha}$ back stress

$$\boldsymbol{\alpha} = \left[\beta\left\langle T - T^{*}\right\rangle + h\|\mathbf{e}^{tr}\| + \gamma\right]\frac{\mathbf{e}^{tr}}{\|\mathbf{e}^{tr}\|}$$

Model completed through the following ingredients

- ✓ Limit function $F(\mathbf{X})$
- ✓ Associative flow rule $\dot{\mathbf{e}}^{tr} = \dot{\zeta}\dfrac{\partial F}{\partial \mathbf{X}}$
- ✓ Kuhn-Tucker condition $\dot{\zeta} \geq 0 \;,\; F \leq 0 \;,\; \dot{\zeta} F = 0$

Limit function example

$$F(\mathbf{X}) = \sqrt{2J_2} + m\frac{J_3}{J_2} - R$$

with J_2 : 2nd X-invariant
J_3 : 3rd X-invariant

$$J_2 = \frac{1}{2}\left(\mathbf{X}^2 : \mathbf{1}\right) \quad , \quad J_3 = \frac{1}{3}\left(\mathbf{X}^3 : \mathbf{1}\right)$$

- Asymmetric experimental behavior in tension / compression (Prager-Lode limit surface)
- *R:* elastic domain radius in deviatoric space; *m < 0.*46 to guarantee limit surface convexity
- *R* and *m* can be related to uniaxial critical stress in tension / compression

Dissipation pseudo-potential : $\phi = \phi(\dot{\mathbf{e}}^{tr}) = \mathrm{R}\left\|\dot{\mathbf{e}}^{tr}\right\|$

Evolutionary equation : $\mathbf{X} \in \dfrac{\partial \phi}{\partial \dot{\mathbf{e}}^{tr}} = \mathrm{R}\,\partial\left\|\dot{\mathbf{e}}^{tr}\right\|$

- ✓ Limit function $F = \|\mathbf{X}\| - R \qquad \mathbf{X} = \mathbf{s} - \alpha$
- ✓ Flow rule $\dot{\mathbf{e}}^{tr} = \dot{\zeta}\,\dfrac{\mathbf{X}}{\|\mathbf{X}\|}$
- ✓ Kuhn-Tucker condition $\dot{\zeta} \geq 0 \qquad \dot{\zeta}\,F(\mathbf{X}) = 0$

Model drawback

non-smooth definition of $\mathbf{X}$ in parent phase ($\mathbf{e}^{tr} = \mathbf{0}$)

$$\mathbf{X} = -\frac{\partial \psi}{\partial \mathbf{e}^{tr}} = \mathbf{s} - \left[\beta \left\langle T - T^{*} \right\rangle + h \, \| \mathbf{e}^{tr} \| + \gamma \right] \frac{\mathbf{e}^{tr}}{\| \mathbf{e}^{tr} \|}$$

Substitute usual Euclidean norm with “regularized norm”

$$\| \mathbf{e}^{tr} \| \quad \sim \quad \overline{\| \mathbf{e}^{tr} \|} = \begin{cases} \| \mathbf{e}^{tr} \| - \dfrac{\eta^{\frac{\eta+1}{\eta}}}{\eta - 1} \left(\| \mathbf{e}^{tr} \| + \eta \right)^{\frac{\eta-1}{\eta}} \\ \\ \sqrt{\| \mathbf{e}^{tr} \|^{2} + \eta} - \sqrt{\eta} \end{cases}$$

where η regularizing factor

Constitutive model: regularization

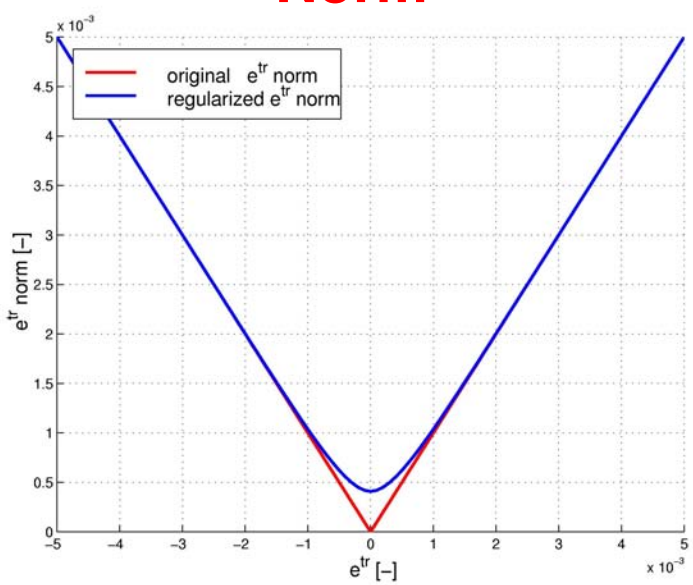
$$\overline{\|\mathbf{e}^{tr}\|} = \|\mathbf{e}^{tr}\| - \frac{\eta^{\frac{\eta+1}{\eta}}}{\eta-1}\left(\|\mathbf{e}^{tr}\|+\eta\right)^{\frac{\eta-1}{\eta}}$$

$$\lim_{\eta\to 0}\overline{\|\mathbf{e}^{tr}\|} = \|\mathbf{e}^{tr}\|$$

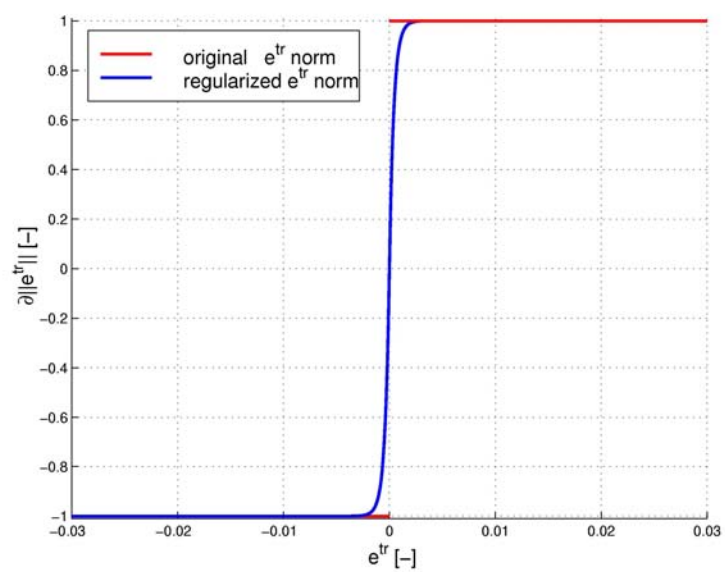
$$\frac{\partial\overline{\|\mathbf{e}^{tr}\|}}{\partial\mathbf{e}^{tr}} = \left[1-\left(\frac{\eta}{\|\mathbf{e}^{tr}\|+\eta}\right)^{1/\eta}\right]\frac{\mathbf{e}^{tr}}{\|\mathbf{e}^{tr}\|}$$

$$\lim_{\eta\to 0}\frac{\partial\overline{\|\mathbf{e}^{tr}\|}}{\partial\mathbf{e}^{tr}} = \frac{\mathbf{e}^{tr}}{\|\mathbf{e}^{tr}\|}$$

Norm



Norm derivative



Recent developments: possible to develop a solution algorithm which <u>does not</u> require any norm regularization (convex analysis tools)

Time-discrete model

Rate equation integrated with implicit backward Euler scheme

$$
\begin{cases}
\mathbf{s} = 2G\left(\mathbf{e} - \mathbf{e}^{tr}\right) \\
\mathbf{X} = \mathbf{s} - \left[\beta\left\langle T - T^{*}\right\rangle + h\overline{\left\|\mathbf{e}^{tr}\right\|} + \gamma\right]\dfrac{\partial\overline{\left\|\mathbf{e}^{tr}\right\|}}{\partial\mathbf{e}^{tr}} \\
\gamma \geq 0 \\
\mathbf{e}^{tr} = \mathbf{e}_{n}^{tr} + \Delta\zeta\dfrac{\partial F}{\partial\mathbf{X}} \\
\overline{\left\|\mathbf{e}^{tr}\right\|} \leq \varepsilon_{L} \\
F\left(\mathbf{X}\right) \leq 0 \\
\Delta\zeta \geq 0 \quad , \quad \Delta\zeta F\left(\mathbf{X}\right) = 0
\end{cases}
$$

Non linear system solution with return-mapping like algorithm

Time-discrete model: state detection & update procedure

TIME-DISCRETE MODEL FRAME

1. **Compute trial state**

$$\begin{cases} \mathbf{e}^{\mathrm{tr},TR} = \mathbf{e}_n^{\mathrm{tr}} \\ \mathbf{s}^{TR} = 2G(\mathbf{e} - \mathbf{e}^{\mathrm{tr},TR}) \end{cases}$$

2. **Check material state**

$$\begin{cases} \text{compute:} \begin{cases} \boldsymbol{\alpha}^{TR} = [\langle \beta(T - M_{\mathrm{f}}) \rangle + h\overline{\|\mathbf{e}^{\mathrm{tr},TR}\|}] \dfrac{\partial \overline{\|\mathbf{e}^{\mathrm{tr},TR}\|}}{\partial \mathbf{e}^{\mathrm{tr}}} \\ \mathbf{X}^{TR} = \mathbf{s}^{TR} - \boldsymbol{\alpha}^{TR} \\ F^{TR} = \sqrt{2J_2(\mathbf{X}^{TR})} + m\dfrac{J_3(\mathbf{X}^{TR})}{J_2(\mathbf{X}^{TR})} - R \end{cases} \\ \text{check } F^{TR}: \begin{cases} \text{if } (F^{TR} < 0) \text{ then} & \textbf{elastic step } (EL) \\ \text{else} & \textbf{active p.t. } (PT_1) \\ \text{end if} \end{cases} \end{cases}$$

3. **Update material state**

BRANCH SOLUTION AND STATE UPDATE

if (CASE EL - Elastic step) then

$$\begin{cases} \mathbf{e}^{\mathrm{tr}} = \mathbf{e}^{\mathrm{tr},TR} \\ \mathbf{s} = 2G(\mathbf{e} - \mathbf{e}^{\mathrm{tr}}) \end{cases}$$

else if (CASE PT_1 - Evolving phase transformation) then

$$\begin{cases} \text{find } \mathbf{e}^{\mathrm{tr}} \text{ solving } \boxed{\text{Equation (21)}} \\ \text{check solution:} \begin{cases} \text{if } \overline{\|\mathbf{e}^{\mathrm{tr}}\|} < \varepsilon_{\mathrm{L}} \text{ then} & \text{exit} \\ \text{else} & \textbf{CASE } PT_2 \\ \text{end if} \end{cases} \end{cases}$$

if (CASE PT_2 - Saturated phase transformation) then

find $\mathbf{e}^{\mathrm{tr}}$, γ solving $\boxed{\text{Equation (22)}}$

end if

end if

- Newton Raphson method
- Algorithmically consistent tangent
  Auricchio & Petrini IJNME 2004

$$\begin{cases} \mathbf{X} - \mathbf{s}^{TR} + 2G\Delta\zeta \dfrac{\partial F}{\partial \boldsymbol{\sigma}} + \left[\beta \left\langle T - T^* \right\rangle + \right. \\ \qquad \left. + h\overline{\left\|\mathbf{e}^{tr}\right\|} \right] \dfrac{\partial \overline{\left\|\mathbf{e}^{tr}\right\|}}{\partial \mathbf{e}^{tr}} = \mathbf{0} \\ F(\mathbf{X}) = 0 \end{cases}$$

$$\begin{cases} \mathbf{X} - \mathbf{s}^{TR} + 2G\Delta\zeta \dfrac{\partial F}{\partial \boldsymbol{\sigma}} + \left[\beta \left\langle T - T^* \right\rangle + \right. \\ \qquad \left. + h\overline{\left\|\mathbf{e}^{tr}\right\|} + \gamma \right] \dfrac{\partial \overline{\left\|\mathbf{e}^{tr}\right\|}}{\partial \mathbf{e}^{tr}} = \mathbf{0} \\ F(\mathbf{X}) = 0 \\ \overline{\left\|\mathbf{e}^{tr}\right\|} - \varepsilon_L = 0 \end{cases}$$

Time-discrete model: algorithm numerical validation

- ✓ Cyclic **uni-axial** tension-compression / torsion tests [proportional / stress control]
- ✓ Cyclic **multi-axial** tests [non proportional / strain control]

Test temperatures :

- **T = 253.15 K :** material stable in martensitic phase, shape memory effect takes place (multiaxial tests)
- **T = 285.15 K :** material stable in austenitic phase, pseudoelastic effect takes place (uniaxial and multiaxial tests)

Material and model data :

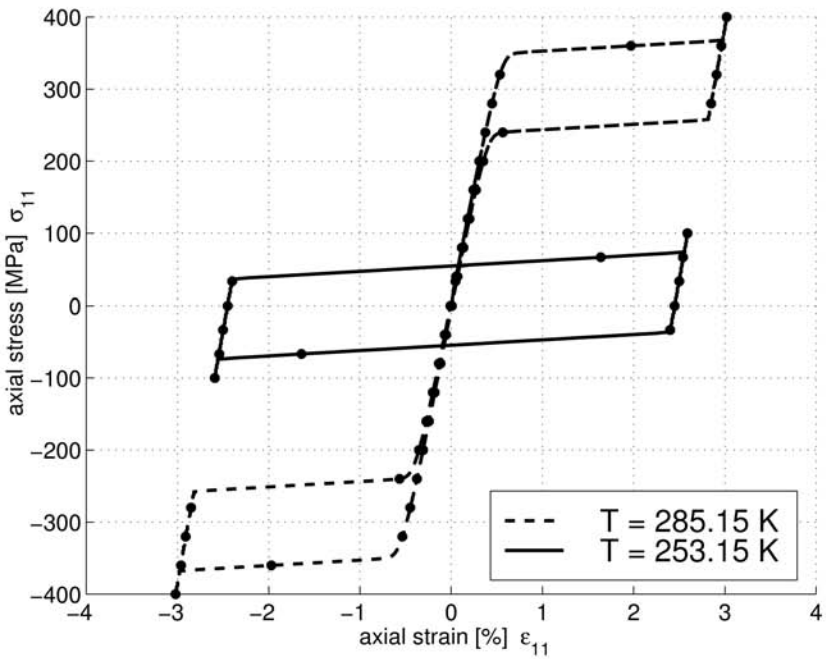
$E = 70000$ Mpa	[Young modulus]
$\nu = 0.3$	[Poisson modulus]
$R = 45$ MPa	[limit surface radius]
$\varepsilon_L = 3$	
$h = 500$ MPa	[hardening modulus]
$T_0 = 254.15$ K	[austenite-martensite transf. temp.]
$\varepsilon = 0.02$	[regularization parameter]

Algorithm numerical validation: uniaxial isothermal test (stress control)

Lines → fine time discretization

Dots → crude time discretization

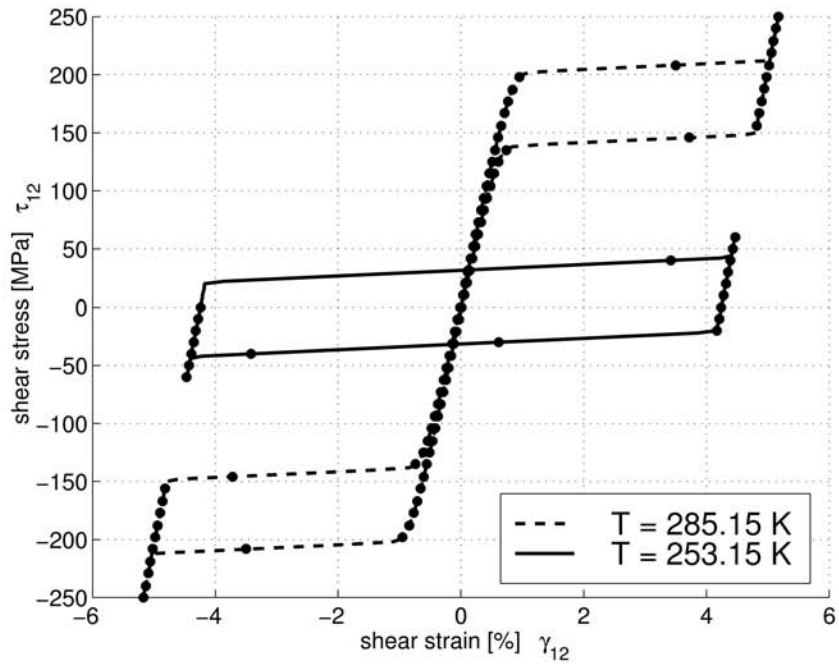
Tension-compression



Stress increment per step :

40 Mpa (dots); 2 Mpa (lines)

Torsion

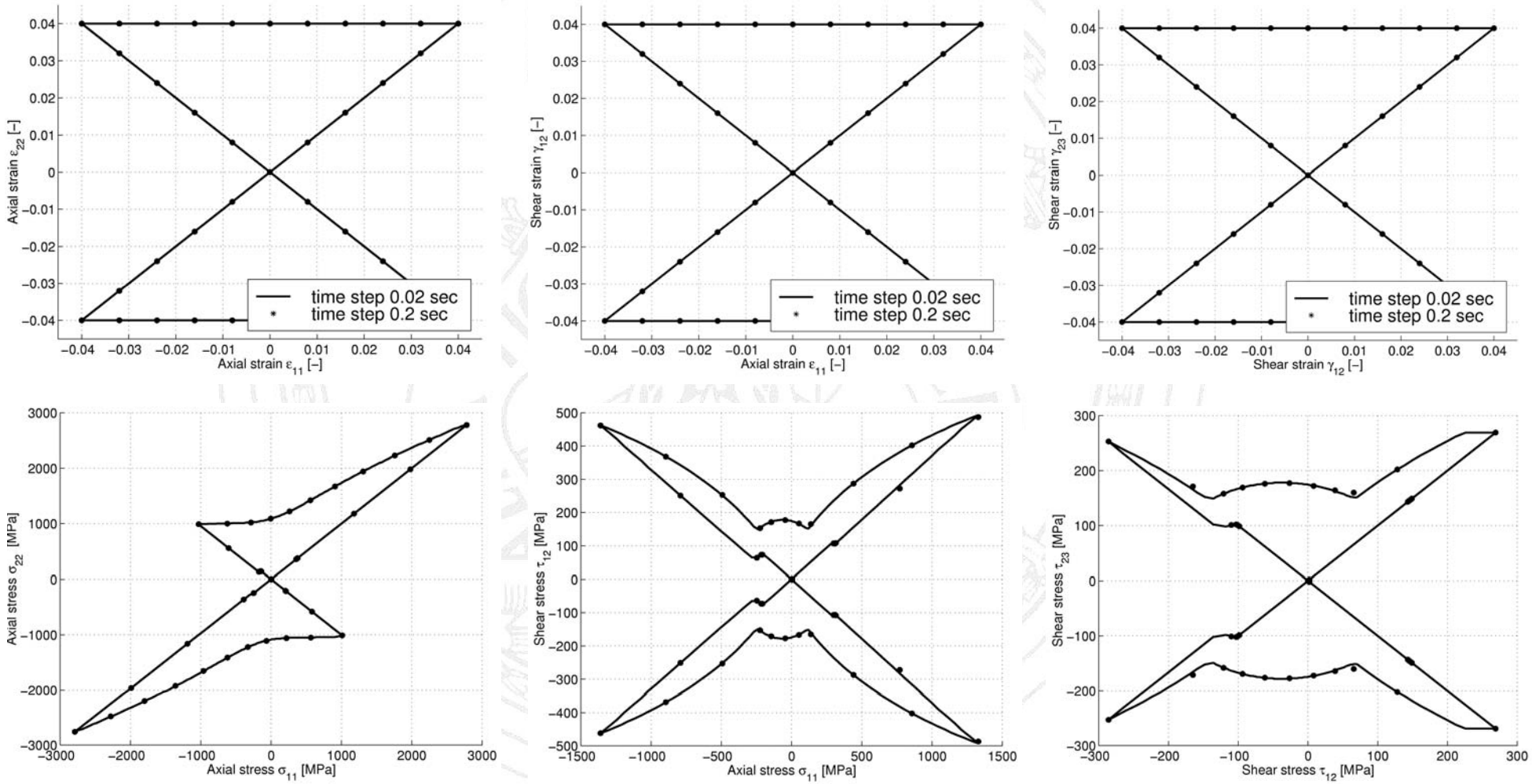


Stress increment per step :

10 Mpa (dots); 2 Mpa (lines)

Algorithm numerical validation: multi-axial isothermal test (strain control)

$\varepsilon_{11} - \varepsilon_{22}$ | $\varepsilon_{11} - \gamma_{12}$ | $\gamma_{12} - \gamma_{23}$

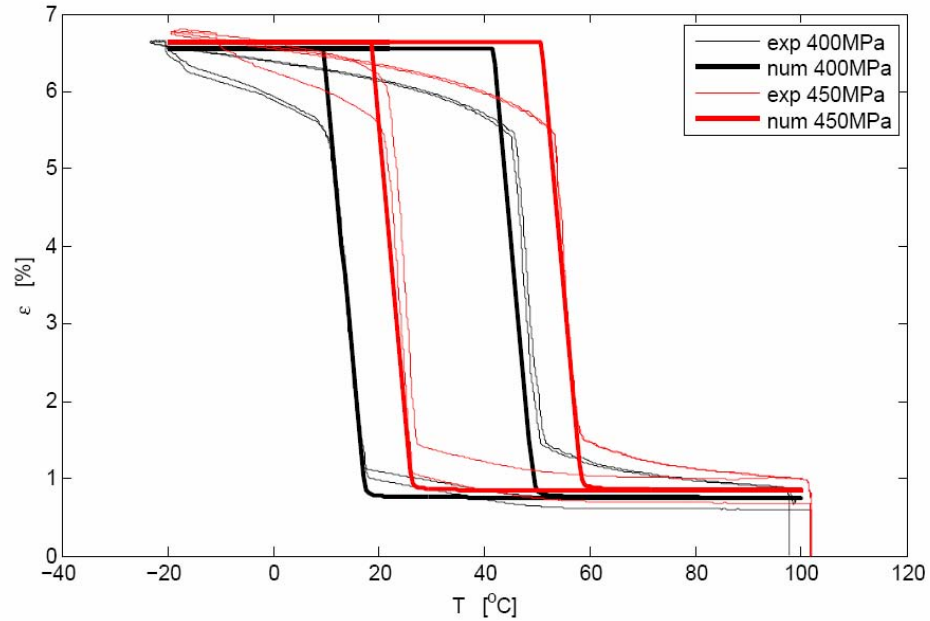


Time step: 0.02 sec (strain increment 8.E-4) fine time discretization
0.2 sec (strain increment 8.E-3) crude time discretization

"International" round-robin test

Parameter identification for roundrobin tests

Employed curves:



Parameters:

kept **constant** throughout **all** the performed numerical simulations

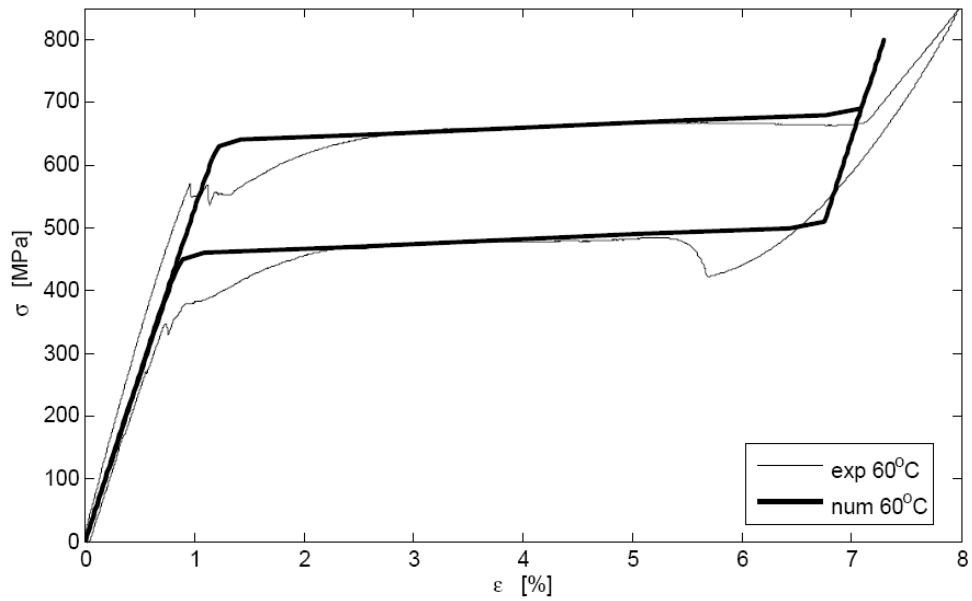
E:	**53600 MPa**
ν:	**0.33**
h:	**750 MPa**
T*:	**-38°C**
β :	**5.6 MPa/°C**
R :	**90 MPa**
ε_L:	**5.8%**

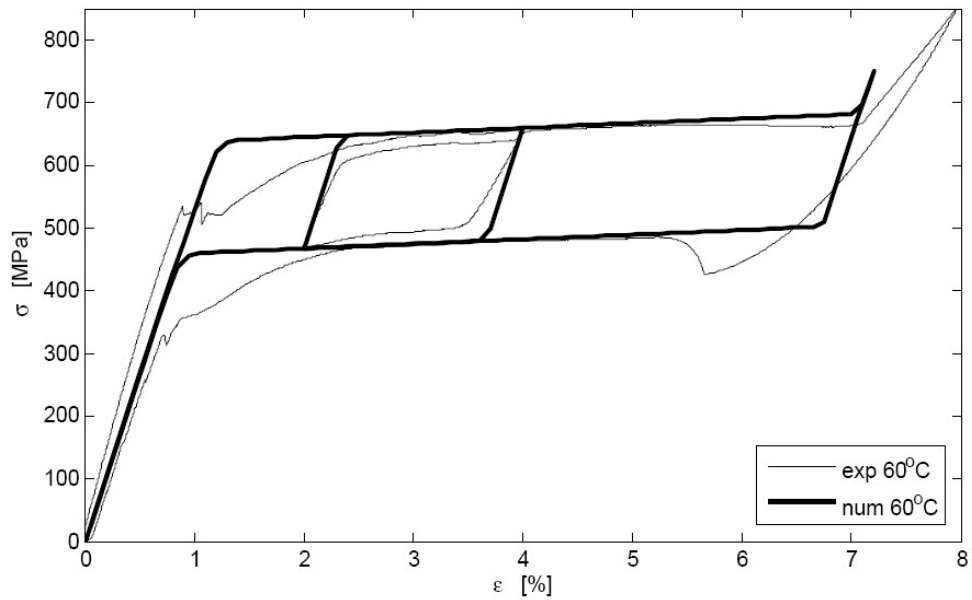
"International" round-robin test: outline

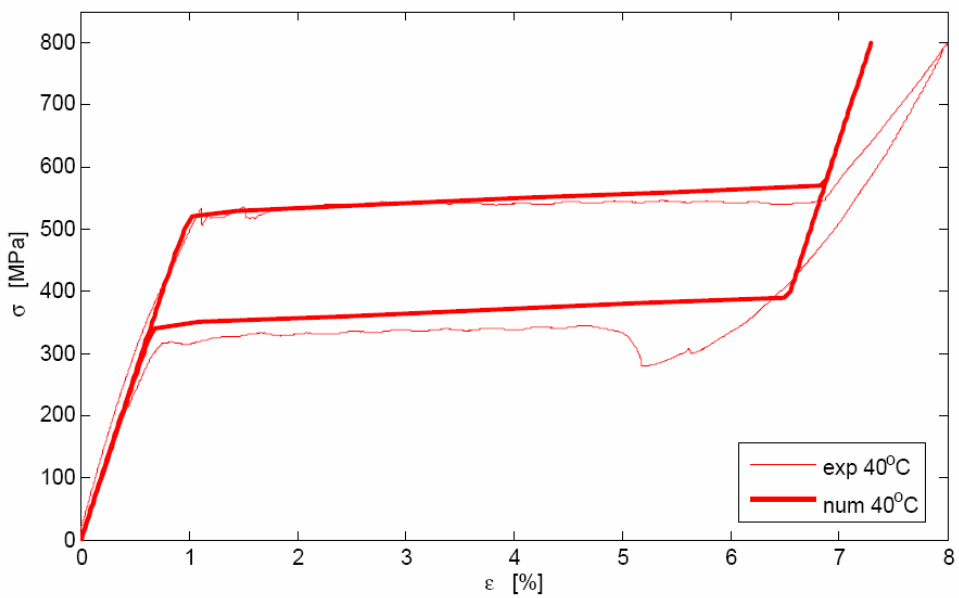
- Numerical results
  - ✓ Tensile tests
  - ✓ Thermal cycling tests at constant stress
  - ✓ Torsion tests
  - ✓ Combined tension-torsion tests
  - ✓ Thermomechanical recovery stress tests

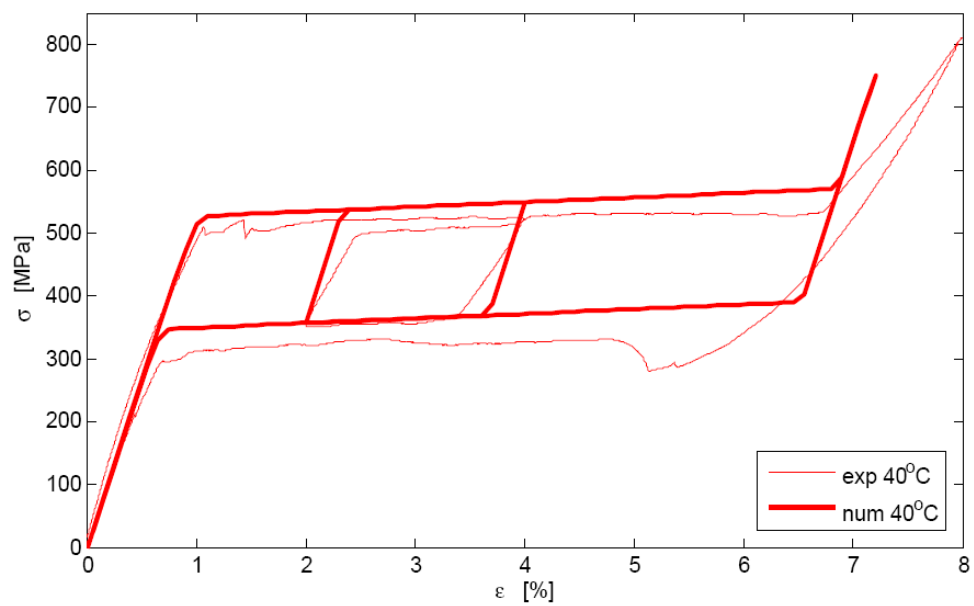
"International" round-robin test: numerical results

Tensile tests (Data set 1)



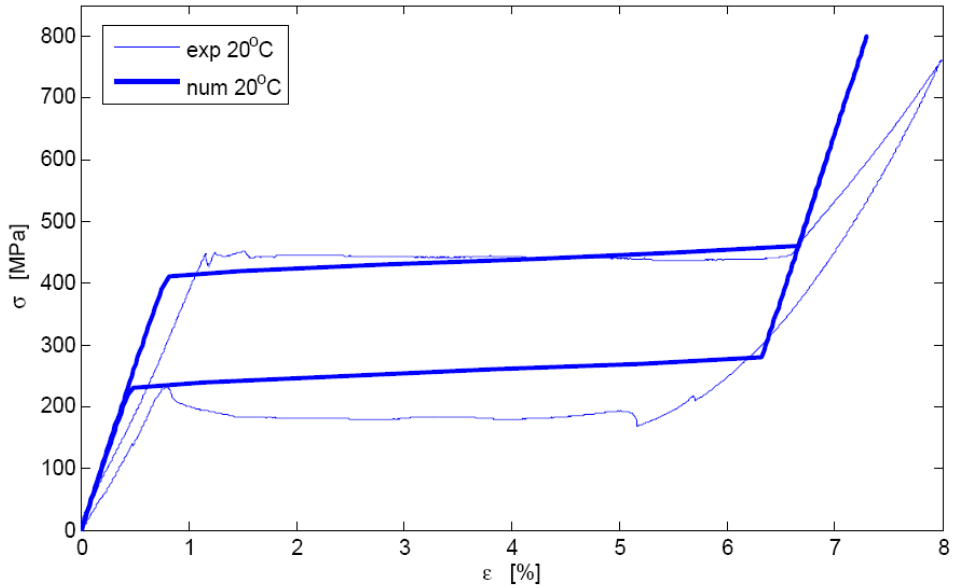


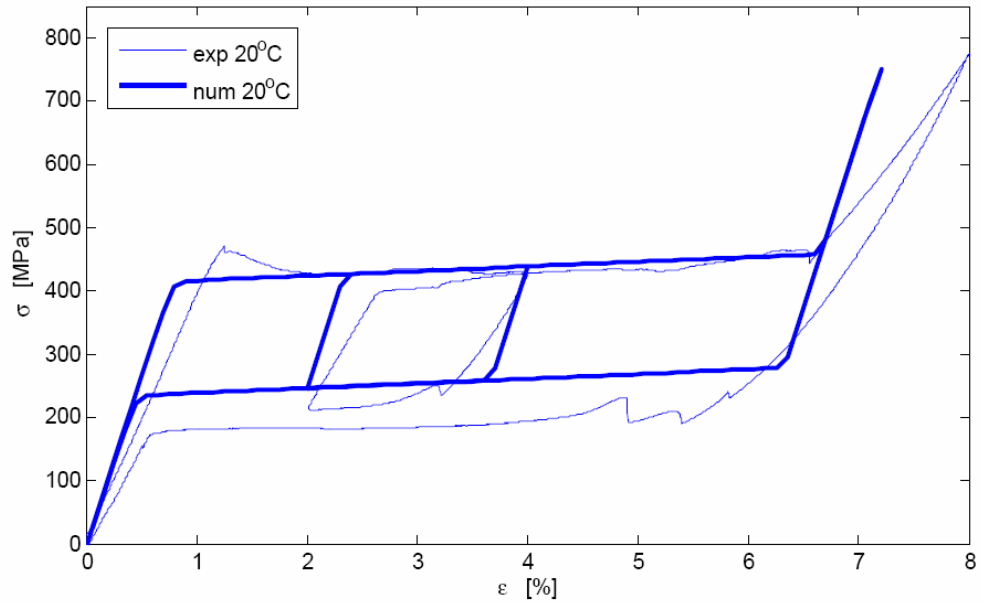


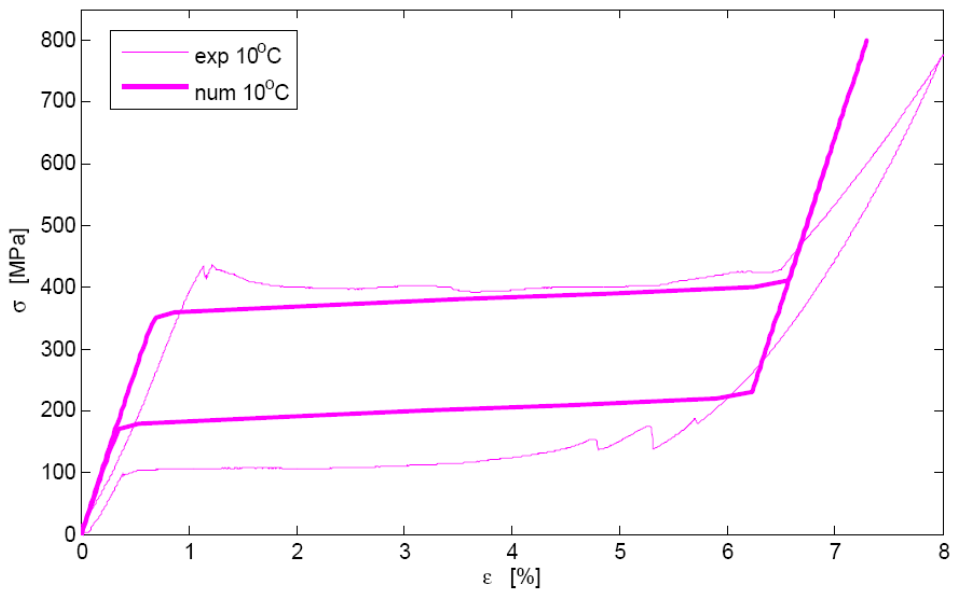


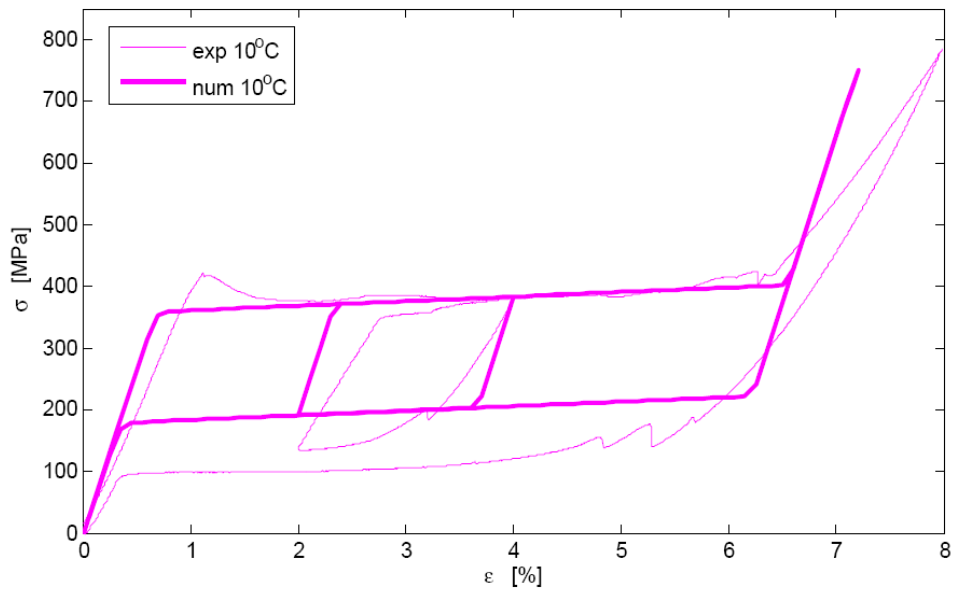
"International" round-robin test: numerical results

Tensile tests (Data set 1)



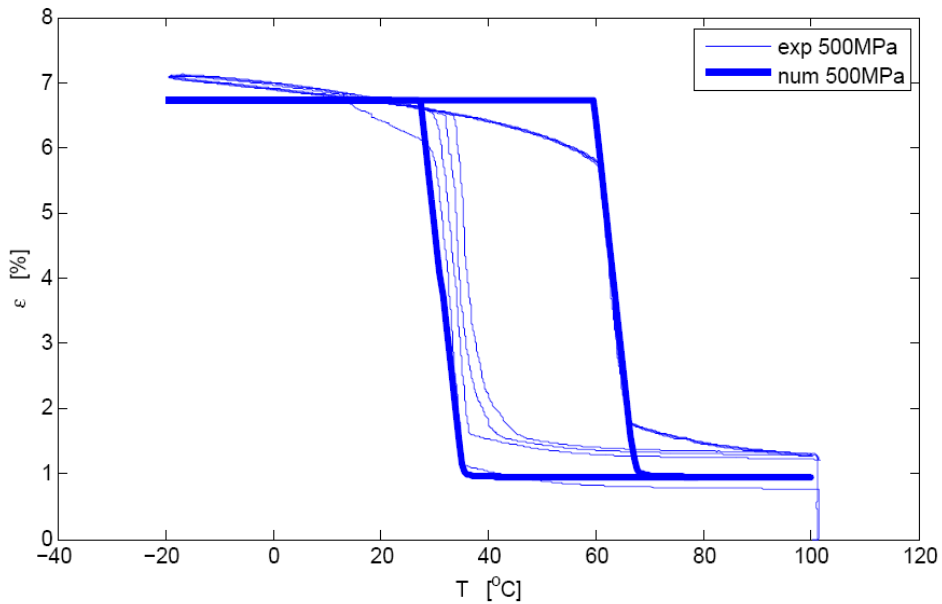


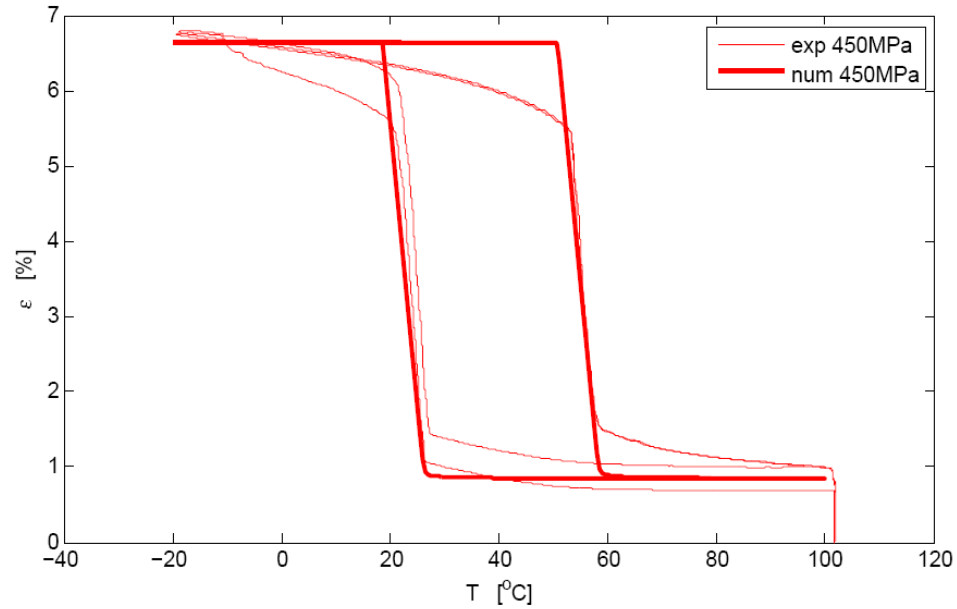


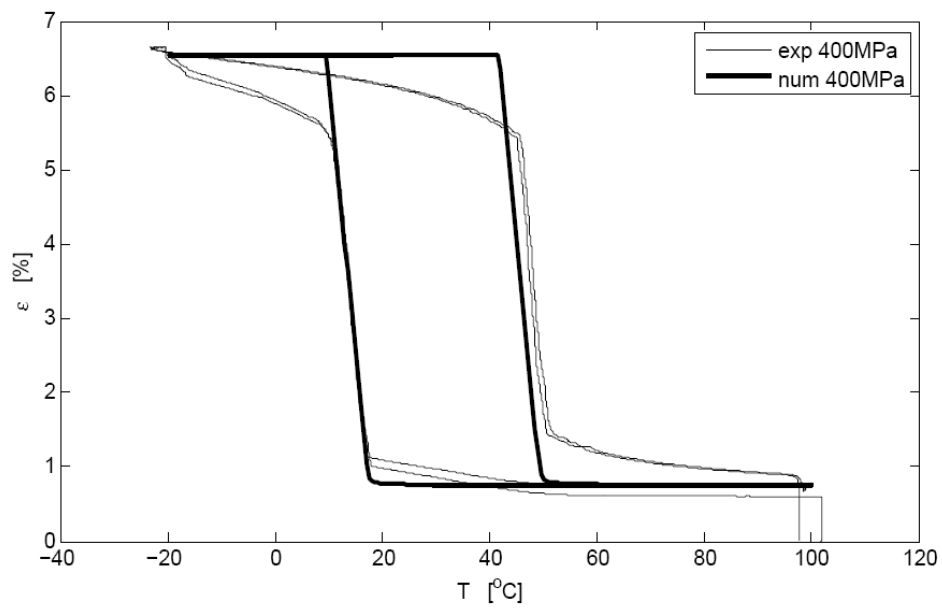


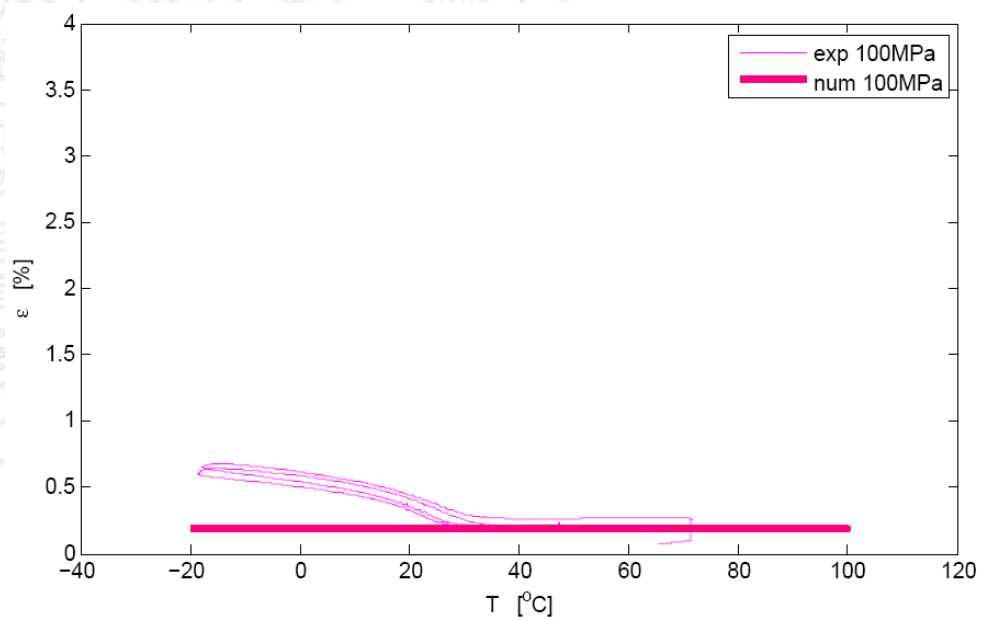
"International" round-robin test: numerical results

Thermal cycling tests at constant stress (Data set 2)



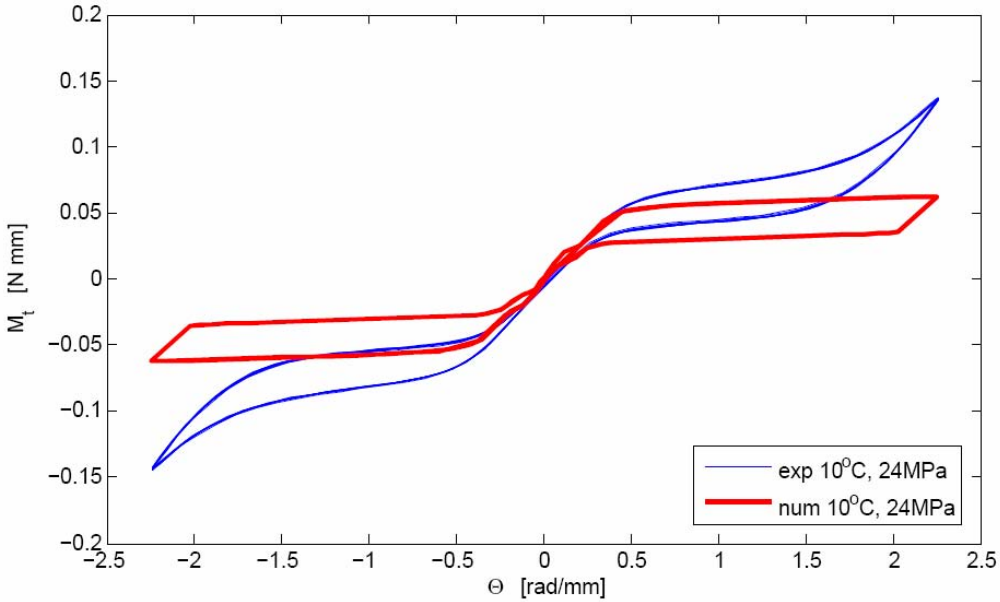


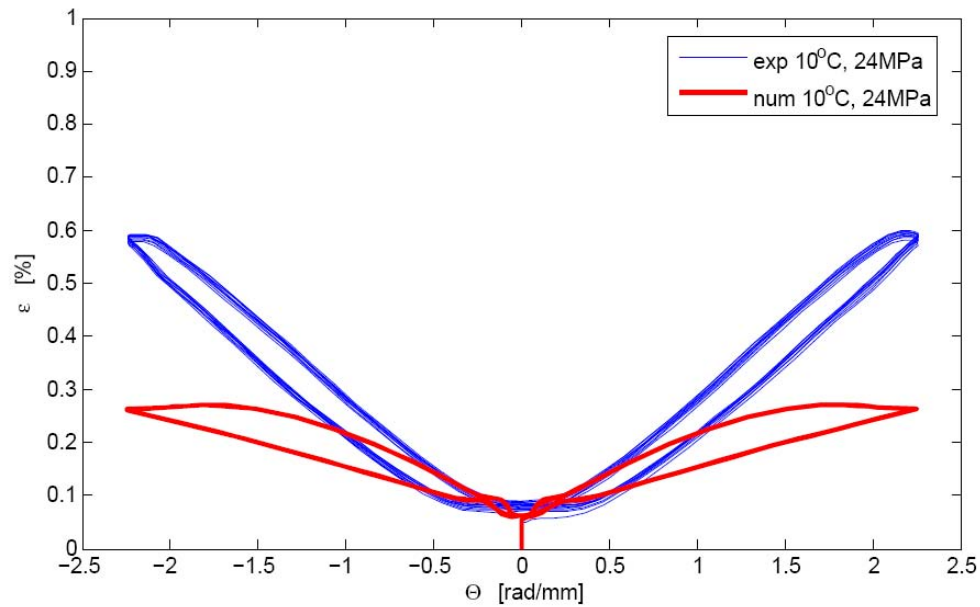


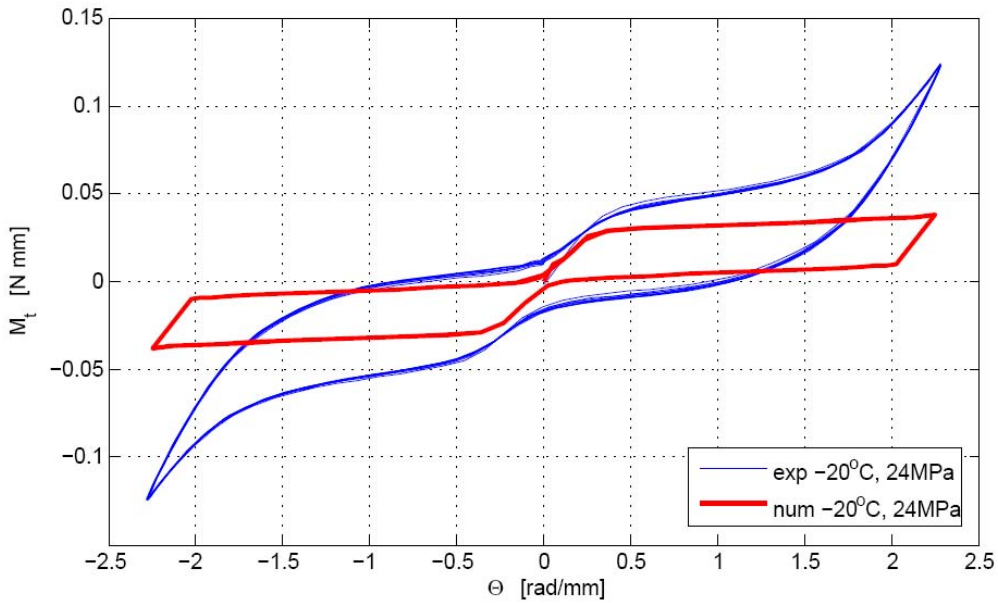


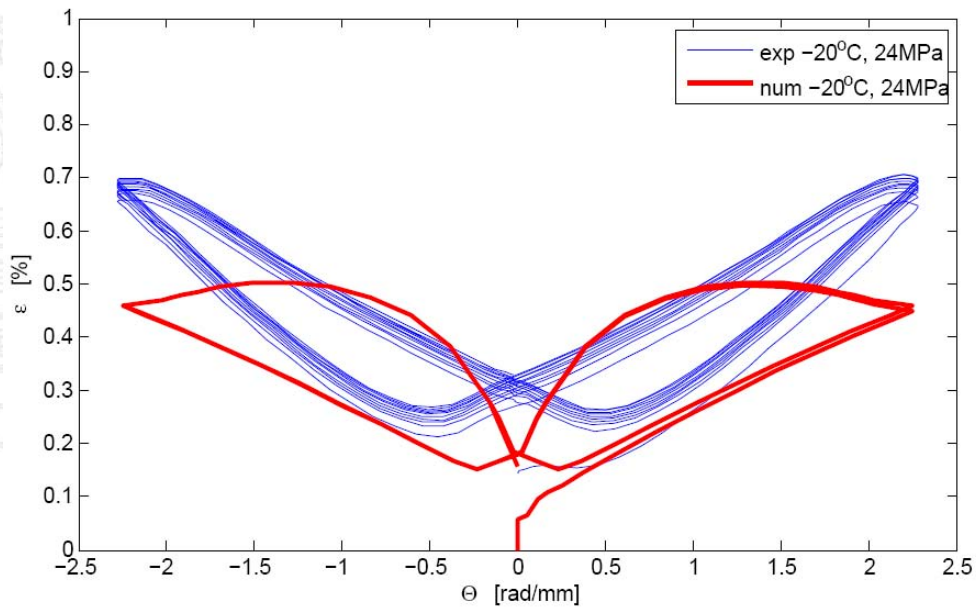
"International" round-robin test: numerical results

Torsion tests (Data set 3)



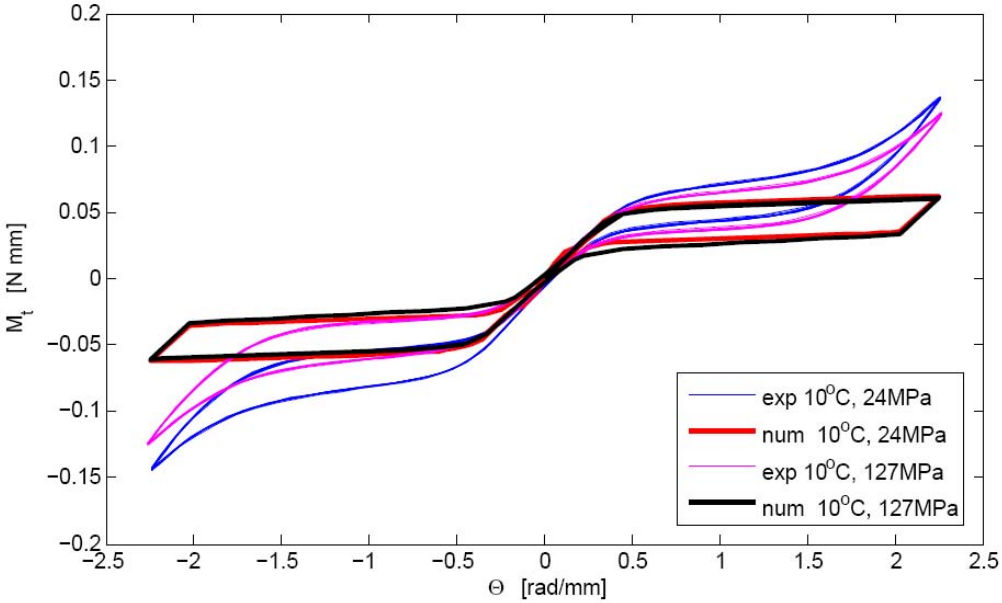


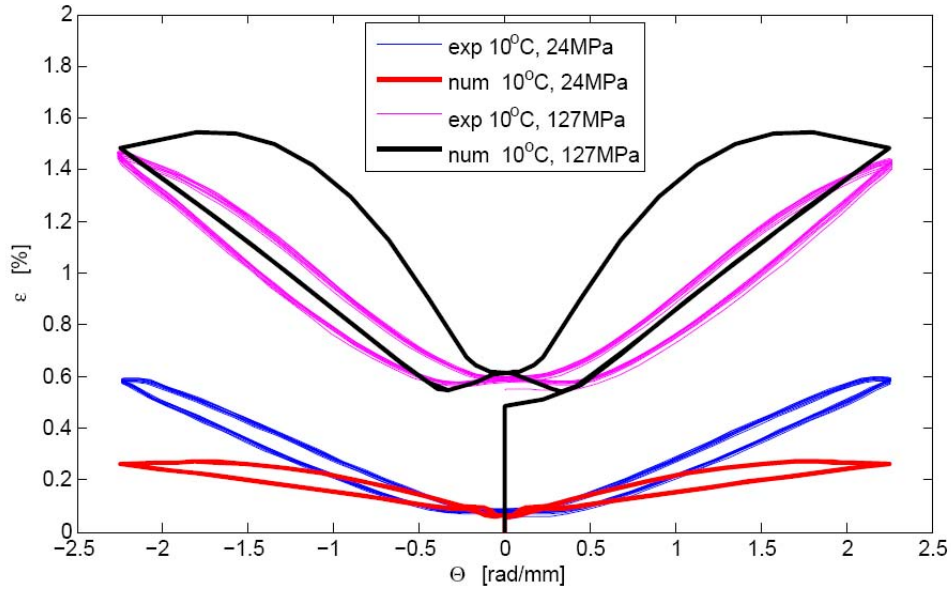


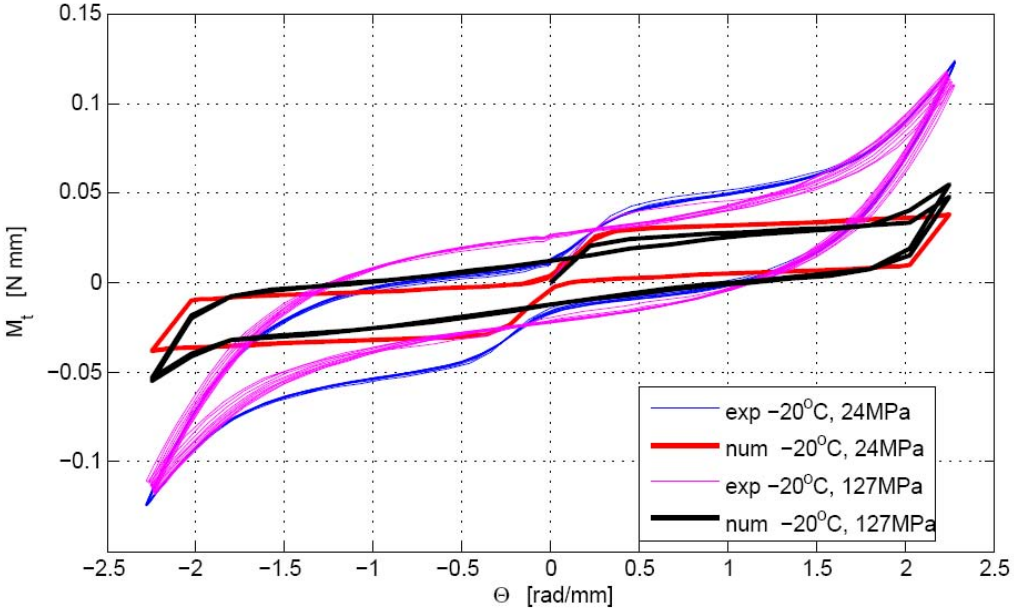


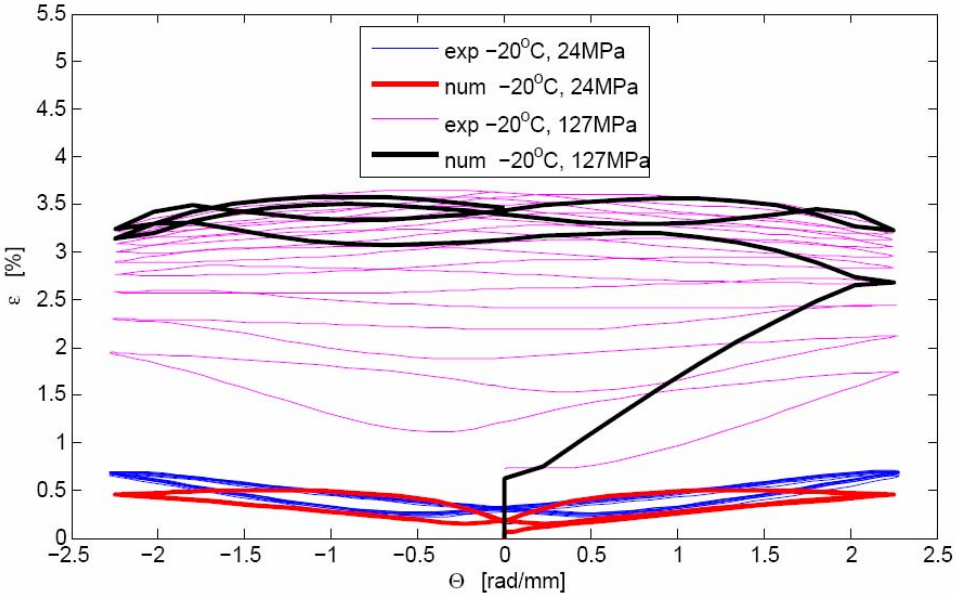
"International" round-robin test: numerical results

Combined tension-torsion tests (Data set 4)









Before addressing more realistic problems … some mathematics

By Convex Analysis, rewrite model in equivalent form

$$\boxed{\begin{pmatrix} -p \\ -\mathbf{s} \\ \eta \\ \partial D\left(\dot{\mathbf{e}}^{tr}\right) \end{pmatrix} + \partial\Psi \begin{pmatrix} \theta \\ \mathbf{e} \\ T \\ \mathbf{e}^{tr} \end{pmatrix} \ni \mathbf{0}}$$

∂D stands for sub-differential of dissipation function D associated to PT

$$D(\mathbf{e}^{tr}) = \sup_{F(\mathbf{A})\leq 0} \left\{\mathbf{A} : \mathbf{e}^{tr}\right\},$$

- Formulation of rate-independent evolution problems in terms of a doubly-nonlinear differential inclusion as above has recently attracted a lot of attention
- Mathematical treatment is nowadays fairly settled and <u>existence, uniqueness, and time-discretization results are available</u>

1. Mielke. Evolution of rate-independent systems. Handbook of Differential Equations. Elsevier, 2006.
2. Auricchio, Mielke, Stefanelli. A rate-independent model for the isothermal quasi-static evolution of shape-memory materials, M3AS 2008

Before addressing more realistic problems … some mathematics

- ✓ Dissipation function has a specific expression

$$D(\mathbf{e}^{tr}) = R||\mathbf{e}^{tr}||$$

- ✓ Easy to check that D is positively 1-homogeneous, i.e.

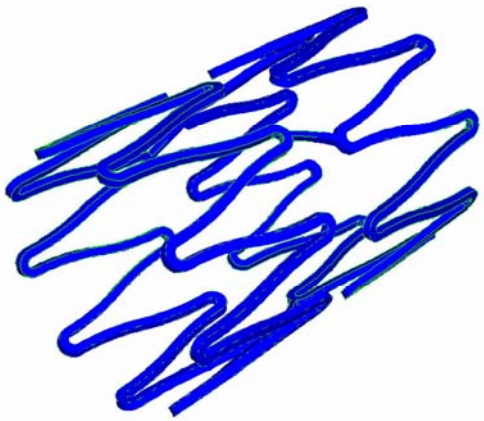
$$D(\lambda \mathbf{e}^{tr}) = \lambda D(\mathbf{e}^{tr}) \quad \forall \lambda > 0.$$

hence, time-evolution of $\mathbf{e}^{tr}$ is rate-independent since

$$\partial D(\lambda \mathbf{e}^{tr}) = \partial D(\mathbf{e}^{tr}) \quad \forall \lambda > 0.$$

- ► **The model is thermodynamically consistent**
- ► **The model undergoes fully reversible phase transformations**

Numerical simulation: SE coronaric stent (free-expansion)

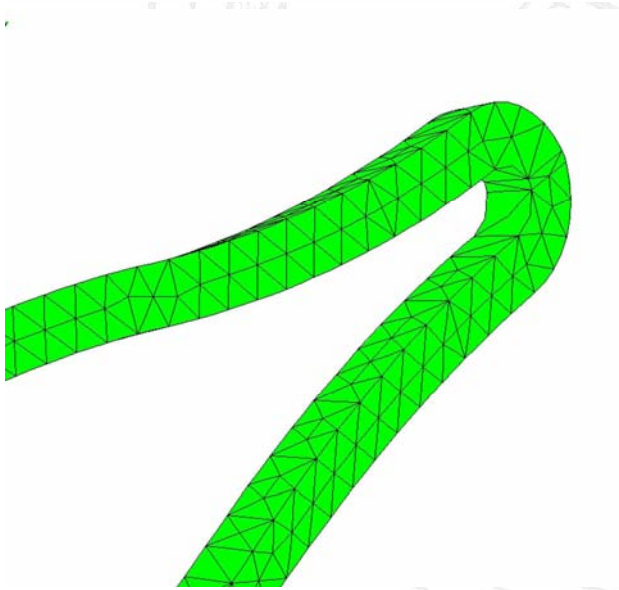


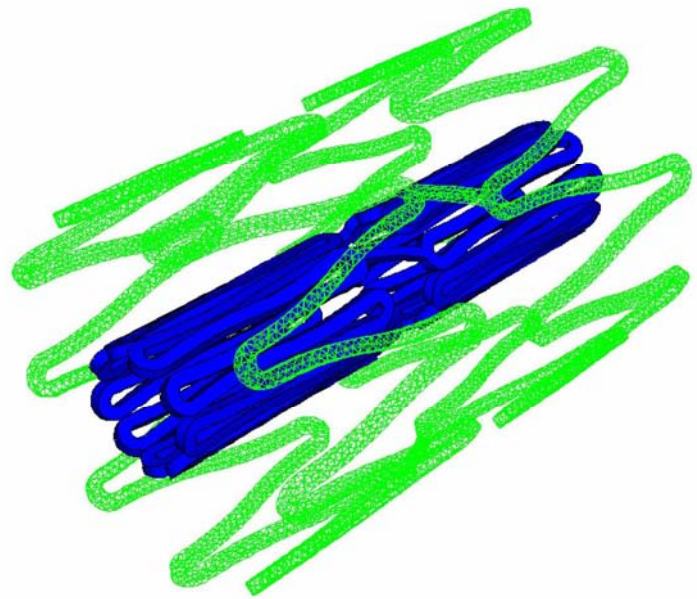
External diameter: 4.25 mm

Stent length: 5.24 mm

Stent thickness: 0.1 mm

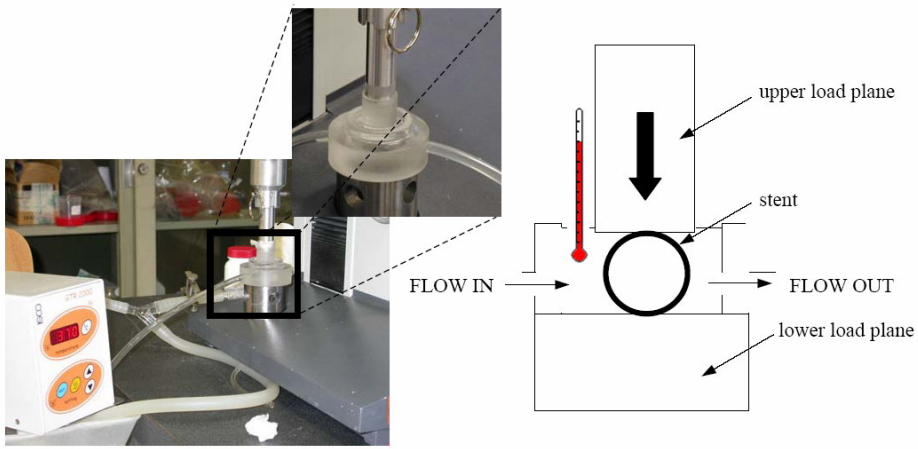
Model: 29263 10-node el.s / 64046 nodes

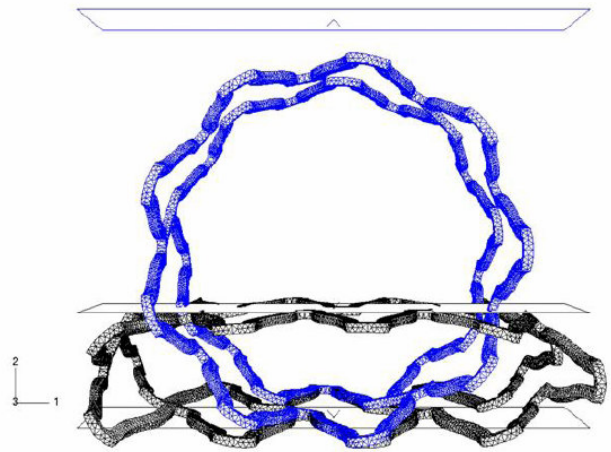


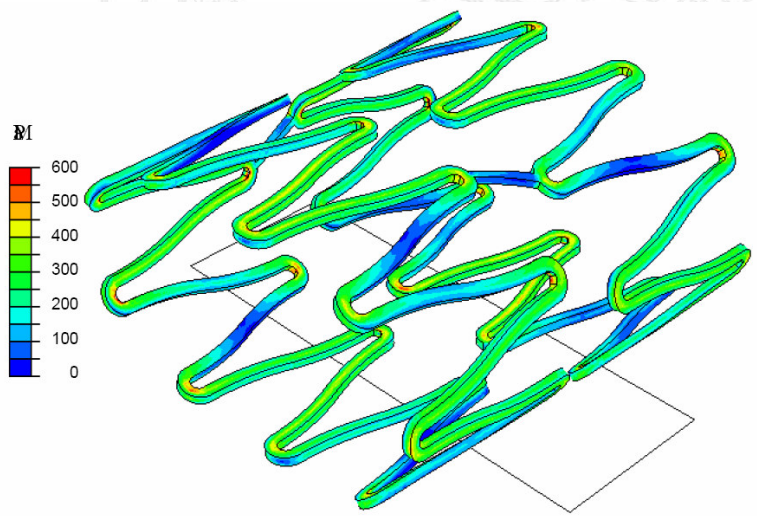


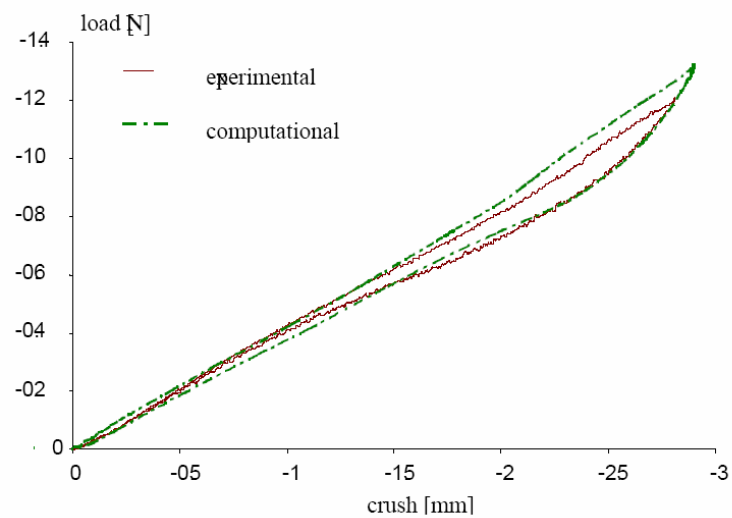
Constitutive model interfaced as “user mat subroutine” into Abaqus

Numerical simulation: SE coronaric stent (crush test)





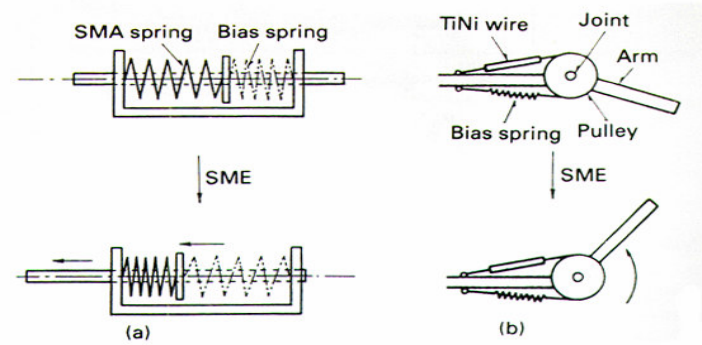


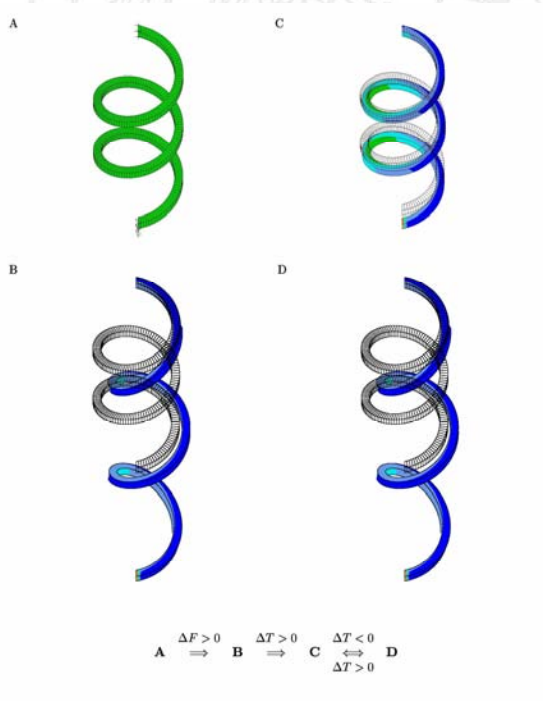


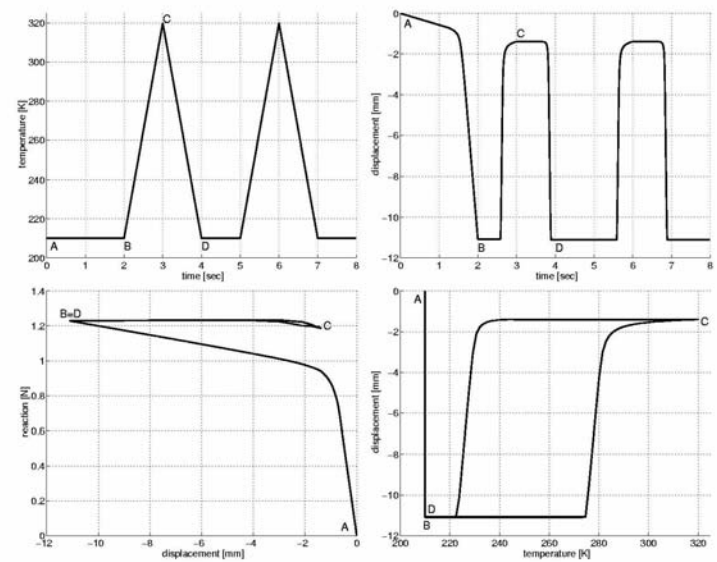
Loading-unloading test: 70% diameter reduction

Numerical simulation: SME spring

SMA actuators based on the shape-memory effect



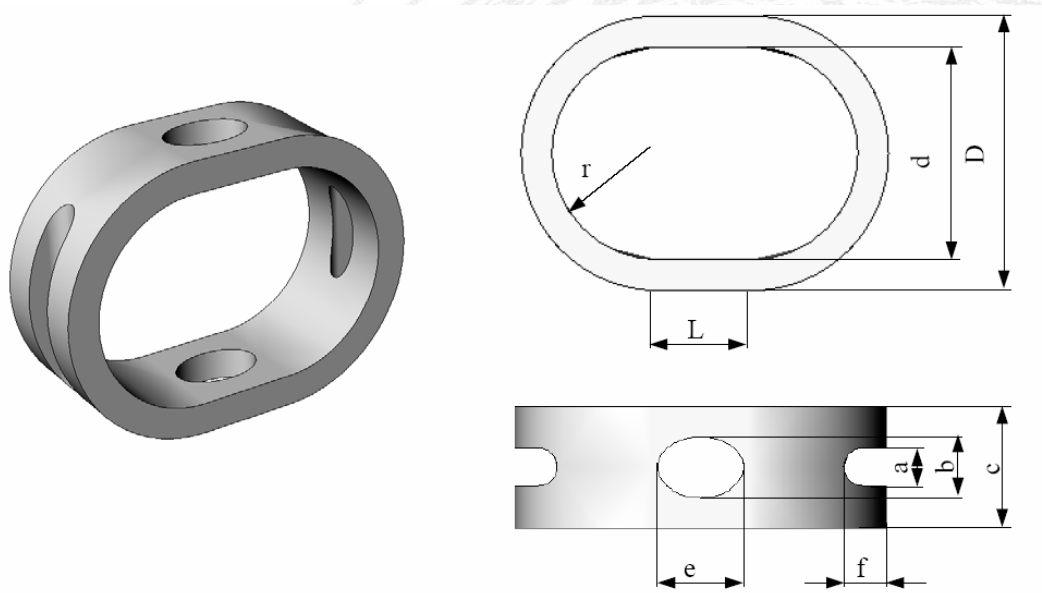




Assume time-dependent temperature known and uniform in the domain

Numerical simulation: SME spinal intervertebral spacer

Device that should be able to substitute a damaged intervertebral disc

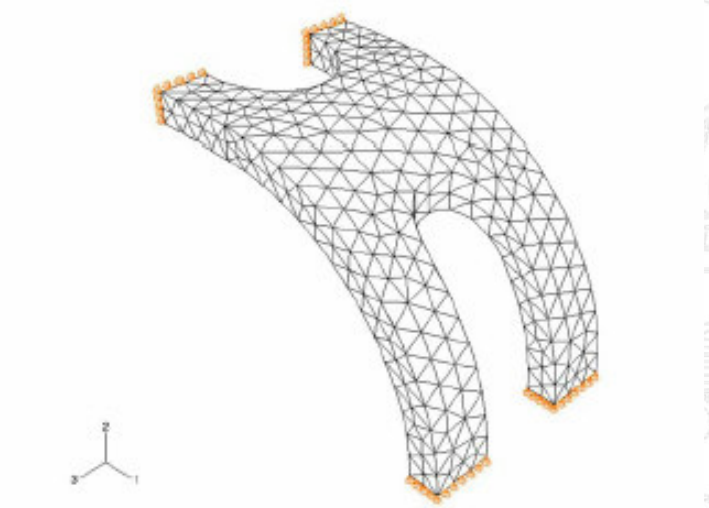


shock absorber & motion unit

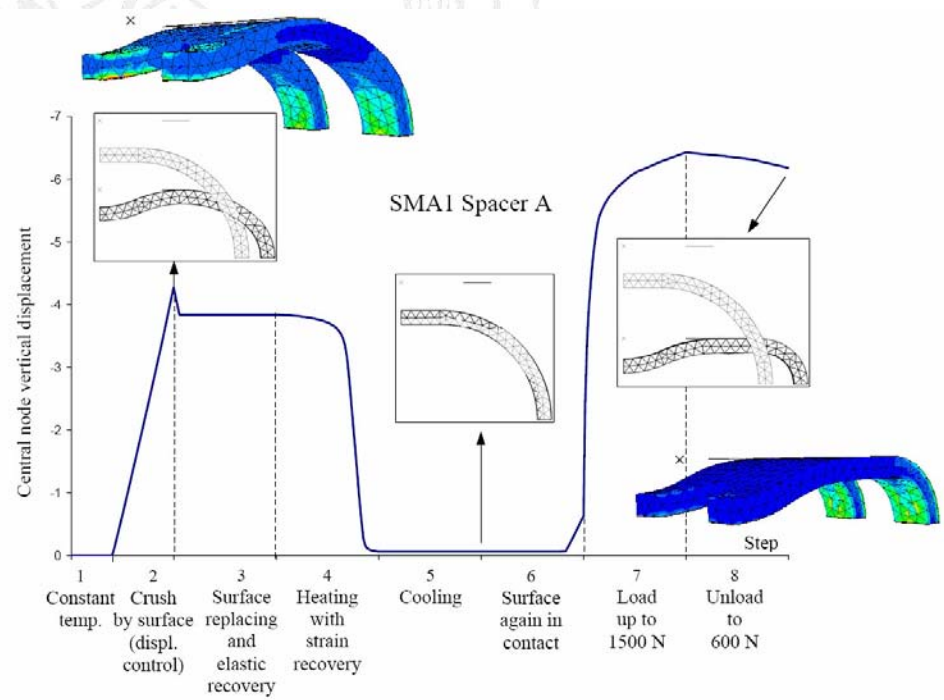
- maintain separation distance between adjacent vertebral elements allowing spine large motion in several directions as well as avoid nerve compression
- allow spine to compress / rebound during activities, resist gravity on head and trunk during prolonged sitting and standing, allow spinal segment to flex, rotate, and bend on side

Numerical simulation: SME spinal intervertebral spacer

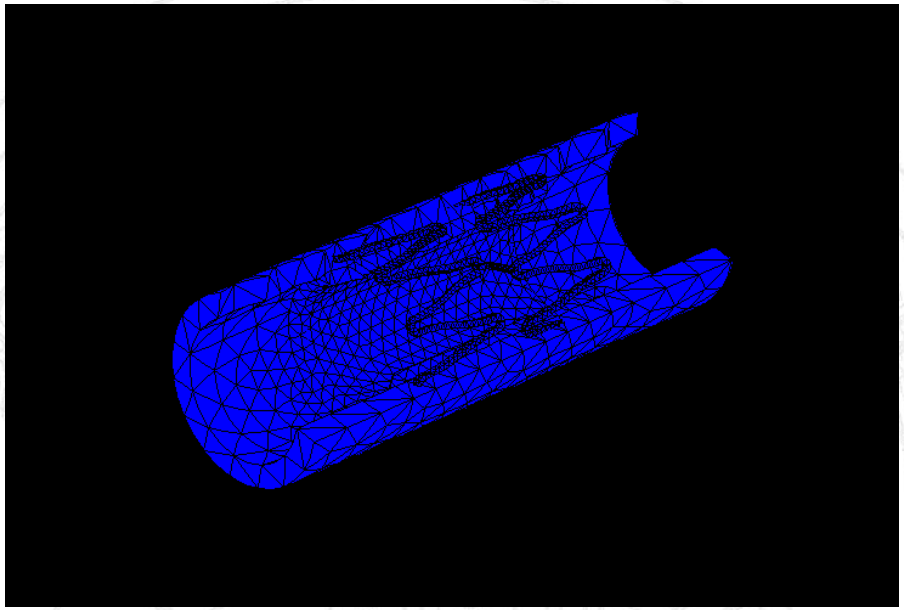
- Device is compressed in martensitic phase, assuming a reduced shape [to help device insertion] .Once positioned in body, it recovers original expanded shape by thermal recovery (shape memory effect) and starts to work opposing force to spinal compressive load
- Compare different Ni-Ti spinal spacers during implant and physiological loading

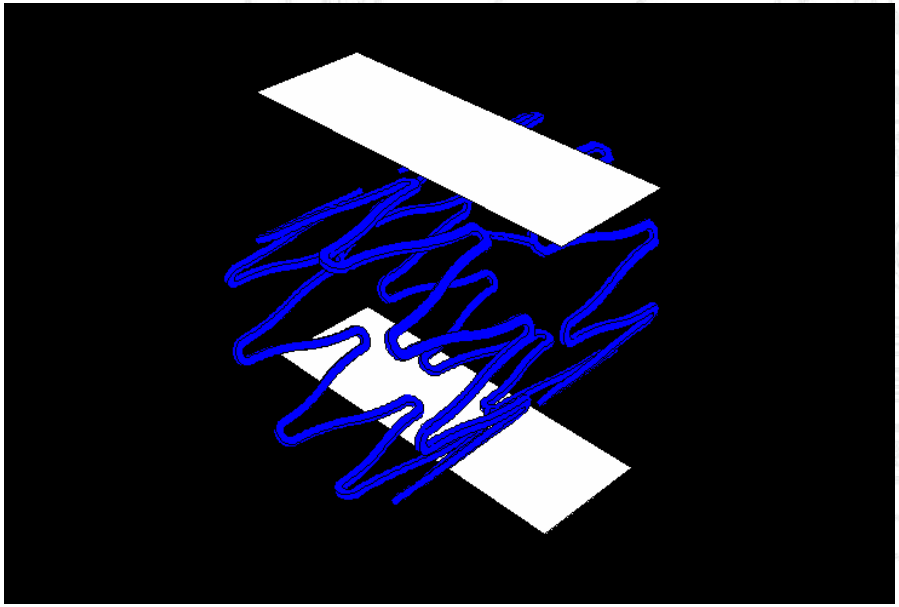


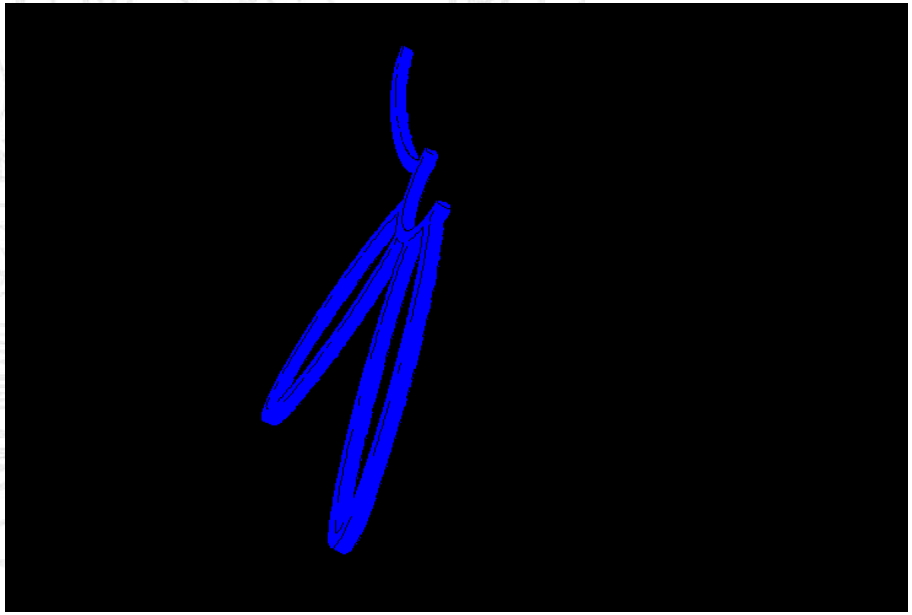
Due to symmetry, only ¼ of the device is simulated



Constitutive model: COFIN funded project



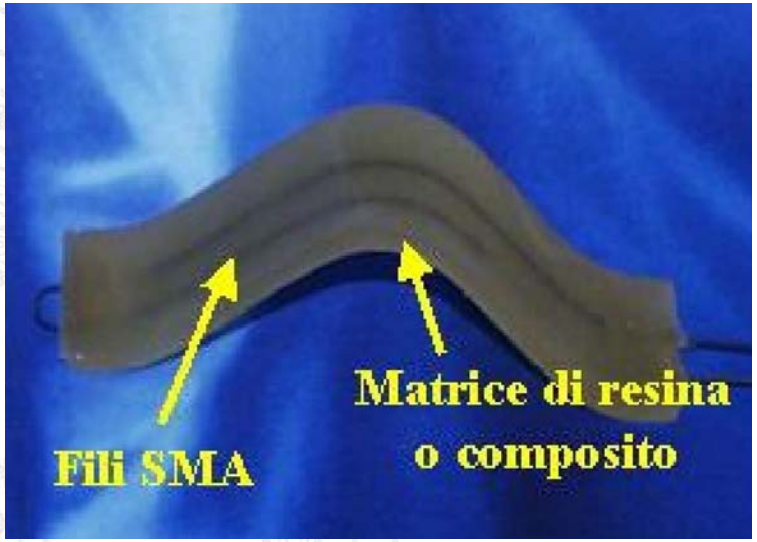




SMA hybrid composites

Example : SMA fibers are surface mounted or embedded in polymeric or metallic matrices

Courtesy of C.R.F.



SMA composites able to : actively control structure shape
actively control structure pre-stress

Application fields : aeronautics, aerospatial, automotive, structural control, medicine

Computational tool useful to support design of hybrid composites

OUR GOAL : 3D finite element frame, modeling SMA & matrix stimulated by electro-thermo-mechanical load

COMPLEXITY: need to deal with three fields

Field equations :

Equations	Mechanical Field	Thermal field	Electrical field
Equilibrium	$\nabla\cdot\boldsymbol{\sigma}+\vec{f}=0$	$\nabla\cdot\vec{q}+C\dot{T}=b$	$\nabla\cdot\vec{j}=0$
Compatibility	$\boldsymbol{\varepsilon}=\dfrac{\nabla\vec{u}+(\nabla\vec{u})^T}{2}$	$\vec{\varepsilon}=-\nabla T$	$\vec{E}=-\nabla V$
Constitutive law	$\boldsymbol{\sigma}=\mathbf{D}:\boldsymbol{\varepsilon}$	$\vec{q}=\mathbf{k}\cdot\vec{\varepsilon}$	$\vec{j}=\boldsymbol{\sigma}\cdot\vec{E}$

SMA hybrid composites: coupling terms

Thermo-mechanical coupling

- dissipative terms
  - ✓ SMA model $b = \mathbf{X} : \dot{\mathbf{e}}^{tr}$
  - ✓ elasto-plastic model $b = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{p}$
- thermal deformation $\boldsymbol{\varepsilon}^{tot} = \boldsymbol{\varepsilon}^{e} + \boldsymbol{\varepsilon}^{p} + \alpha \, \Delta T \cdot \mathbf{1}$

Thermo-electrical coupling

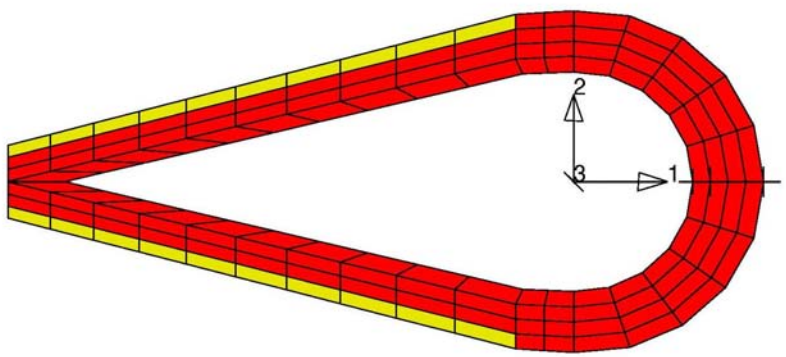
- Joule effect $b = \vec{E} \cdot \vec{j}$

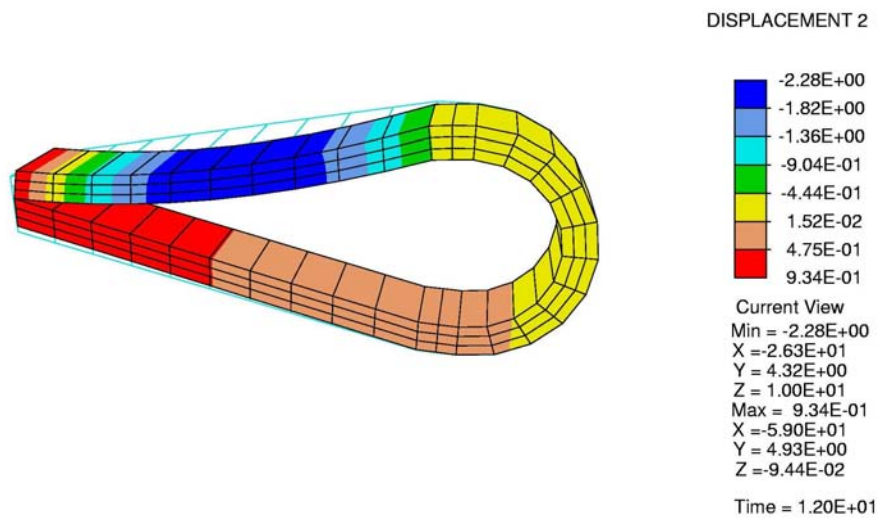
Thermo-electro-mechanical coupling

- SMA model $b = \mathbf{X} : \dot{\mathbf{e}}^{tr} + \vec{E} \cdot \vec{j}$
- elasto-plastic model $b = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{p} + \vec{E} \cdot \vec{j}$

SMA hybrid composites: numerical example

Helicopter blade: SMA hybrid composite could be useful employed to reduce helicopter blade vibration by edge flap active control





Simplified model of flap activated by embedded Ni-Ti fibers in martensitic phase

The upper pre-strained SMA fiber activated by heating induces flap shape changes that are recovered heating the lower fiber

Micro-actuators: analysis & design

- Development of a method for designing SMA micro-actuator
- Investigation & optimization of a micro-gripper already discussed in literature

M. Kohl. Shape Memory Microactuators. Springer, 2002.

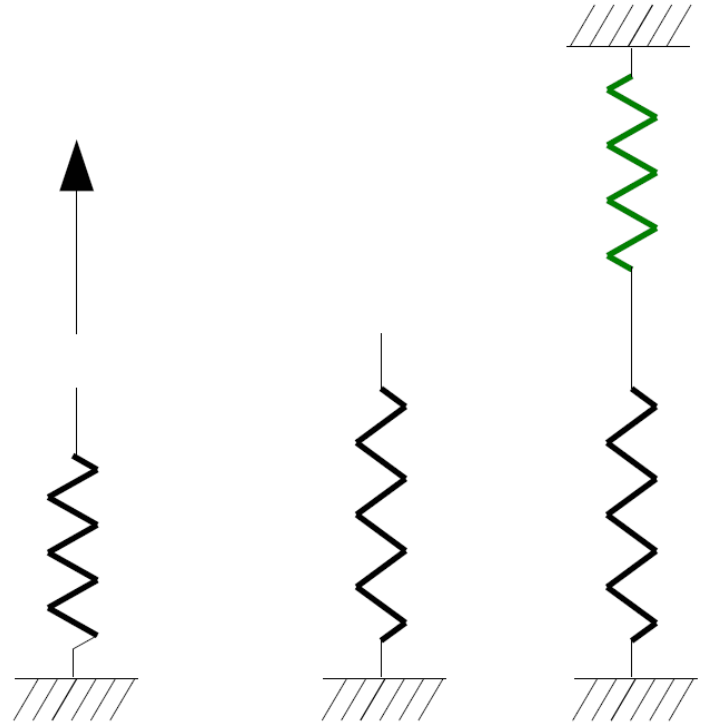
- Standard design method: based on austenite/martensite elastic modulus difference
- GOAL: develop a design method taking advantage of effective material response
- DRAWBACK: adopted model does not take into account elastic modulus change with phase transformation

Micro-actuators: analysis & design

Focus on the design of a simple antagonist mechanism made of two SMA springs

Activation procedure

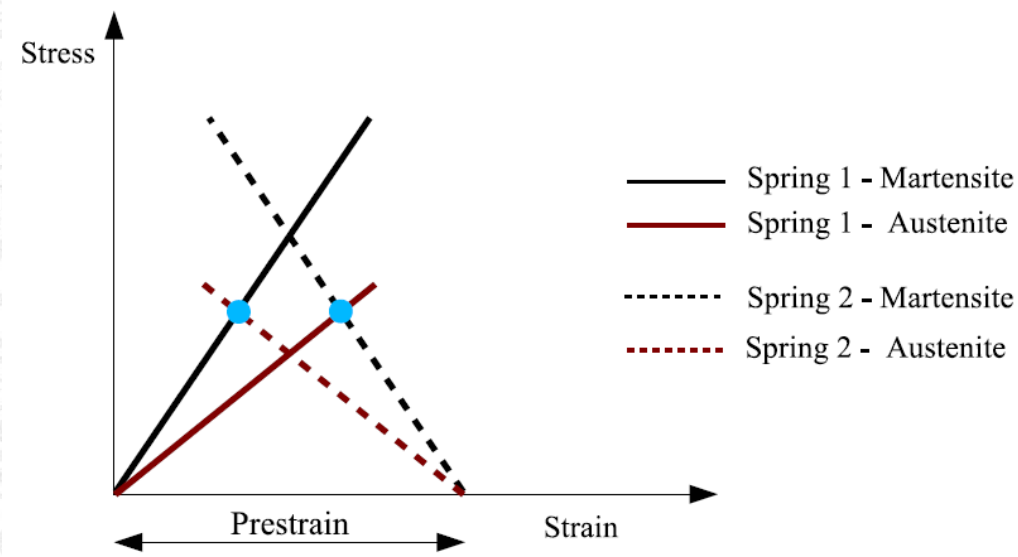
1. Lower spring deformed applying a force to one end
2. Constrain both ends of series system
3. Activate system by heating each spring alternatively



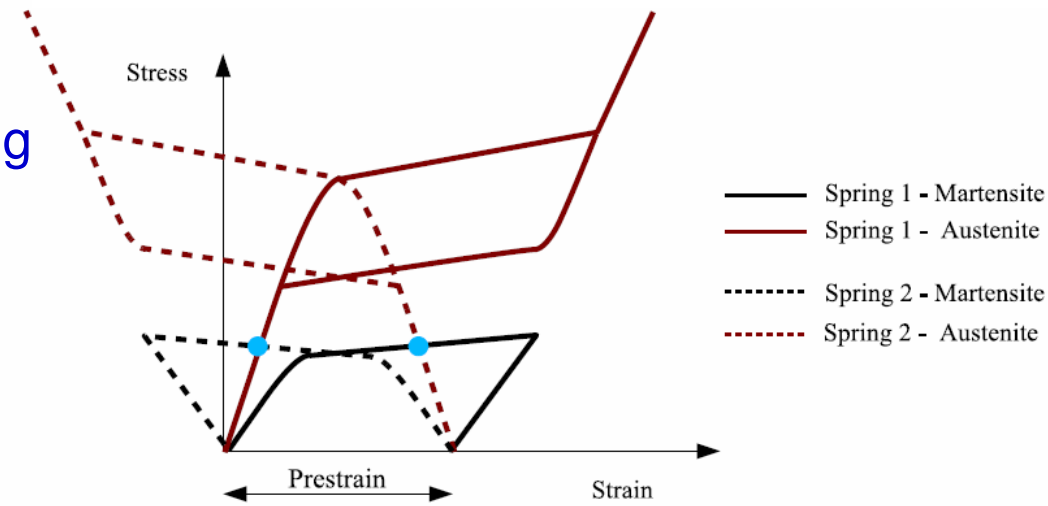
- Design parameter: pre-deformation induced in lower spring
- Design output: overall system response. Obtainable characterizing both springs in terms of force-displacement and “properly” superimposing single response in a force-displacement plane
- Since there are two possible – martensitic and austenitic – conditions for each springs, we need two different characterization for each part of the system

Micro-actuators: analysis & design

Classical design approach: assume material as always working in elastic range.
There are 4 different equilibrium points. Interested only in two of them, i.e. intersection between lower spring in austenitic condition and upper spring in martensitic condition and vice versa.



More appropriate design: consider spring effective response and hysteresis.
Consider only unloading branches in austenitic condition and only loading branches in martensitic condition!!



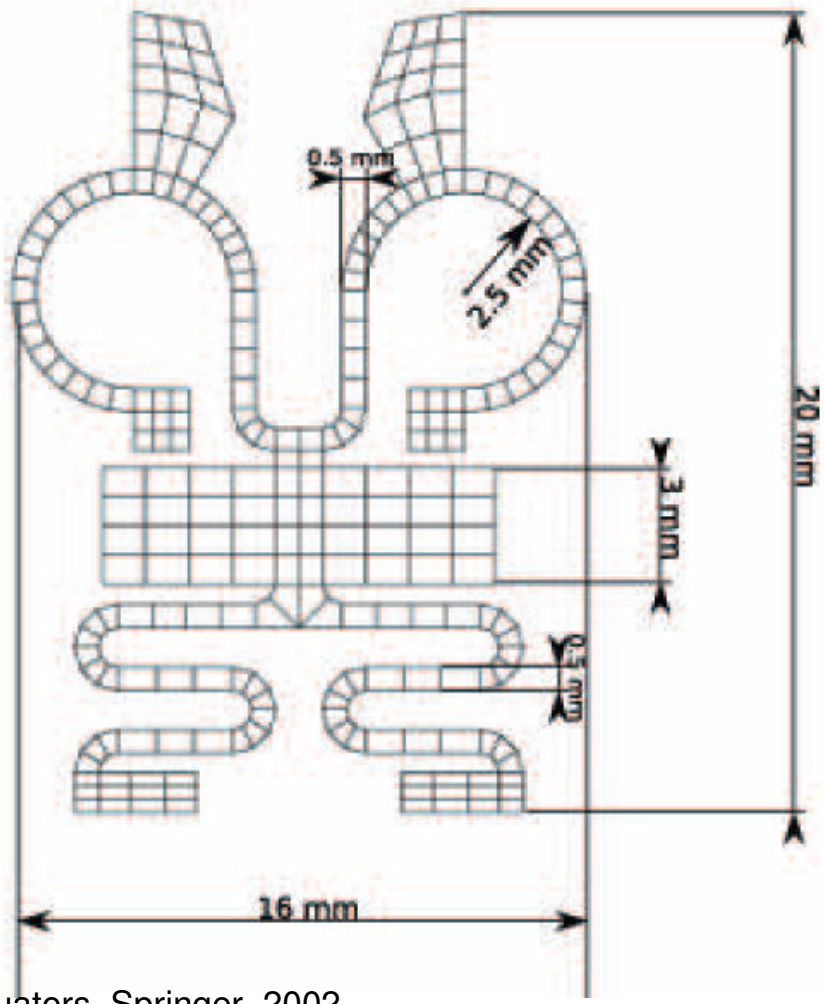
Micro-actuators: analysis & design

Once procedure is defined, we can design a micro-gripper [Koh02]

Micro-gripper composed by two main components:

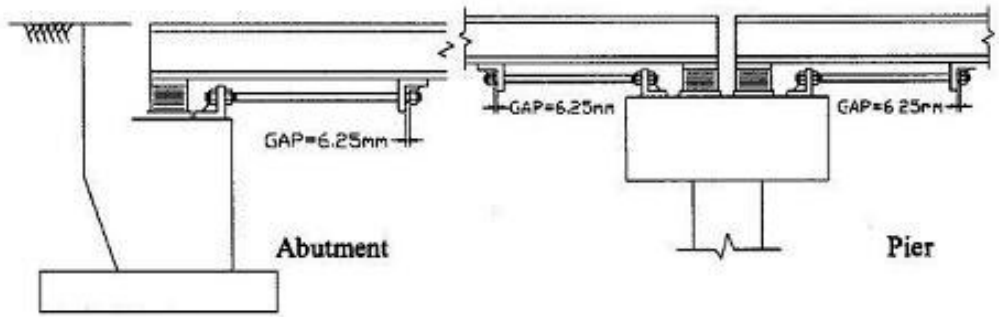
- Gear Actuator: transform a linear force into a gripping force between jaws
- Linear Actuator: provide a linear force to Gear Actuator

Activation: predeform linear actuator

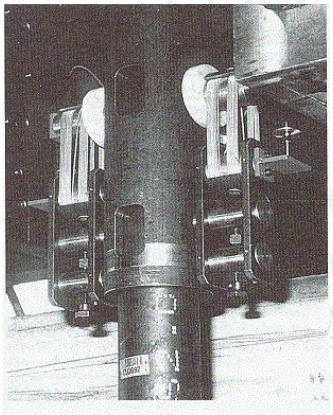


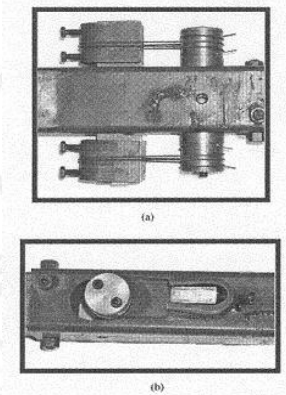
M. Kohl. Shape Memory Microactuators. Springer, 2002.

SMA: structural applications

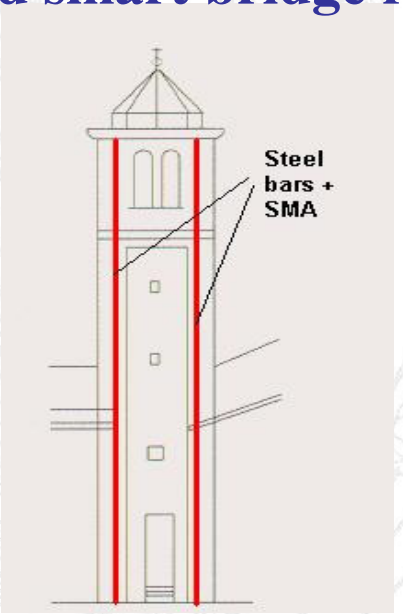


(1)Innovative connections (left) and smart bridge restrainers (right)





(2)Bracing system with SMA wires



(3)Retrofit with SMA rods

(1) DesRoches et al., 2003
(2) Dolce et al., 2000
(3) Indirli, 2000

SMA: structural applications

- **IENI wires (Lecco, ITALY)**

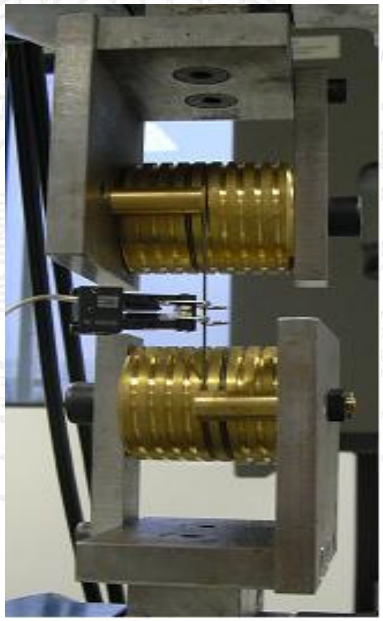
  Superelastic wires, 1.00 mm diameter

- **IENI bars (Lecco, ITALY)**

  Superelastic bars, 8.00 mm diameter

- **MEMRY wires (Menlo Park, USA)**

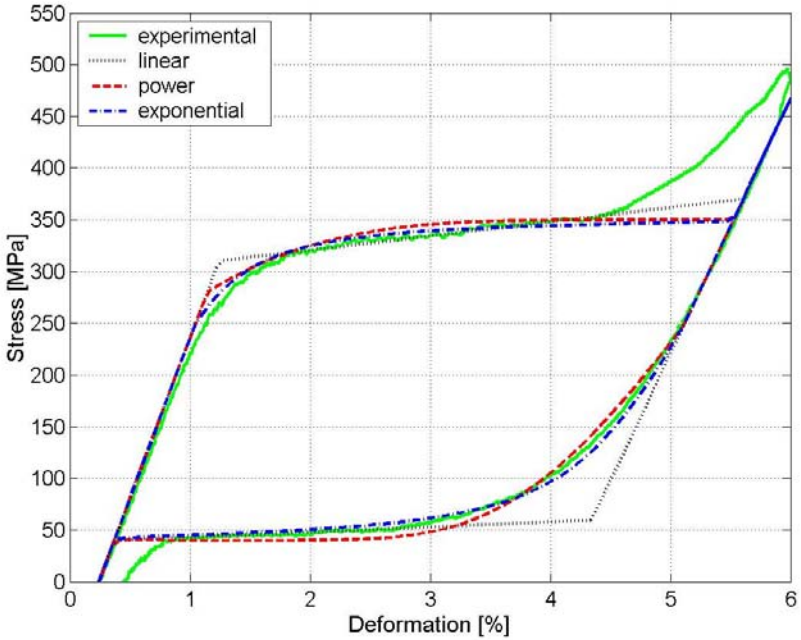
  Superelastic wires, 0.75 mm diameter



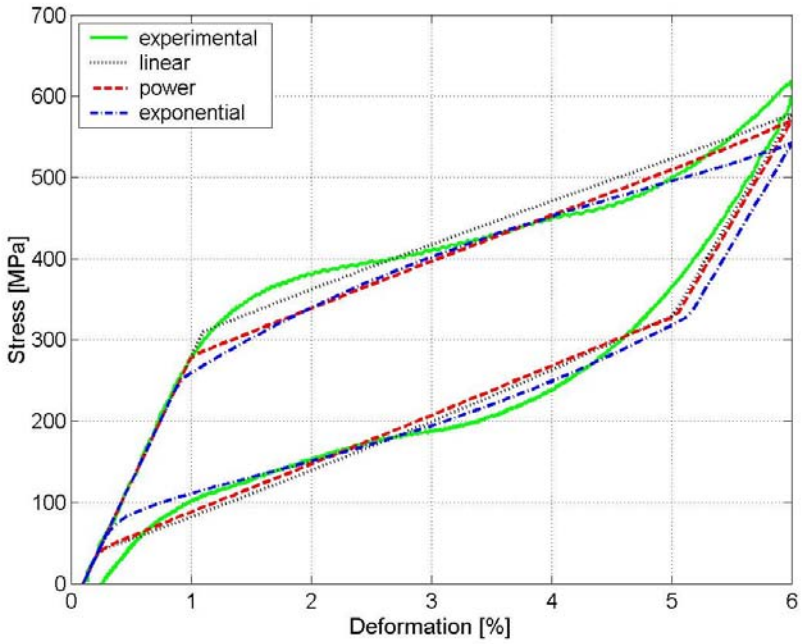
Different testing frequecies

0.001, 0.1 and 1 Hz

Comparison with experiments (IENI wires, Italy)



From static tests….



…. to dynamic tests

VISCOUS Model

A collaboration with another Italian company: AGOM

AGOM® METAL RUBBER ENGINEERING

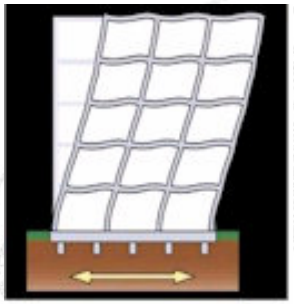
Ideas, engineering and manufacture

EXISTING SEISMIC ISOLATION DEVICES: LEAD RUBBER BEARINGS

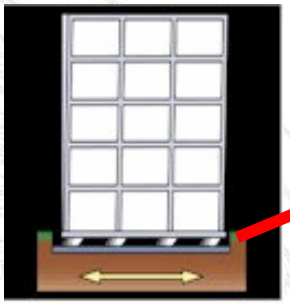
SMA Isolation Bearing

Isolation Effectiveness:

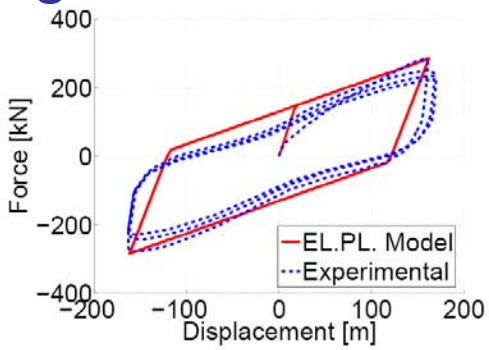
Fixed-base structure lateral displacement:



Isolated structure lateral displacement:

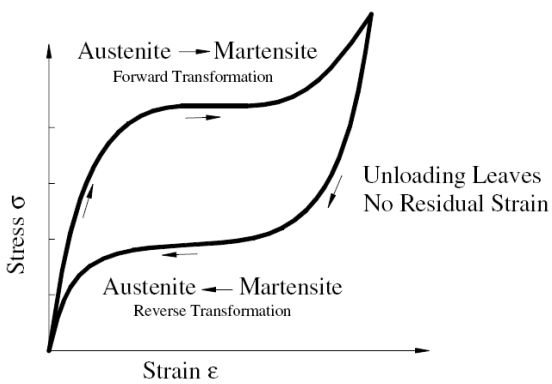


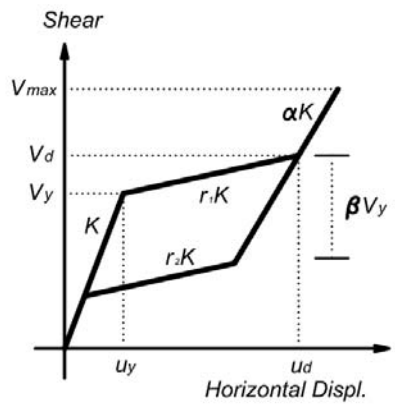
Lead Rubber Bearing:



LRB 500	
diameter	**500 mm**
effective horizontal stiffness	**1.62 kN/mm**
seismic comb. vertical load	**1653 kN**
seismic design displacement	**162 mm**
hysteretic damping ratio	**28%**

Superelastic *Equivalent* Isolator:

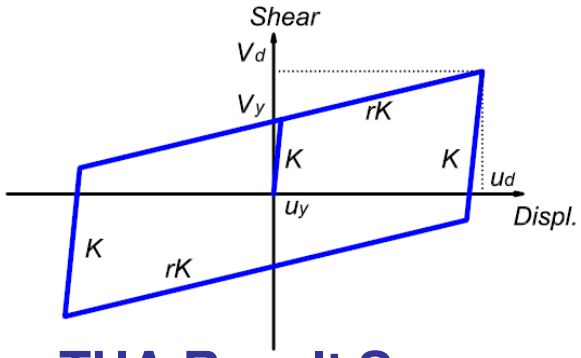


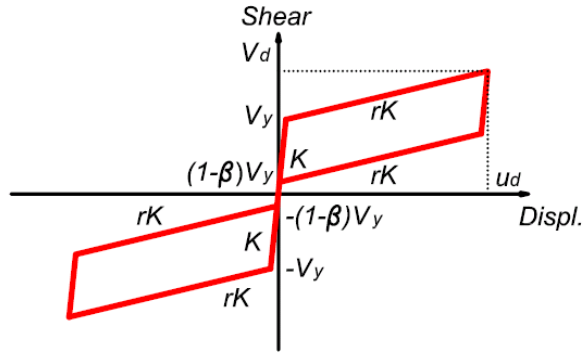


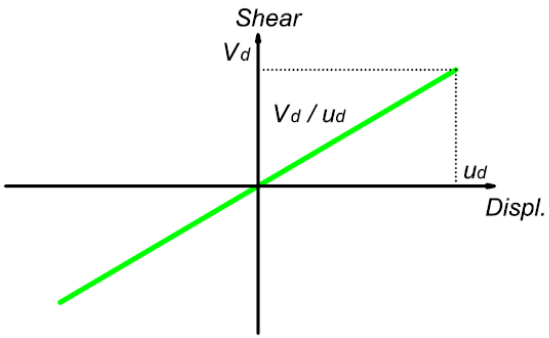
SMA eq. LRB500		
yielding shear	V_y	**147 kN**
design shear	V_d	**262 kN**
yielding displacement	u_y	**17.5 mm**
design displacement	u_d	**162 mm**
initial stiffness	k	**8.4 kN/mm**
second stiffness	rk	**0.8 kN/mm**

SMA Isolation Bearing: Time History Analysis Response

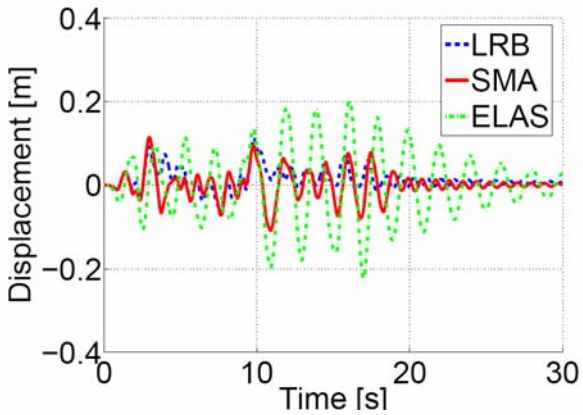
Force-Displacement Relations:

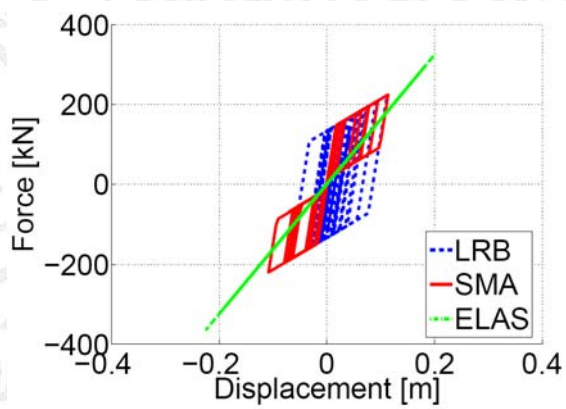


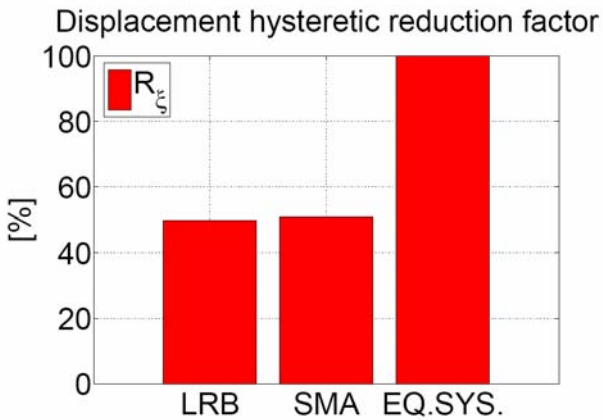


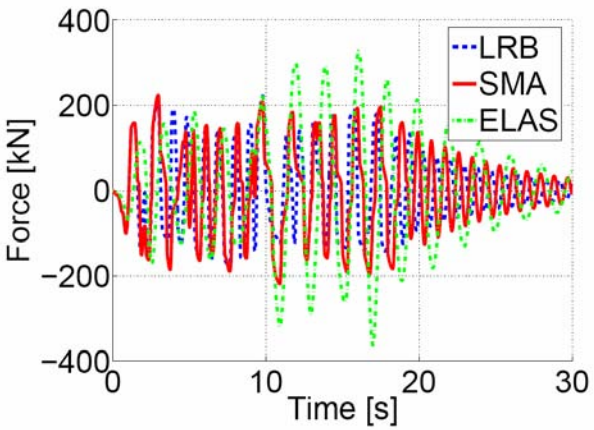


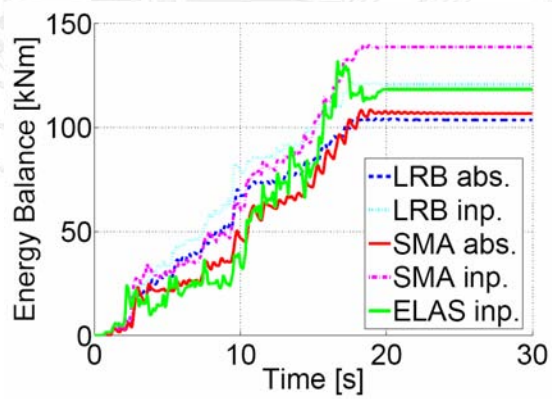
THA Result Summary:

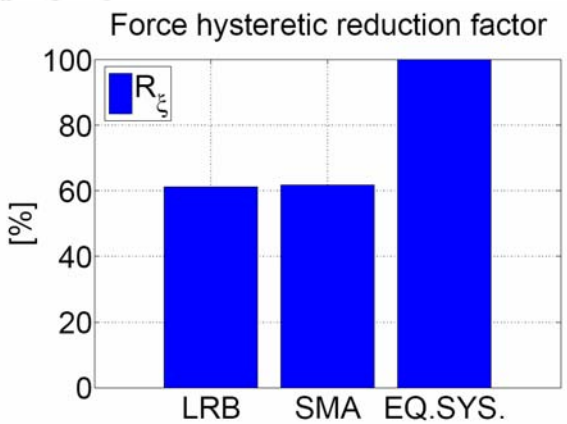










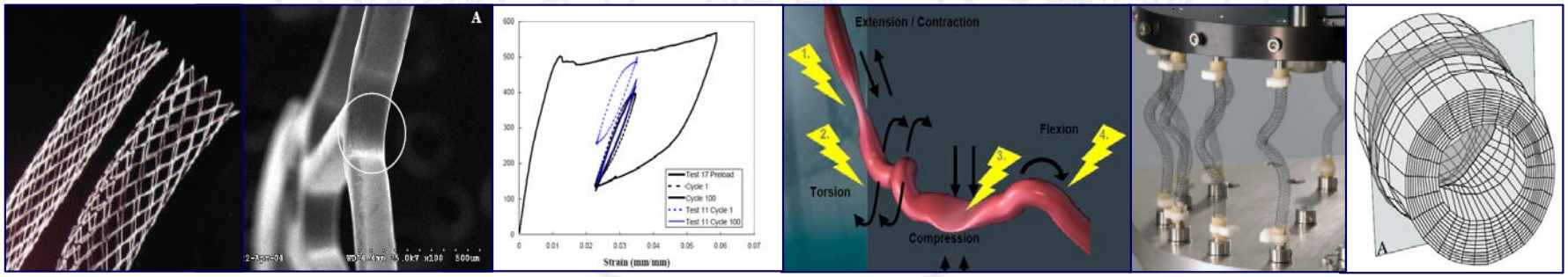


Cost Action

Reducing nitinol stent fracture in highly deformable arteries

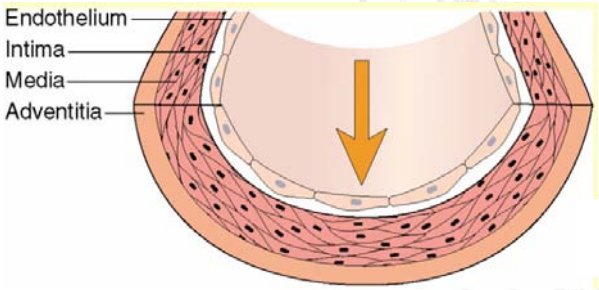
SMArtcare

Promoter: Prof. Ferdinando Auricchio



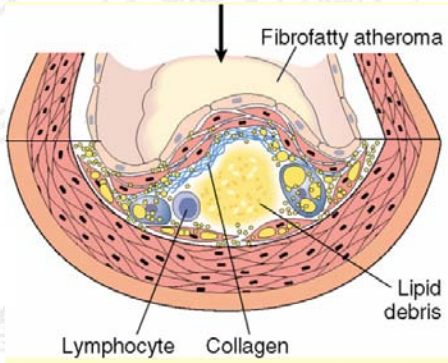
Cardiovascular disease: a real emergency

- **Atherosclerosis: the silent killer**
- **Cardiovascular disease (CVD): 49% of all deaths in 2005 in Europe**
- **Overall CVD is estimated to cost the EU economy €169 billion a year !!!**
- **Coronaries: well-known example of stenosis situ**



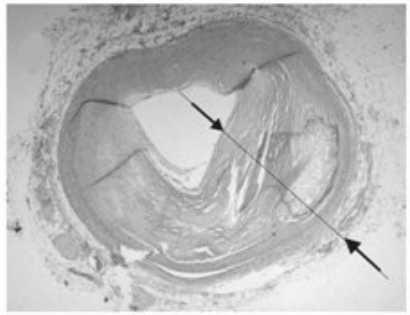
Healthy vessel

Chronic endothelial injury

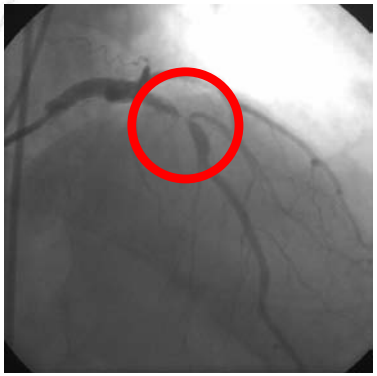


Atherosclerotic vessel

Stenosis

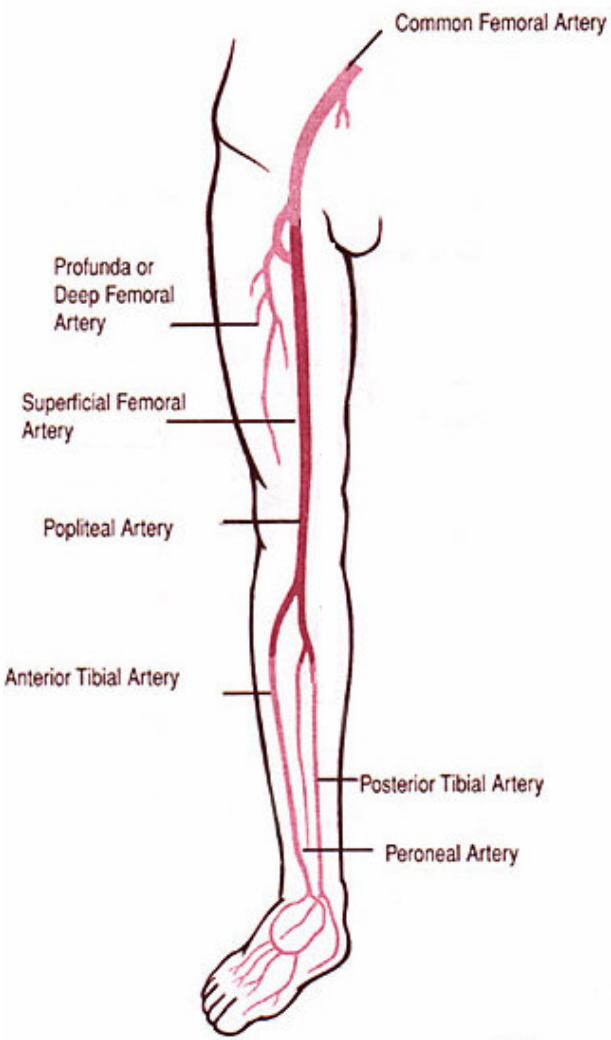


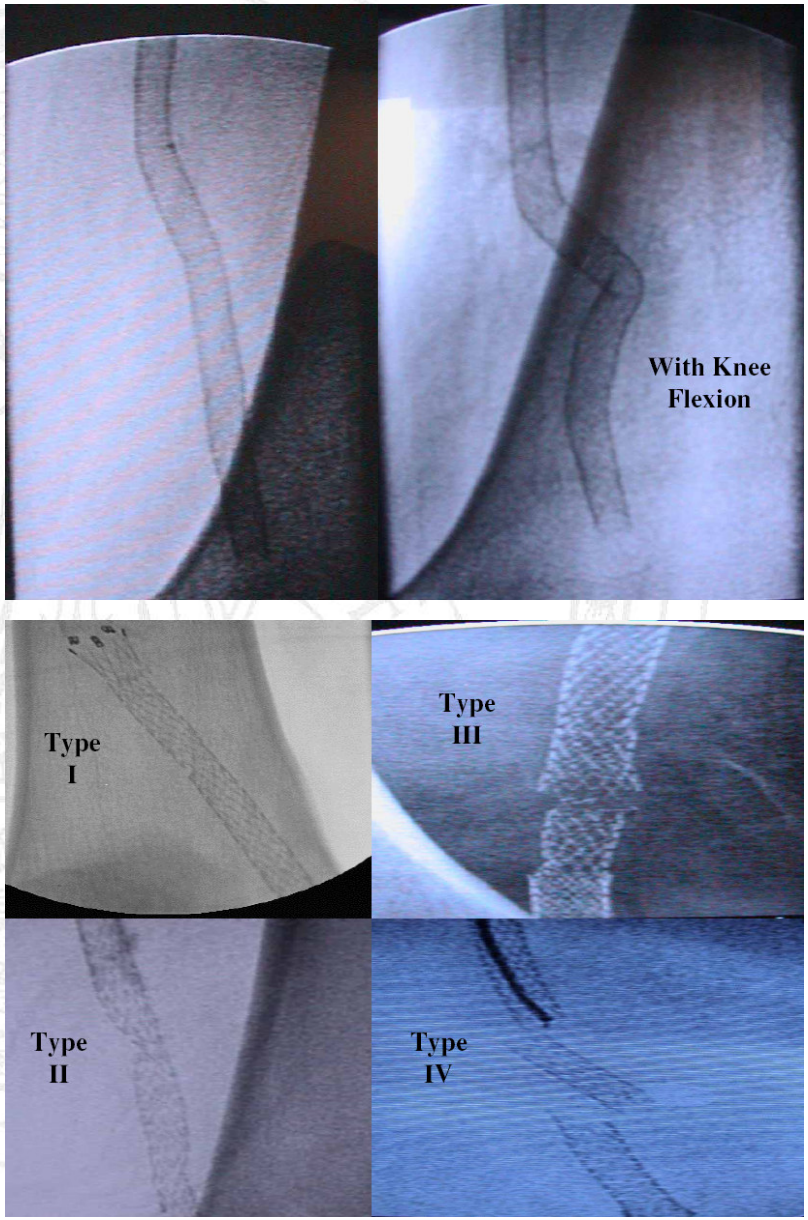
Blood flow blockage



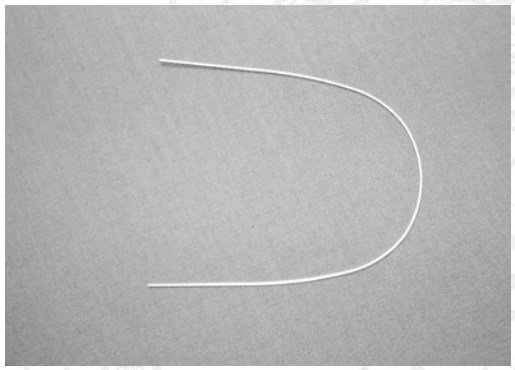
Cardiovascular disease: stent … peripheric procedures

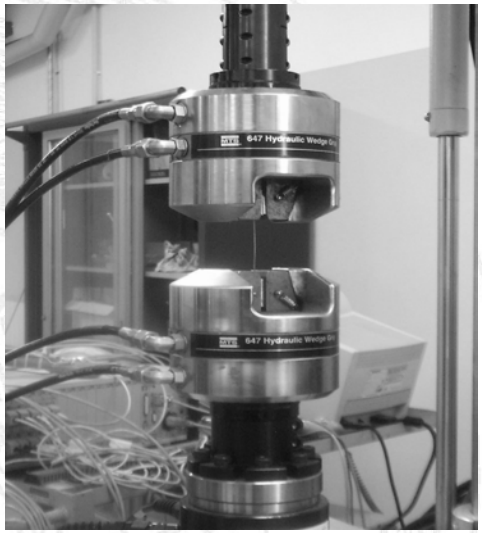
Focus on Superficial Femoral Artery (SFA)

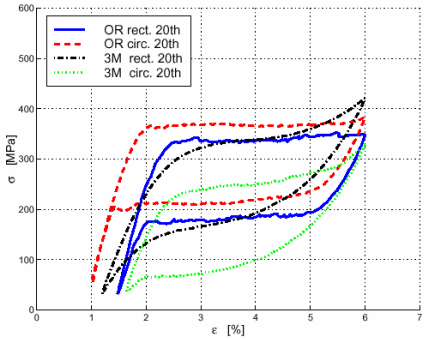


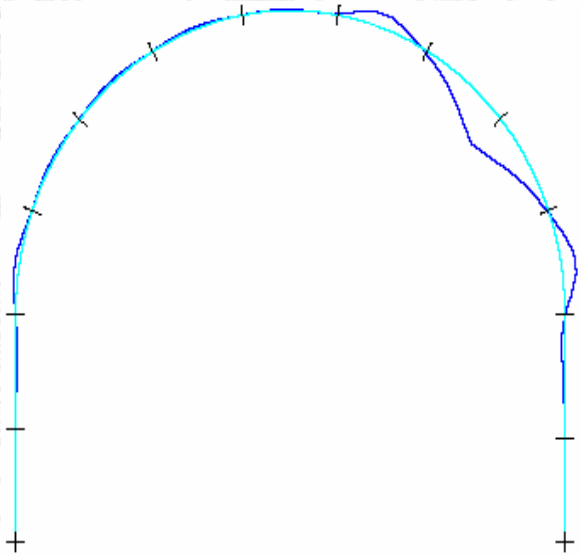


SMA for orthodontics



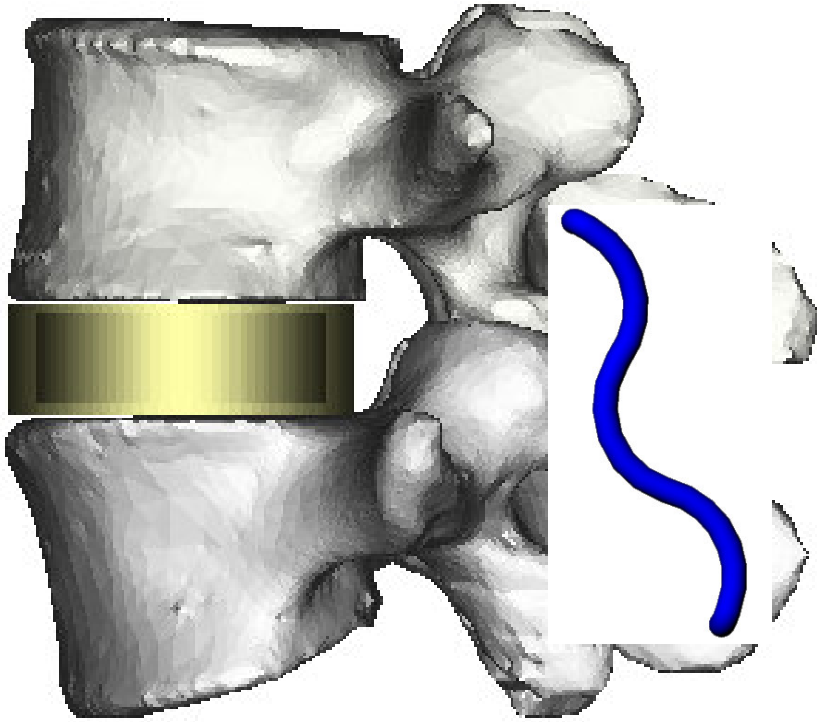




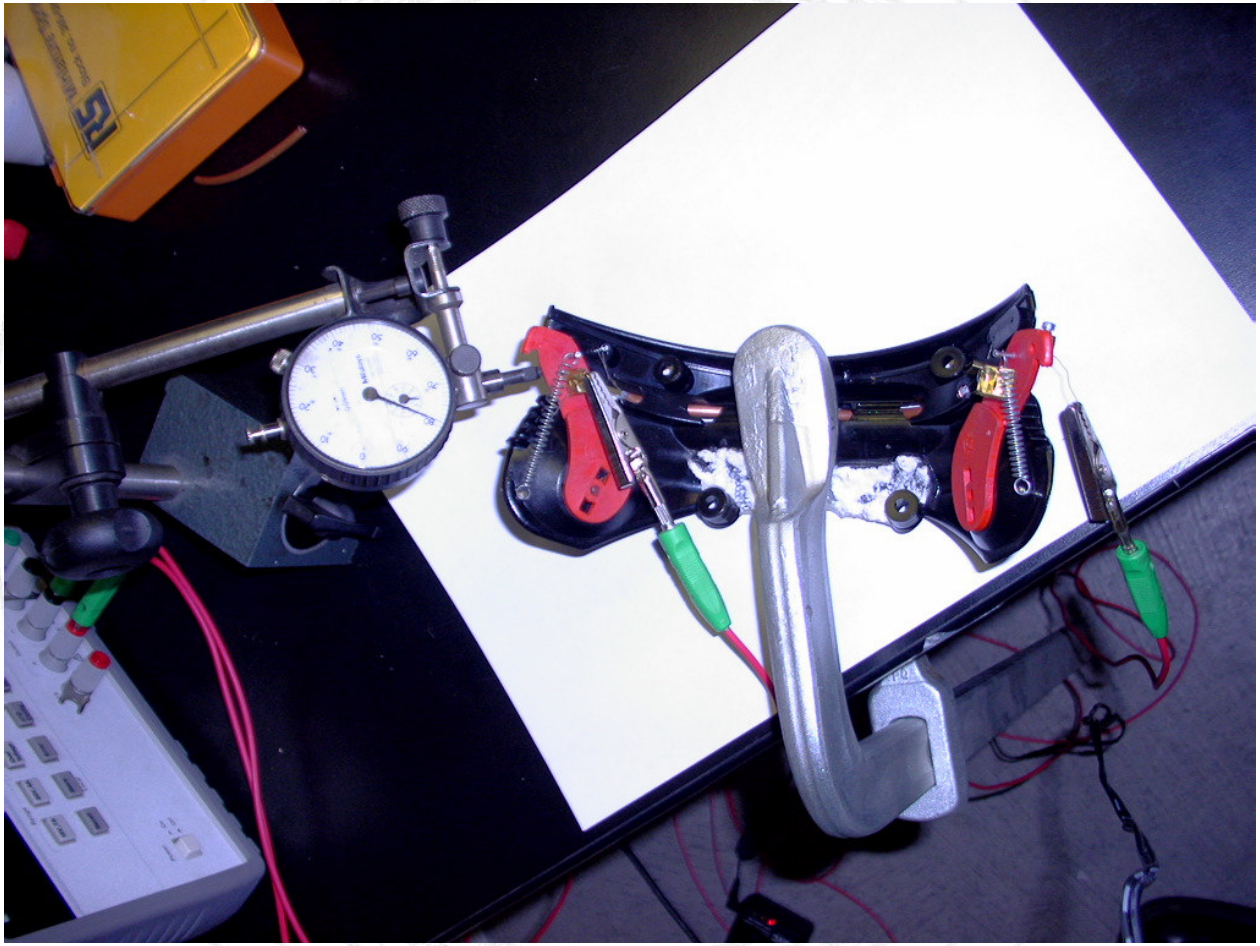


In collaboration with Politecnico di Milano

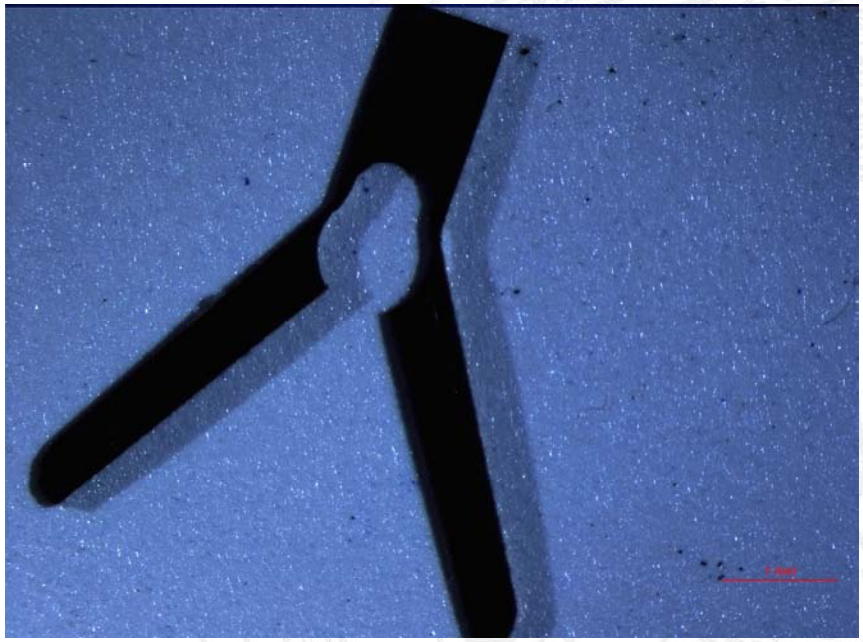
SMA for orthopedics

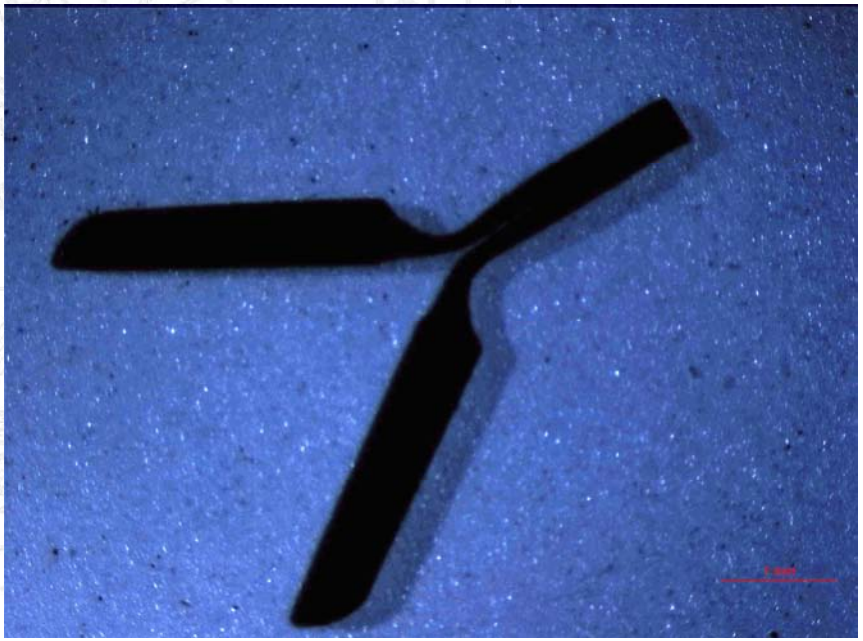


SMA for race helmets

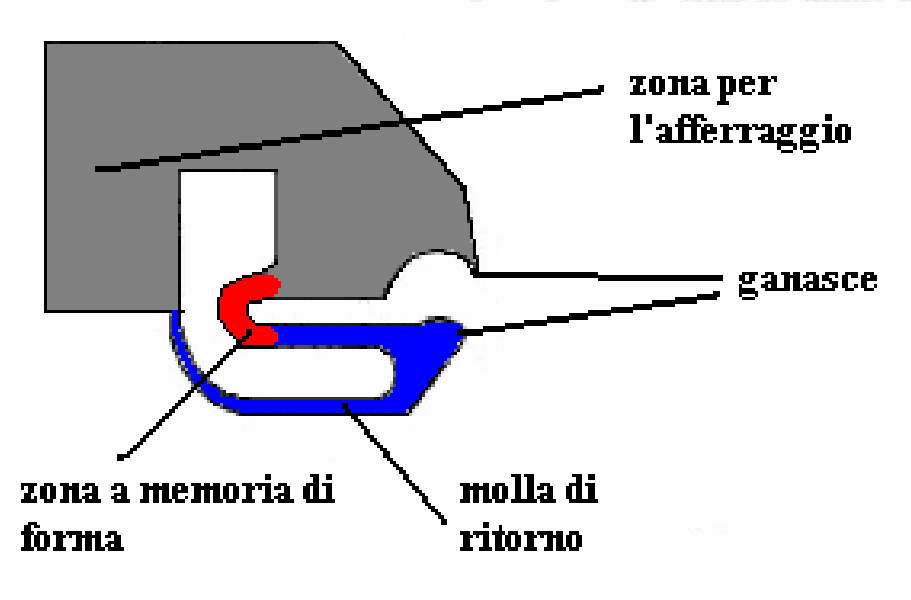


SMA laser cut & biomedical devices

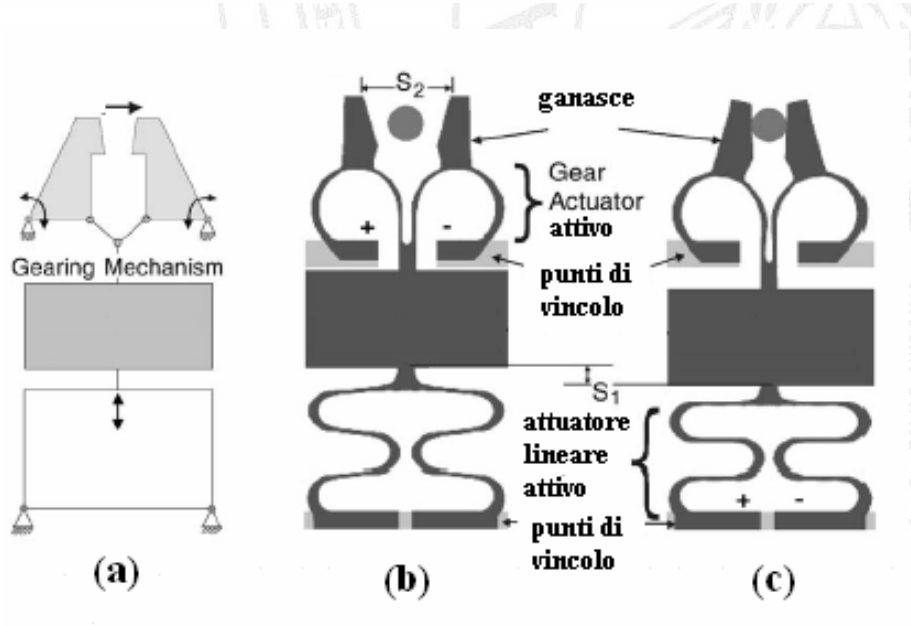




SMA laser cut & biomedical devices

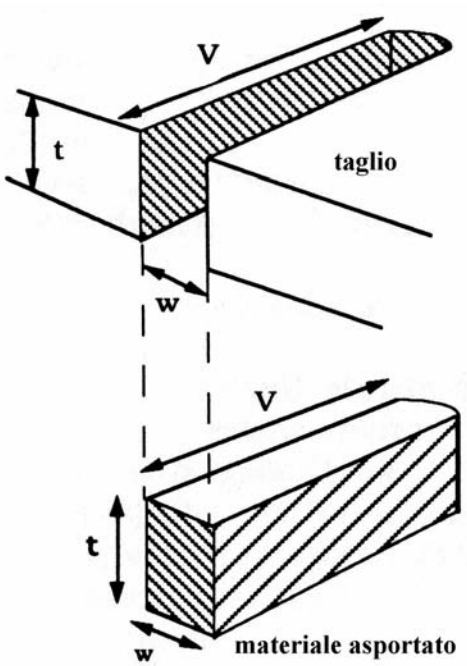


gripper e le zone attive e passive
(Bellouard et al., 1999)



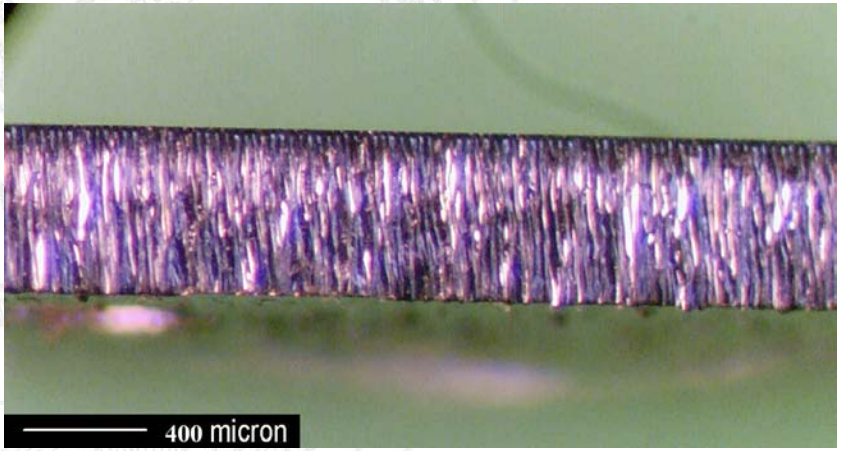
(a) schema del meccanismo
(b) gripper in posizione aperta
(c) Gripper in posizione chiusa
(Kohl et al., 2002)

SMA laser cut & biomedical devices



volume fuso e asportato durante il taglio (Steen, 1998)

Bave di materiale fuso non asportato



Striature del taglio di campione con spessore 0,5 mm